Predicting Extra-Fire-Fighting Costs in the Province of Ontario

Rob McAlpine
Aviation and Forest Fire Management,
Ontario Ministry of Natural Resources
And
Ou Feng, Ian McLeod and David Stanford
Department of Statistical and Actuarial Sciences
University of Western Ontario

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Outline

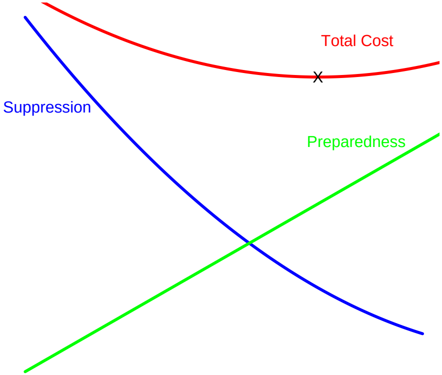
1. Overview of Forest Fire Funding Management in Ontario
2. Review of Existing MNR Method
3. Multivariate Regression Analysis
4. Out-of-Sample Comparisons
5. Conclusions and Future Work

Forest Fire Management Program Funding

Forest Fire Management Program funding comprises 4 main components:

- AFFM - Infrastructure and Management
- AFFM - Preparedness
- EFF - Preparedness
- EFF - Suppression

EFF Costs



- The EFF suppression cost is a stochastic variable which can vary widely from year to year and week to week.
- At most, a single fund request for EFF suppression can be made, and there are penalties associated with asking for too much or too little.
- At the same time, it is impossible to remove all uncertainty for the remainder of the fire season when placing a mid-season request for supplementary funds.
- Fire activity is correlated with temperatures, wind speeds, precipitation amounts and other highly volatile factors.
- Our goal is to develop the best prediction possible for the EFF suppression cost of the remain year given all cost data up to date, while realizing that substantial uncertainty will remain.

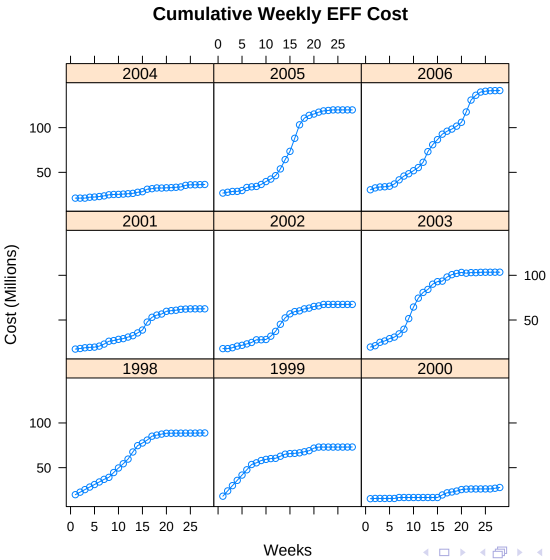
Daily EFF Cost Reporting System (DECRS)

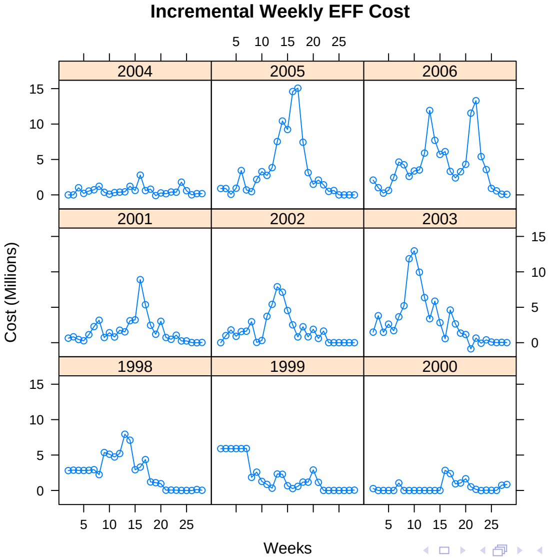
DECRS consists of 4 components as follows:

- DECRS/DFOSS [1] Personal Cost Estimate
- Aircraft Information Management System
- Service and Supplies Estimate Factor
- Base-camp and Equipment Commitment

Pull all together to compile Daily EFF Cost Estimate Report

[1] Daily Fire Operations Support System


Cumulative Weekly EFF Cost
2004
2005
2006
2001
2002
2003
1998
1999
2000
Cost (Millions)
100
50
Weeks
0 5 10 15 20 25

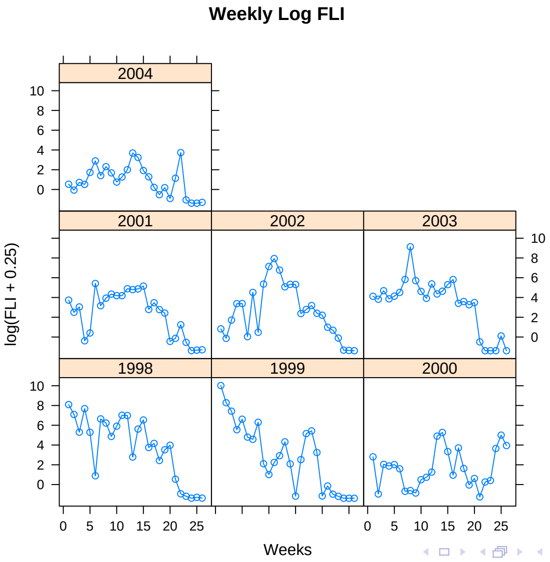

Incremental Weekly EFF Cost
2004
2005
2006
2001
2002
2003
1998
1999
2000
Cost (Millions)
Weeks
0
5
10
15
5
10
15
20
25

Fire Load Index

Fire load "The number and magnitude (i.e., fire size class and frontal fire intensity) of all fires requiring suppression action during a given period within a specified area" (Merrill, 1987).

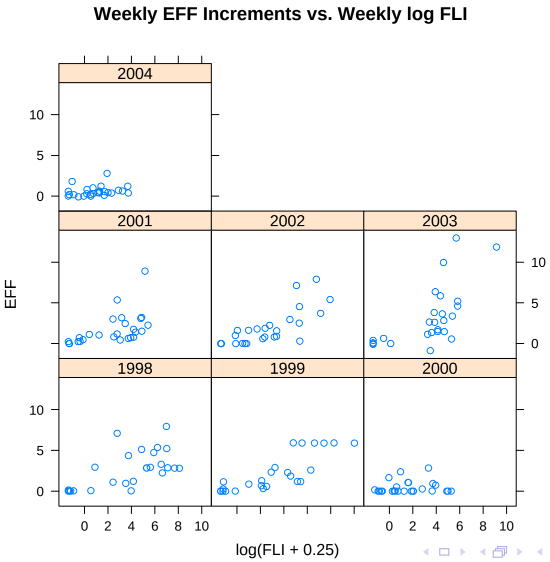
Fire load index (FLI) summed for a period of time and/or across a geographic area to provide an aggregate measure of fire load. The index is a measure of work in watts.

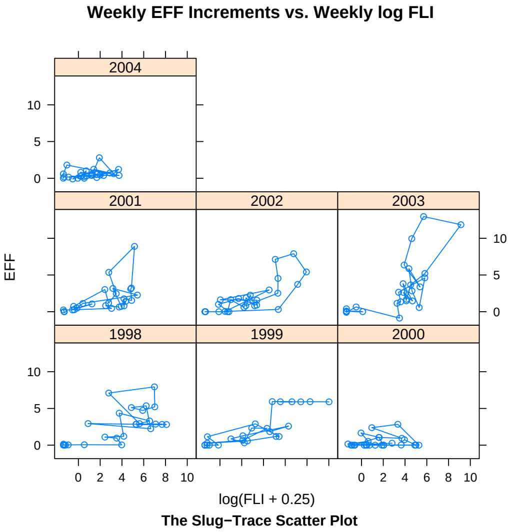
Fire Load Index



EFF vs Fire Load Index

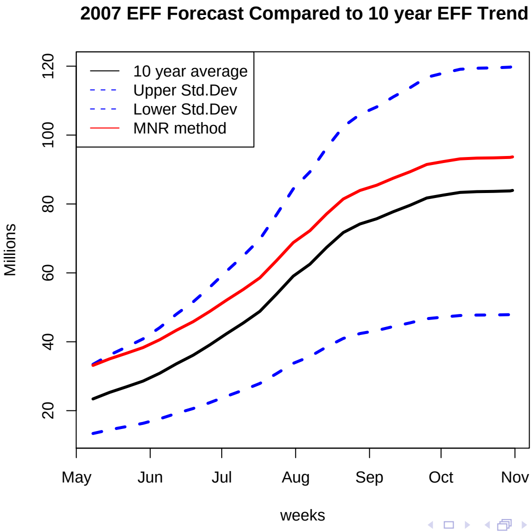
- The relationship between the fire load index and the EFF costs will be examined.
- It is expected that they will be highly correlated.
- The fire load index is useful in helping to quantify the intrinsic variability that is associated with the EFF costs.


Weekly EFF Increments vs. Weekly log FLI
2004
2001
2002
2003
1998
1999
2000
EFF
log(FLI + 0.25)
0
5
10
0
2
4
6
8
10


Weekly EFF Increments vs. Weekly log FLI
2004
2001
2002
2003
1998
1999
2000
EFF
0
5
10
0
2
4
6
8
10
log(FLI + 0.25)
The Slug−Trace Scatter Plot

10-Year Average Approach

- The average of last 10-year final EFF spending is calculated and allocated through the entire fire season according to the proportion of each week that was accounted in the total amount of 10-year spending.
- The standard deviation of last 10-year final spending is also calculated in order to produce a upper and a lower boundaries for the average. The boundaries are distributed weekly the same way as the average.
- **Example** If on July 1st of a given year, the amount of spending to date is 7 million higher than the 10-year average, then the forecast for the remainder of the season is obtained by displacing the 10-year average curve 7 million upwards.


2007 EFF Forecast Compared to 10 year EFF Trend
10 year average
Upper Std.Dev
Lower Std.Dev
MNR method
Millions
20
40
60
80
100
120
May
Jun
Jul
Aug
Sep
Oct
Nov
weeks

Notations

- Z_t^T the cumulative cost for year T and week t; $T = 1, 2, \ldots, 9, \ldots$ and $t = 1, \ldots, 28$.
- CPI^T Consumer Price Index for year T; $T = 1, 2, \ldots, 9, \ldots$
- $Y_{t,l}^T = Z_{t+l}^T - Z_t^T$ is the incremental cost l weeks later than week t, where l is the remaining lead time $l = 1, \ldots, 28 - t$.
- Alternative $R_{t,l}^T = \log Z_{t+l}^T - \log Z_t^T \approx \frac{Z_{t+l}^T - Z_t^T}{Z_t^T}$
- Given the data completely for years $T = 1, 2, \ldots, T_0$ and weeks $t = 1, \ldots, 28$. We need projections for year $T_0 + 1 = \tau$. Assume that for year τ we have data for weeks t=1,...,n where $n < 28$, then we need to forecast Z_{n+l}^τ for the remaining lead time.

Multivariate Regression

- For each l, we run a linear regression $Y_{n,l}^{T} \sim Z_n^{T} + CPI^{T}$, where $T = 1, 2, \ldots, T_0$; $l = 1, \ldots, 28 - n$.
- $$\hat{Y}_{n,l}^{\tau} = \alpha_0^{(l)} + \alpha_1^{(l)} Z_n^{\tau} + \alpha_2^{(l)} CPI^{T_0} \qquad (1)$$

  where $\alpha_0^{(l)}$,$\alpha_1^{(l)}$ and $\alpha_2^{(l)}$ are the coefficients from the regression and vary with each l.
- Then $\hat{Z}_{n+l}^{\tau} = \hat{Y}_{n,l}^{\tau} + Z_n^{\tau}$.

Alternative Method

- Naive and Minimum Mean Square Error (MMSE) log transformation for the explanatory variables.
- The new respond variable will be changed to $R^T_{t,l} = \log Z^T_{t+l} - \log Z^T_t \approx \frac{Z^T_{t+l} - Z^T_t}{Z^T_t}$ as shown before.
- The new regression $R^T_{n,l} \sim \log Z^T_n + \log CPI^T$.
-

$$\hat{R}^\tau_{n,l} = \beta^{(l)}_0 + \beta^{(l)}_1 \log Z^\tau_n + \beta^{(l)}_2 \log CPI^{T_0} \quad (2)$$

where $l = 1, \ldots, 28 - n$; $\beta^{(l)}_0$,$\beta^{(l)}_1$ and $\beta^{(l)}_2$ are the coefficients from the new regression and vary with each l.

Alternative Method

- For Naive, $\hat{Z}^{\tau}_{n+l} = \exp(\hat{R}^{\tau}_{n,l}) \cdot Z^{\tau}_{n}$.
- For MMSE, $\hat{Z}^{\tau}_{n+l} = \exp\left(\hat{R}^{\tau}_{n,l} + (\sigma^2)^{(l)}/2\right) \cdot Z^{\tau}_{n}$, where $(\sigma^2)^{(l)}$ is the residual variance depending on l.

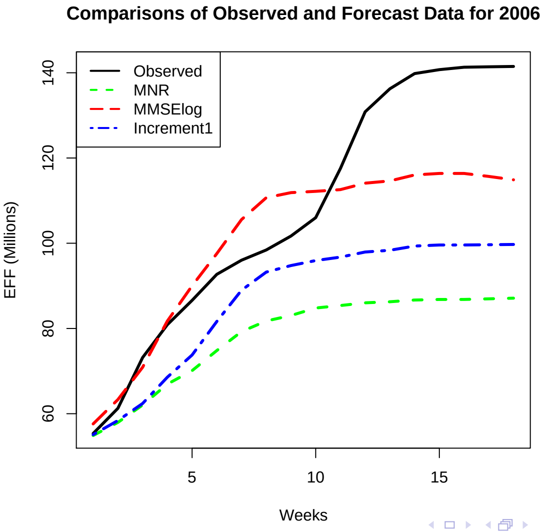
Sample Data

Below is the bottom portion of the dataset we created from the past data and we will predict the rest of EFF spending for year 2006.

	1998	1999	2000	2001	2002	2003	2004	2005	2006
Week 23	88.64	73.02	26.17	61.99	67.43	102.94	33.68	119.24	NA
Week 24	88.65	73.03	26.21	62.23	67.43	103.34	35.48	119.86	NA
Week 25	88.66	73.03	26.23	62.48	67.43	103.43	36.07	119.86	NA
Week 26	88.66	73.03	26.24	62.50	67.43	103.44	36.08	119.86	NA
Week 27	88.80	73.05	26.98	62.49	67.43	103.49	36.26	119.86	NA
Week 28	88.83	73.10	27.83	62.52	67.43	103.50	36.44	119.86	NA

Observed and Forecast Data

	Observed	MNR	Untransformed	NaiveLog	MMSElog
Week 11	55.36	54.89	57.82	57.53	57.61
Week 12	61.26	57.96	62.63	62.93	63.31
Week 13	73.18	62.05	67.71	69.77	70.90
⋮	⋮	⋮	⋮	⋮	⋮
Week 23	136.24	86.27	105.86	108.58	114.62
Week 24	139.81	86.68	106.75	110.37	116.03
Week 25	140.73	86.81	106.93	110.84	116.38
Week 26	141.30	86.81	106.94	110.84	116.38
Week 27	141.40	86.96	106.85	110.29	115.68
Week 28	141.48	87.11	106.73	109.64	114.89

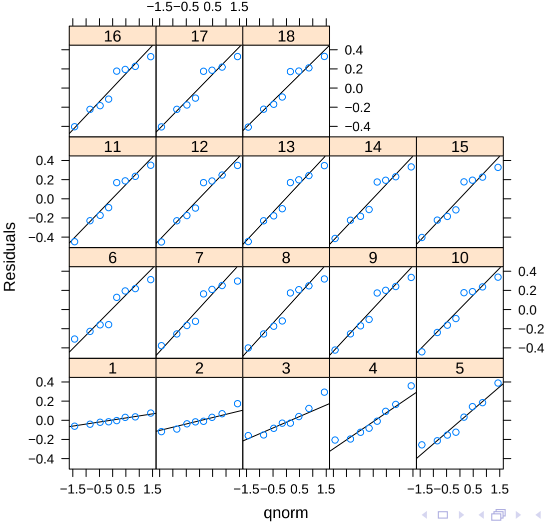

Comparisons of Observed and Forecast Data for 2006
Observed
MNR
MMSElog
Increment1
EFF (Millions)
60
80
100
120
140
5
10
15
Weeks

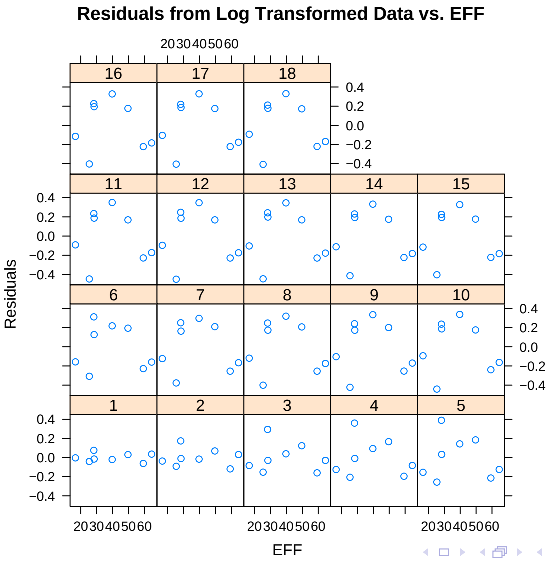
Comparisons under Three Criteria

	MNR	Untransformed	Naive	MMSE
MAPE [2]	0.24	0.12	0.10	0.10
Accuracy[3]	0.76	0.88	0.90	0.90
RMSE[4]	35.30	20.66	18.53	15.55
MAE [5]	29.65	15.00	13.35	12.36

[2] Mean Absolute Percentage Error
[3] =Max(1-MAPE, 0)
[4] Root Mean Square Error
[5] Mean Absolute Error

Normal QQ Plot for Residuals from Logged Data




Residuals from Log Transformed Data vs. EFF
16
17
18
11
12
13
14
15
6
7
8
9
10
1
2
3
4
5
0.4
0.2
0.0
−0.2
−0.4
20 30 40 50 60
Residuals
EFF

Durbin-Watson and Jarque-Bera Tests

Durbin-Watson Test Used to as a dynamic check for the presence of autocorrelation in the residuals.

Jarque-Bera Test Applied to the residuals as a goodness-of-fit measure of departure from normality assumption.

	pDW [6]	pJB [7]	pF.test	$R^2(\%)$
1	0.91	0.79	0.15	53.10
2	0.83	0.80	0.24	43.50
3	0.99	0.82	0.36	33.70
4	0.99	0.76	0.37	33.00
⋮	⋮	⋮	⋮	⋮
14	0.77	0.85	0.41	30.30
15	0.76	0.85	0.40	30.70
16	0.76	0.85	0.40	30.70
17	0.77	0.85	0.40	30.40
18	0.78	0.86	0.41	30.10

Table: P values and R^2 Calculated Using the Untransformed Data

[6] the Durbin-Watson Test
[7] the Jarque-Bera Test

Conclusions

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- New forecasts seem to improve upon existing method
- Fundamentally, little predictive power beyond one week
- Various years appear to follow differing regimes (i.e. quiet, typical and extreme years)

Future Work

- Scenario Generation or Bootstrapping
- STL/ARMA Decomposition
- Credibility Theory $P_c = Z\overline{X} + (1 - Z)M$, where $Z = \frac{n}{n+k}$