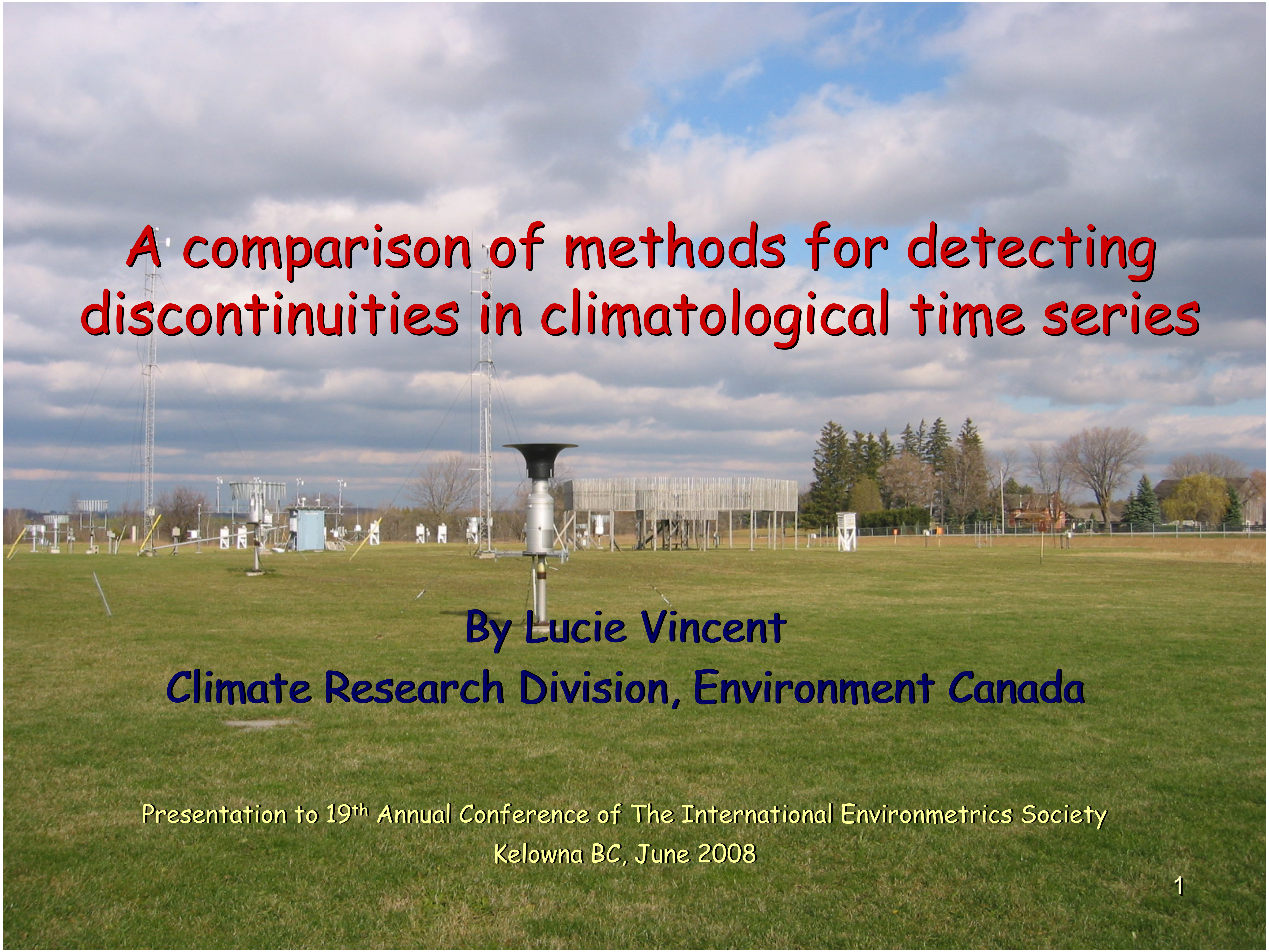

A comparison of methods for detecting discontinuities in climatological time series

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Objectives

- To improve our knowledge about the different issues and difficulties related to the detection of discontinuities in climatological time series
- To compare the ability of different methods to identify
  - Homogenous series (series with no steps)
  - Series with a single step
  - Series with a random number of steps

- Methods based on statistical test to facilitate comparison
- Many others methods presented in scientific literature
- Ducré-Robitaille, Vincent and Boulet, 2003: Comparison of techniques for detection of discontinuities in temperature series Int. J. Climatol. 23, 1087-1101

Four methods for detecting discontinuities in climatological time series

1. Standard Normal Homogeneity Test (SNHT)
   Alexandersson 1986: Journal of Climatology
   Dep. of Meteorology, Sweden

2. Two-phase regression (TPR)
   Easterling and Peterson 1995: International Journal of Climatology
   National Data Climate Center, US

3. Multiple Linear Regression (MLR)
   Vincent 1998: Journal of Climate
   Climate Research Branch, Environment Canada

4. Wilcoxon Ran-Sum (WRS)
   Karl and Williams 1987: Journal of Climate & Applied Climatology
   National Data Climate Center, US

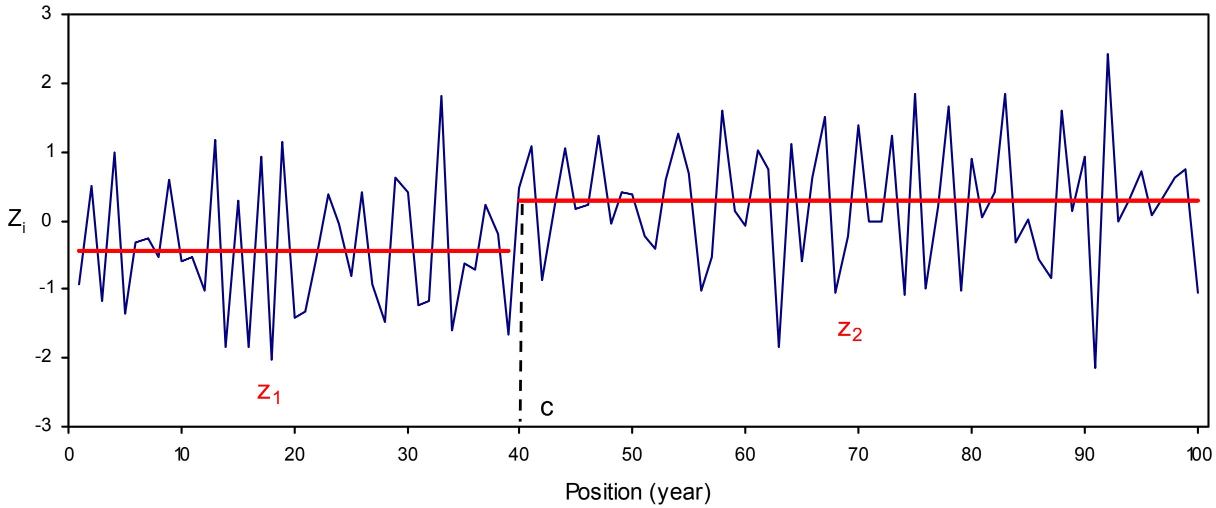
Standard Normal Homogeneity Test (SNHT)

Let X_{1i} be the annual mean temperature of the tested site
X_{2i} be the annual mean temperature of a reference series
$Q_i = X_{1i}/X_{2i}$ for $i = 1,\ldots,n$ (years)
$Z_i = (Q_i - Q)/s$ where Q is the mean and s the standard deviation

Assumption: Z_i i.i.d. ~ N(0,1)

H_0: $Z_i \sim N(0,1)$ for $1 \le i \le n$
H_1: $Z_i \sim N(\mu_1,1)$ $1 \le i < c$
$Z_i \sim N(\mu_2,1)$ $c \le i \le n$

Find c that max{T_c} for $T_c = cz_1+(n-c)z_2$ (z_1 & z_2 means bef & aft c)
If $T_c > T_{0.95}$ reject H_0



Two-phase regression (TPR)

Let X_{1i} be the annual mean temperature of the tested site
X_{2i} be the annual mean temperature of a reference series
$Y_i = X_{1i} - X_{2i}$

Assumption: Y_i i.i.d. ~ $N(\mu,\sigma)$

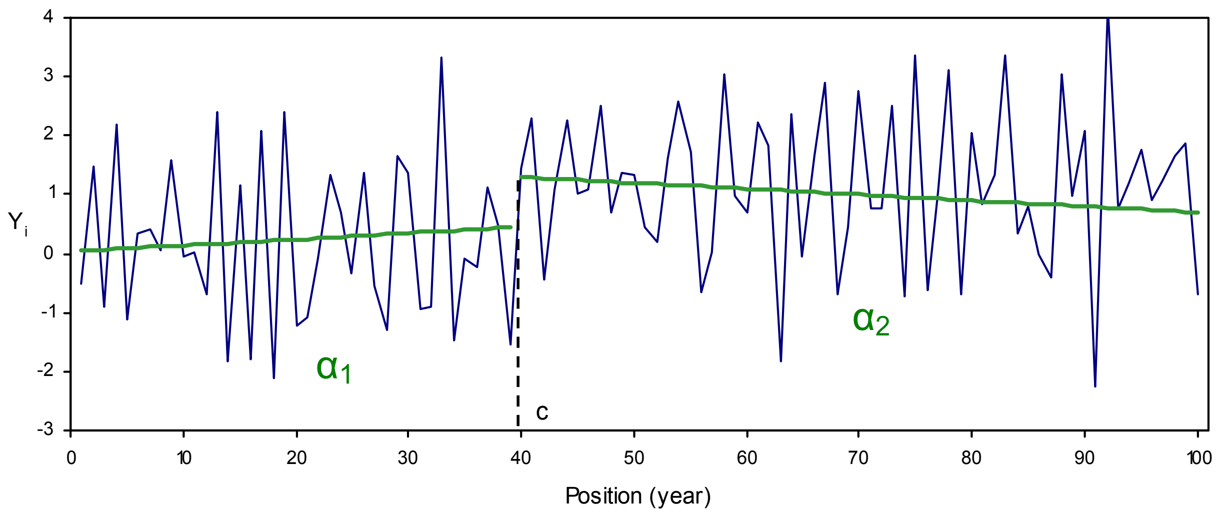
Model 1: $Y_i = \mu+\alpha i+e_i$ for $1 \le i \le n$
Model 2: $Y_i = \mu_1+\alpha_1 i+e_i$ $1 \le i < c$
$Y_i = \mu_2+\alpha_2 i+e_i$ $c \le i \le n$

Find c that min SSE_2

$F_c = [(SSE_1-SSE_2)/2] / [SSE_2/(n-4)]$

If $F_c > F_{0.95;2,n-4}$ reject H_0

Model 2 revised by Wang 2003 to keep the same trend bef & aft c



Multiple Linear Regression (MLR)

Let $Y_i = X_{1i}-X_{2i}$

Assumption: Y_i i.i.d. ~ $N(\mu,\sigma)$

Model 1: $Y_i = \mu+e_i$ for $1 \le i \le n$

Model 2: $Y_i = \mu+\alpha i+e_i$ $1 \le i \le n$

Model 3: $Y_i = \mu+\beta I_i+e_i$ $I=0$ for $1 \le i< c$

$I=1$ for $c \le i \le n$

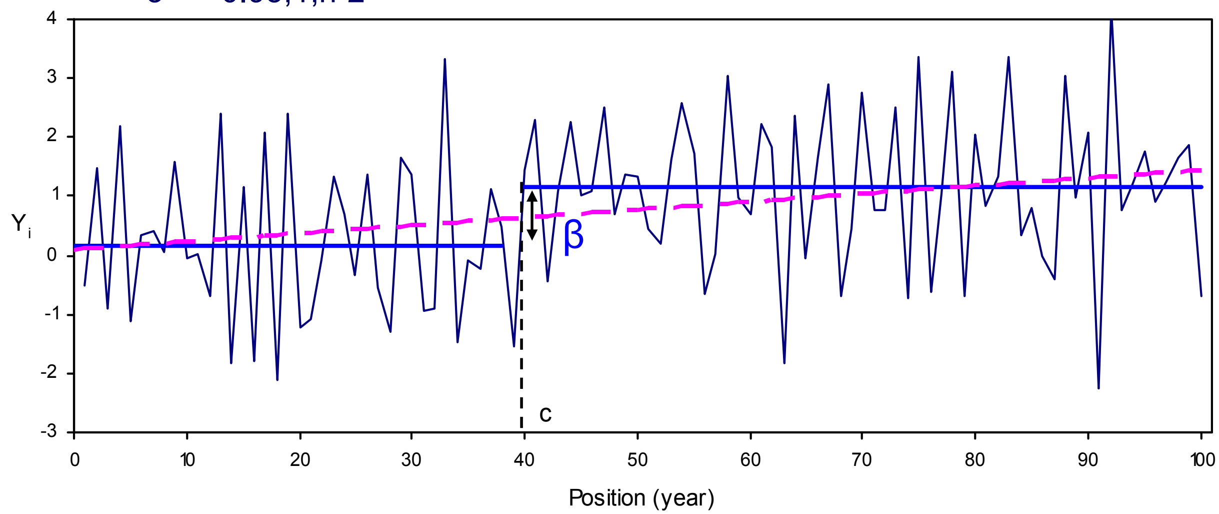
Find c that min SSE_3

$F_2 = [(SSE_1-SSE_2)/1] / [SSE_2/(n-2)]$

$F_3 = [(SSE_1-SSE_3)/1] / [SSE_3/(n-2)]$

If $F_2>F_{0.95;1,n-2}$ keep Model 2

If $F_3>F_{0.95;1,n-2}$ keep Model 3



Wilcoxon Rank-Sum (WRS)

Often the assumption of normality is debatable in climatological series

Assumption: Y_i i.i.d. ~ N(0,1) is not respected

Non-parametric test

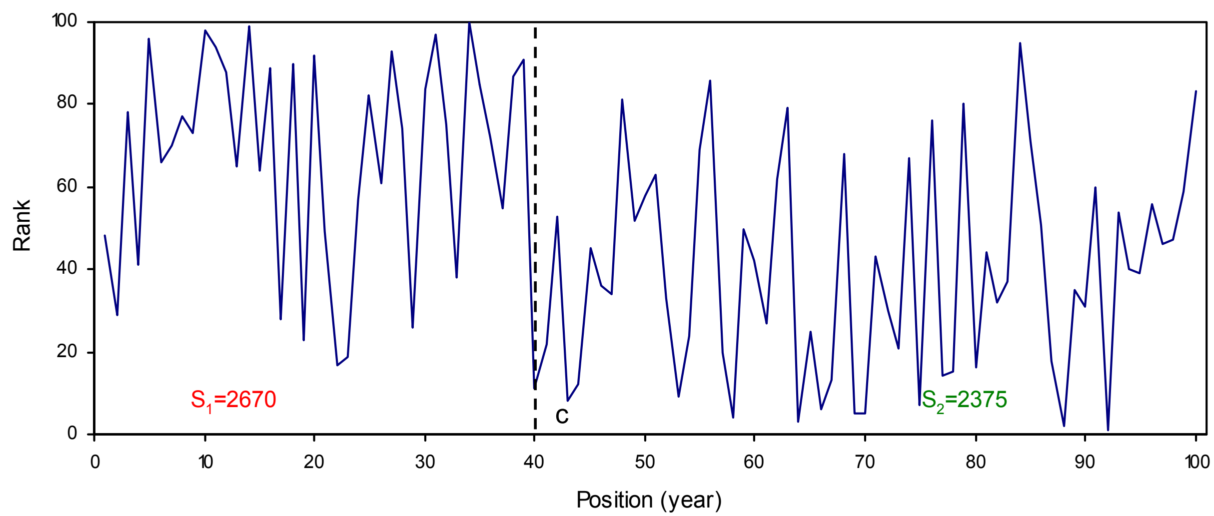
Series is divided in two groups (i=1,…,c-1 and i=c,…,n)

Each value is ranked & the sum of the ranked is obtained for each group

$S_1 = \Sigma_{i=1,..,c-1} r_i$ and $S_2 = \Sigma_{i=c,..,n} r_i$

Find c that max{W_c} where $W_c = 12[S_1 - c(n+1)/2]^2 / [c(n-c)(n+1)]$

If prob(W_c) > 0.05 keep H_0 (S_1 and S_2 are not different)



Simulation of annual mean temperature

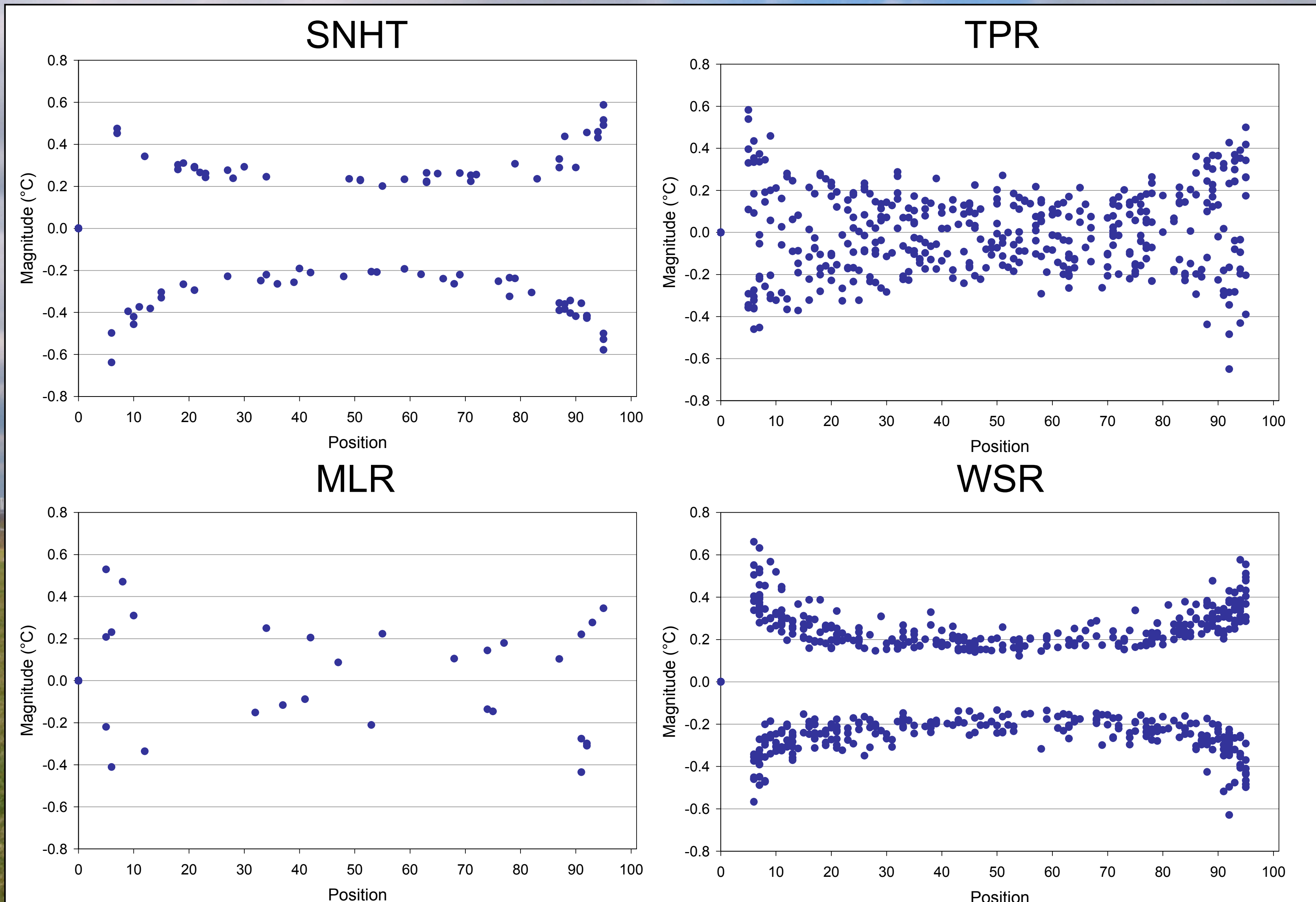
- Homogeneous Series (series with no steps)
  - Random numbers ~ N(0,1) with AR(1)=0.1
  - 1000 homogeneous series of 100 values (years)
- Series with one step
  - Step of magnitude 0.25, 0.50, 0.75, …, 2.00 σ
  - Position 5, 10, 15, 20, 35, 50
  - 48 000 series with a single step
- Series with a random number of steps
  - Step of magnitude ∂ = 0.5 to 2.0 σ; $\partial \sim N(0,1)$
  - Position $\Delta t = \exp(0.05)$, $\Delta t \geq 10$
  - 25 000 series with a random number of steps (0 to 7 steps)
- Reference series
  - Reference series cross-correlated with candidate series with correlation factor ~ 0.8 and re-standardized

Identification of homogeneous series

Magnitude (σ)	SNHT	TPR	MLR	WRS
0.0 - 0.1	0.2	12.9	0.2	6.6
0.1 - 0.2	1.8	15.0	1.0	22.0
0.2 - 0.3	4.0	8.2	1.4	16.7
0.3 - 0.4	1.4	3.9	0.6	8.1
0.4 - 0.5	0.9	1.0	0.3	1.7
0.5 - 0.6	0.3	0.2	0.1	1.0
> 0.6	0.0	0.1	0.0	0.2
Total	8.6	41.3	3.6	56.3

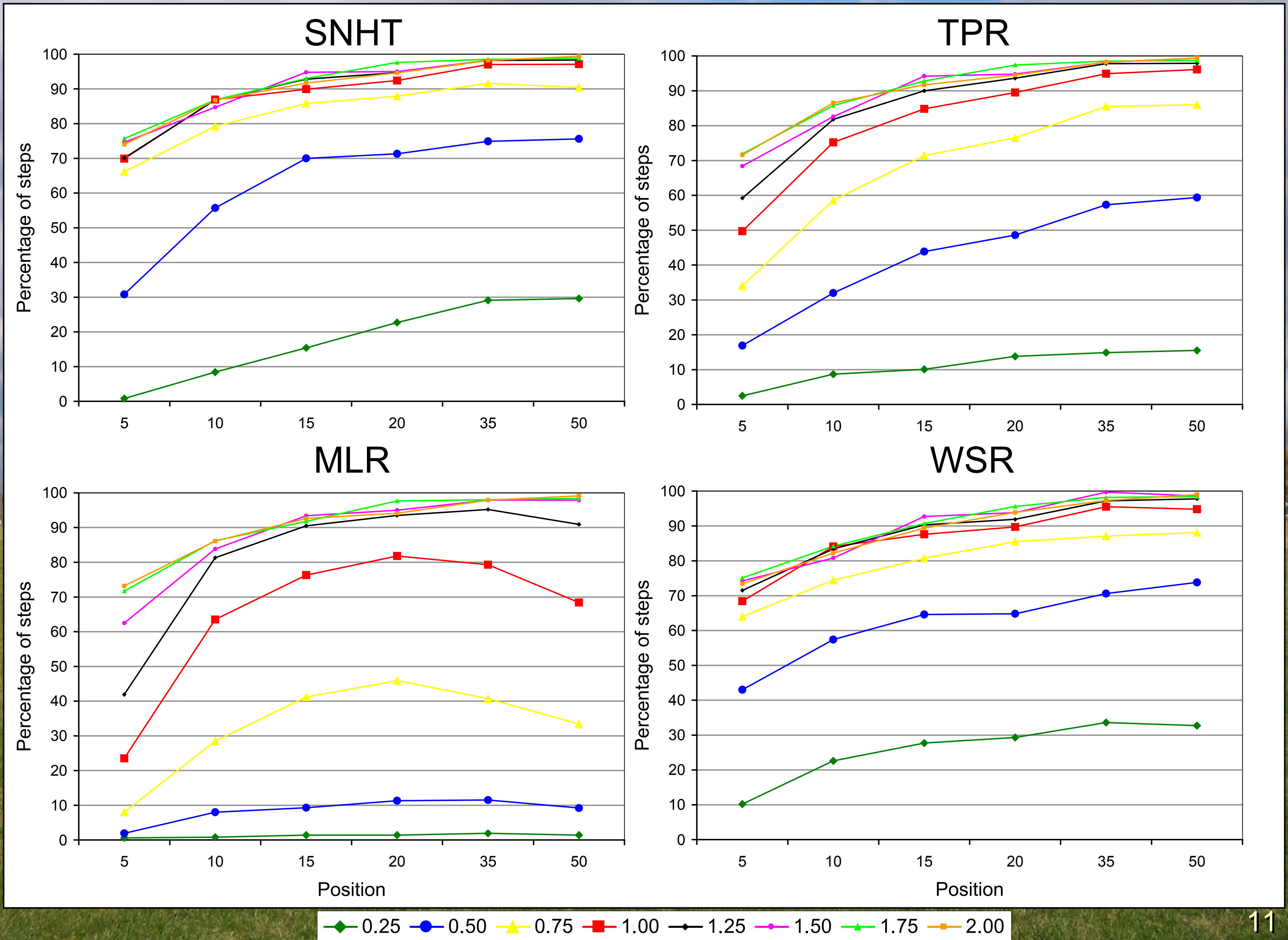
Percentage of steps falsely detected by each method
when applied to 1000 homogeneous series

Identification of homogeneous series



Position and magnitude of the steps falsely detected when applied to 1000 homogeneous series

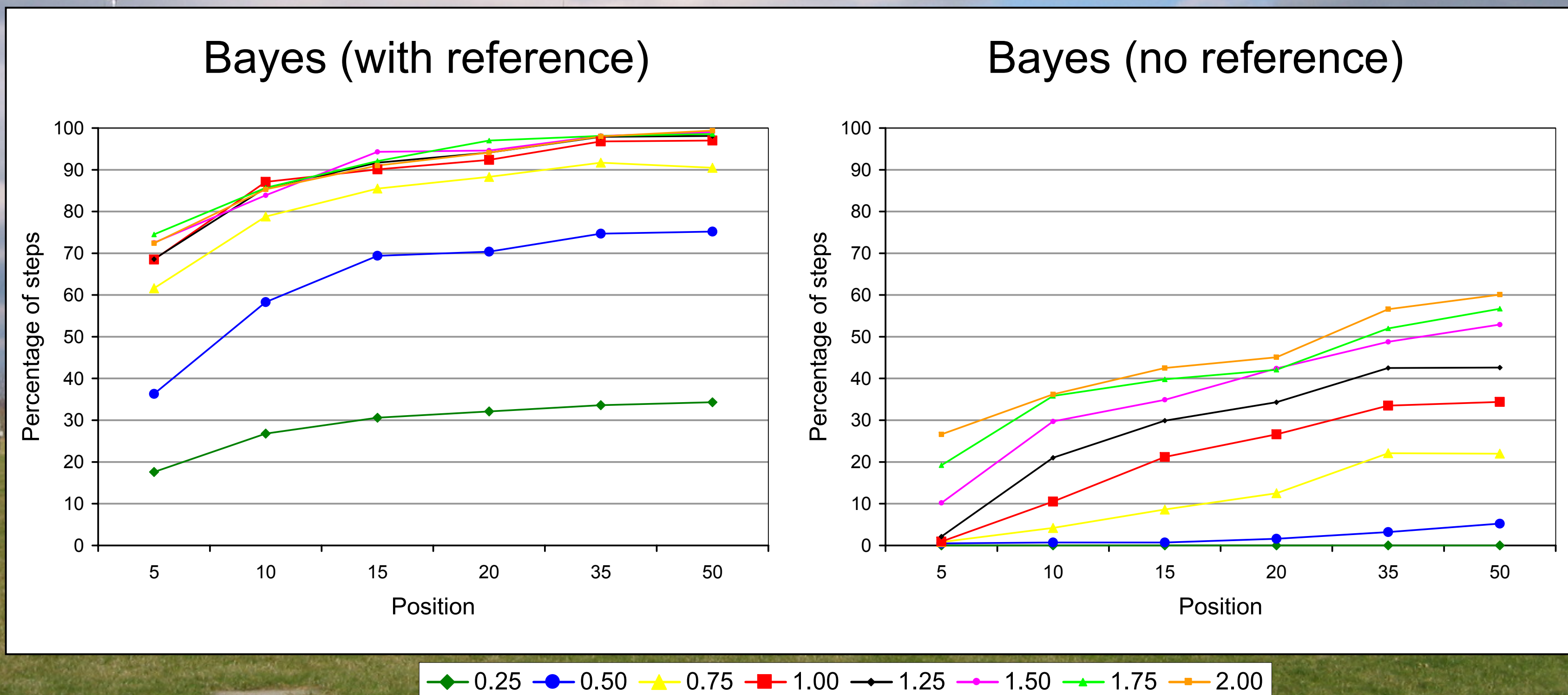
Identification of a single step



Percentage of steps identified when one step is introduced in the candidate series

Identification of a single step

(impact of using a reference series)



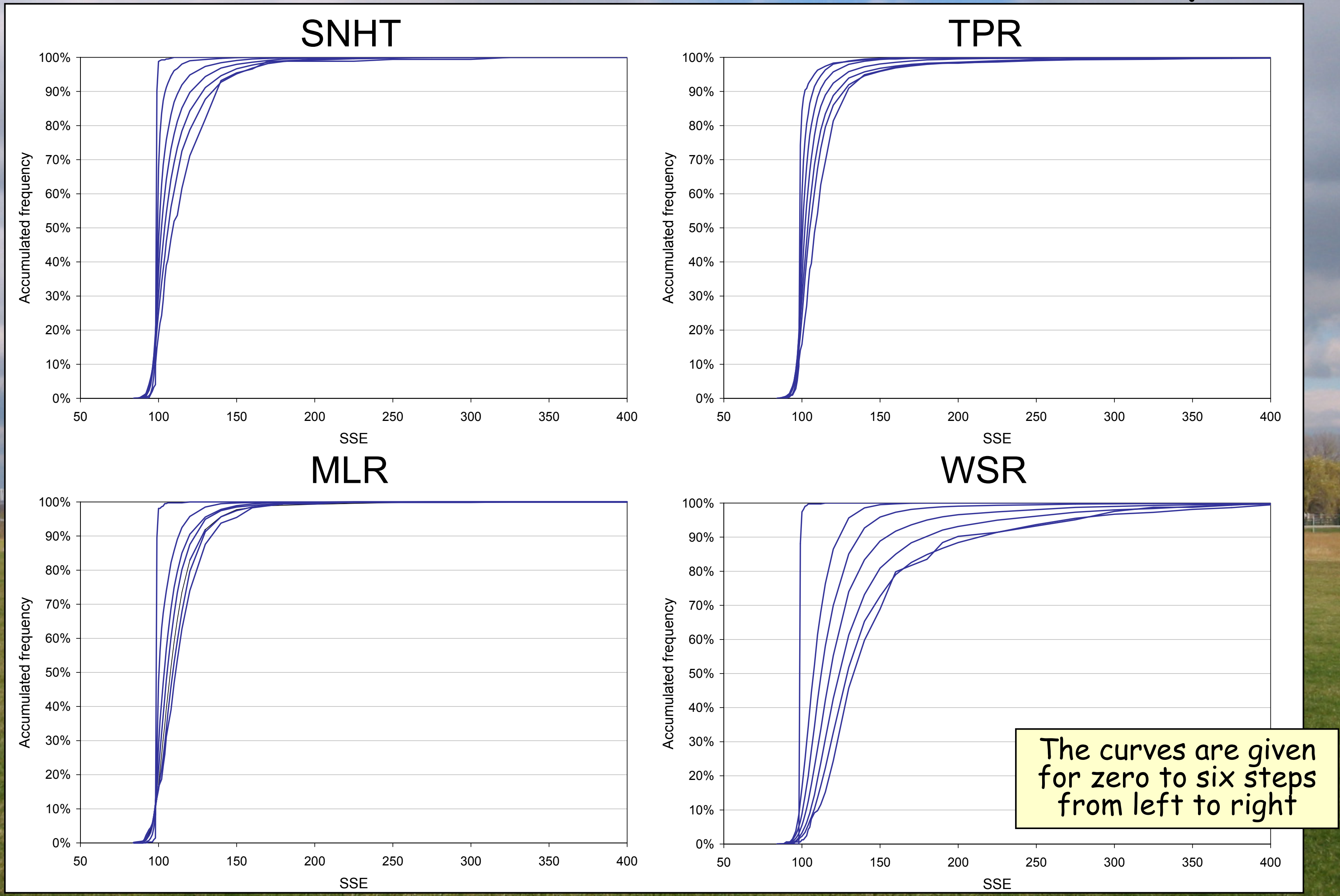
Percentage of steps identified when one step is introduced in the candidate series

Identification of a random number of steps

Method	Steps detected	Number of steps artificially introduced in the series: 0	1	2	3	4	5	6	7
SNHT	0	**93.9**	0.1	1.3	0.4	0.4	1.1		
	1	5.8	**92.5**	1.2	4.2	2.2	1.2	3.4	
	2	0.3	7.0	**89.2**	4.3	9.1	4.4	4.0	
	3		0.4	7.8	**84.0**	9.1	14.7	6.2	16.7
	4			0.4	6.8	**74.6**	14.8	23.2	33.3
	5			0.1	0.3	4.5	**61.6**	18.6	16.7
	6					0.1	2.2	**44.0**	
	7							0.6	**33.3**
TPR	0	**96.5**	29.0	8.3	2.5	1.1	0.3		
	1	0.6	**56.7**	39.2	16.6	6.3	2.8	0.6	
	2	0.9	8.1	**34.4**	38.7	25.4	13.3	5.7	
	3	0.9	3.2	11.3	**27.3**	36.2	33.6	14.7	16.7
	4	0.5	1.4	4.7	10.5	**21.4**	30.5	41.8	66.6
	5	0.2	0.8	1.4	3.6	7.6	**15.7**	31.6	16.7
	6	0.2	0.6	0.6	0.7	1.9	3.6	**5.6**	
	7	0.2	0.2	0.1	0.1	0.2	0.2		
MLR	0	**96.5**	0.1	0.1	0.1	0.1	0.1	0.6	
	1	3.5	**69.6**	9.5	1.1	0.7	2.5	2.3	
	2		20.6	**64.9**	16.5	6.2	9.1	2.8	
	3		7.5	20.1	**63.9**	20.0	23.8	6.2	
	4		2.0	4.7	15.9	**57.6**	26.8	52.5	50.0
	5		0.2	0.6	2.4	14.9	**28.8**	29.9	50.0
	6			0.1	0.1	0.5	8.7	**5.7**	
	7						0.2		
WRS	0	**94.2**	12	19.3	17.8	16.6	15.6	13.5	16.7
	1	5.6	**21.5**	7.1	5.3	3.3	4.2	7.9	
	2	0.2	29.9	**14.4**	7.8	7.1	4.3	1.1	
	3		22.1	23.1	**15.8**	12.7	12.9	11.3	
	4		12.4	19.2	19.7	**16.3**	13.2	11.9	
	5		1.8	10.8	16.6	17.5	**17.2**	7.3	33.2
	6		0.2	4.6	10.7	14.2	14.6	**18.1**	16.7
	7		0.1	1.4	4.7	8.2	10.3	14.1	**16.7**

Percentage of steps detected versus number of steps introduced

Identification of a random number of steps



Percentage of series with SSE less than a fixed value according to the number of steps introduced in the candidate series

Summary

False detection (Type I error)

- Methods that clearly described a step (SNHT & MLR) have lower rate of false detection
- Methods including trends bef & aft step (TPR) and based on non-parametric test (WRS) allow detection of false steps

Detection of a single step

- Steps ≥ 1.0 σ are easy to detect
- Methods allowing an overall trend (TPR & MLR) incorrectly identify a trend instead of a small step
- It is easier to identify a step when a reference series is used

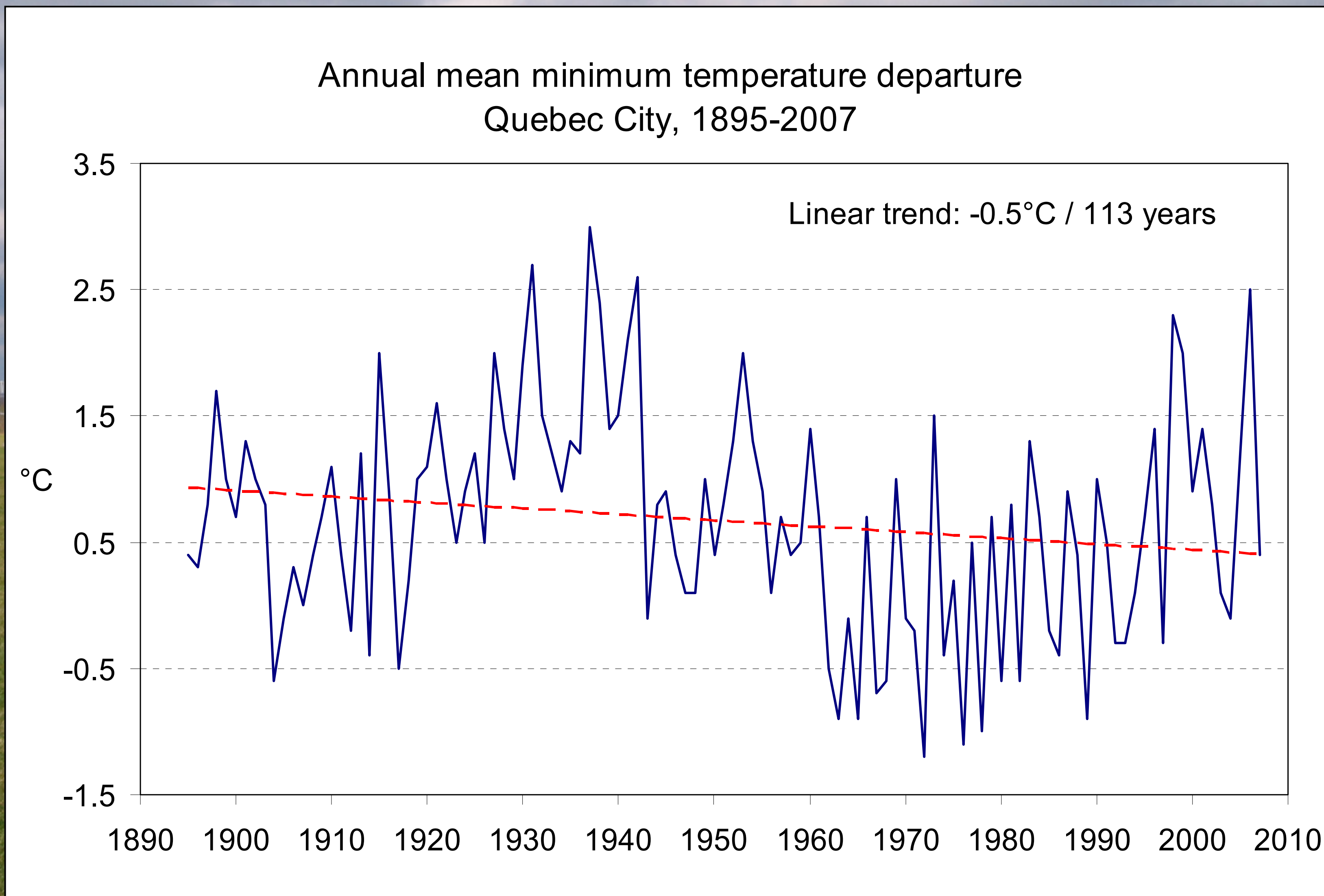
Detection of a random number of steps

- Methods that clearly described a step (SNHT & MLR) are more successful to identify the correct number and magnitude of steps
- It is more difficult to identify a step when the interval is smaller

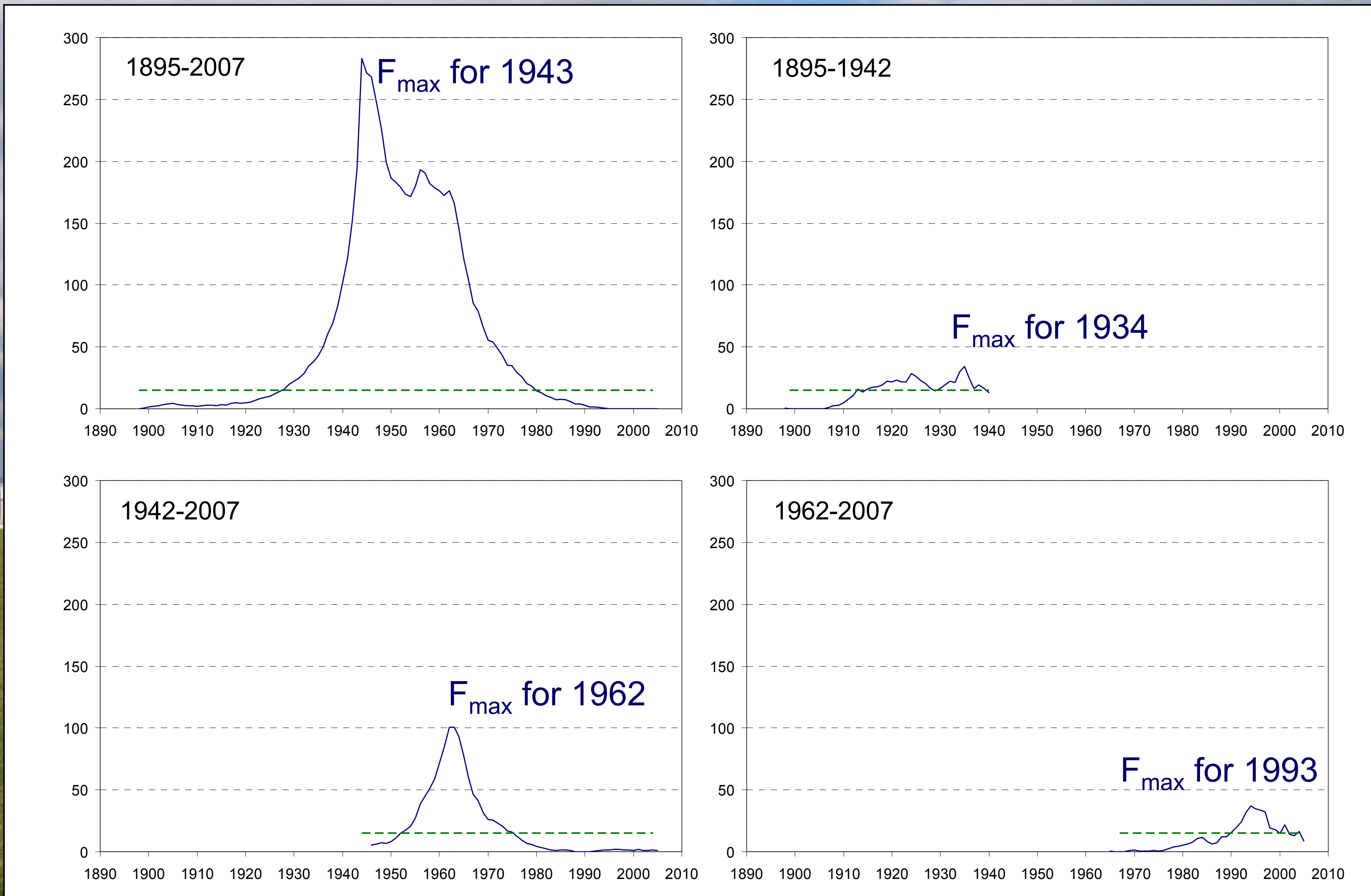
Overall, it seems that SNHT performs better!

Application of MLR to real data

Does this time series represent the temperature variations of this location?

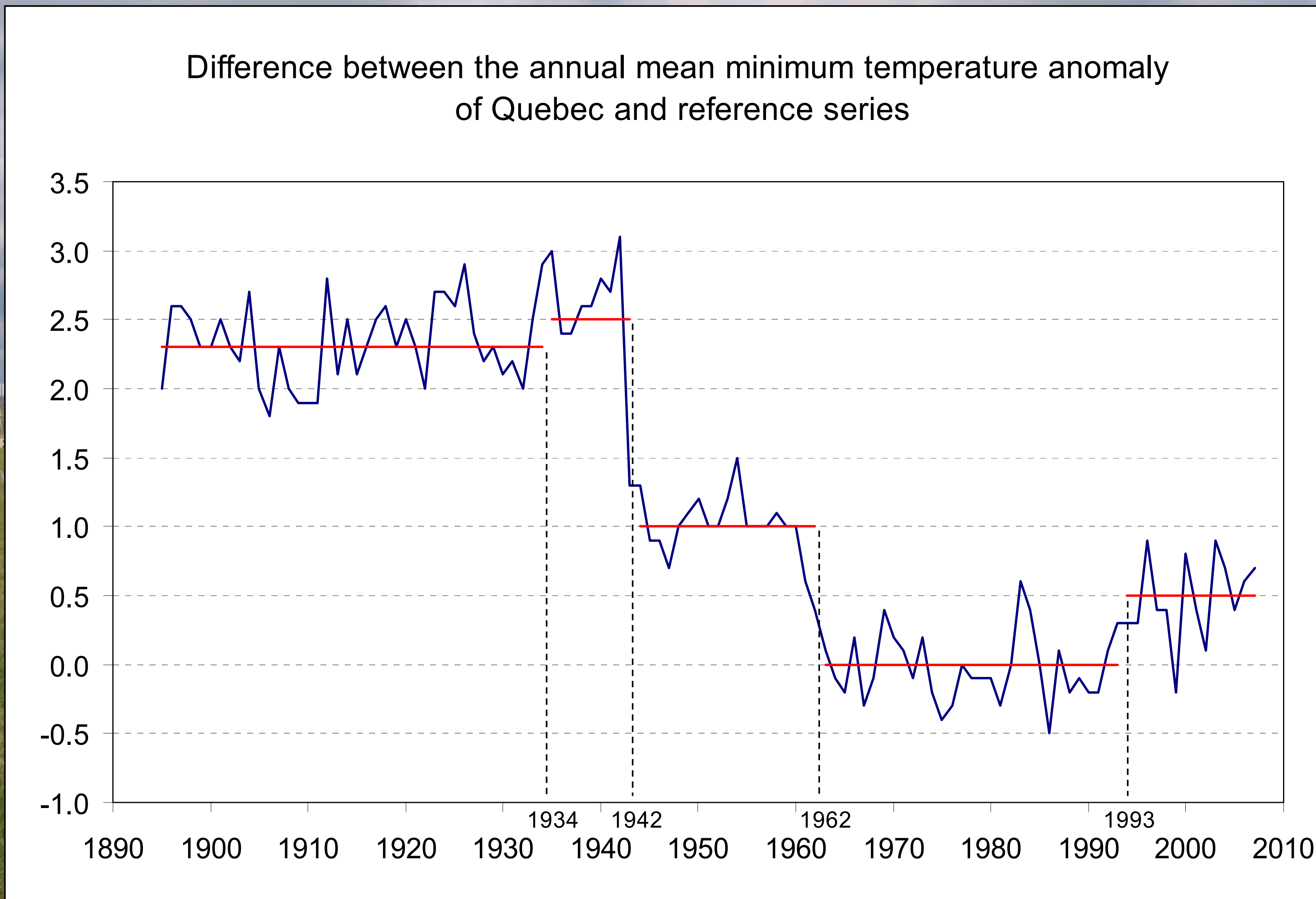


Search for discontinuities



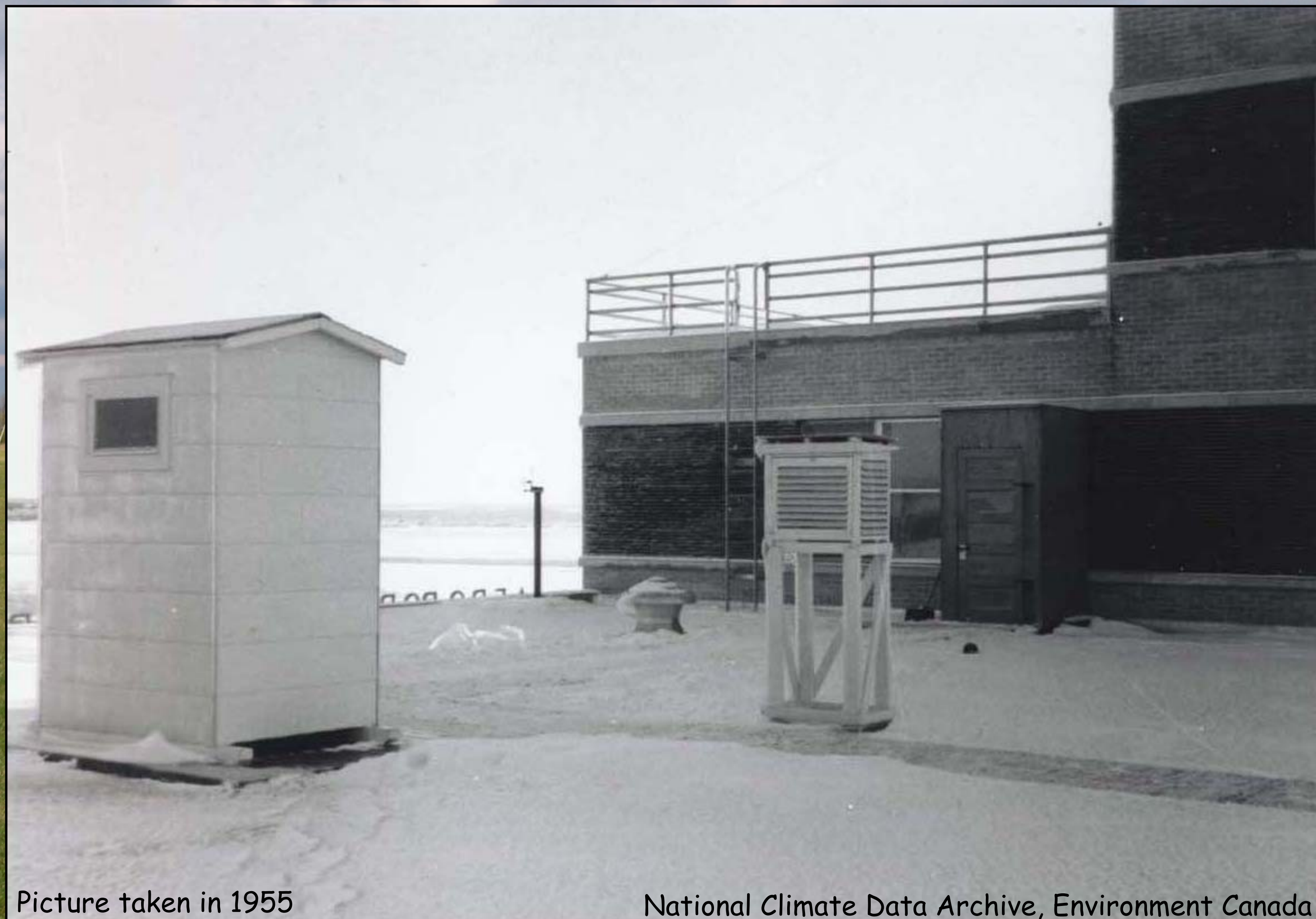
F-values for different intervals

Position of each discontinuity



Instruments located on the roof of the main building

Quebec City Airport



Picture taken in 1955

National Climate Data Archive, Environment Canada

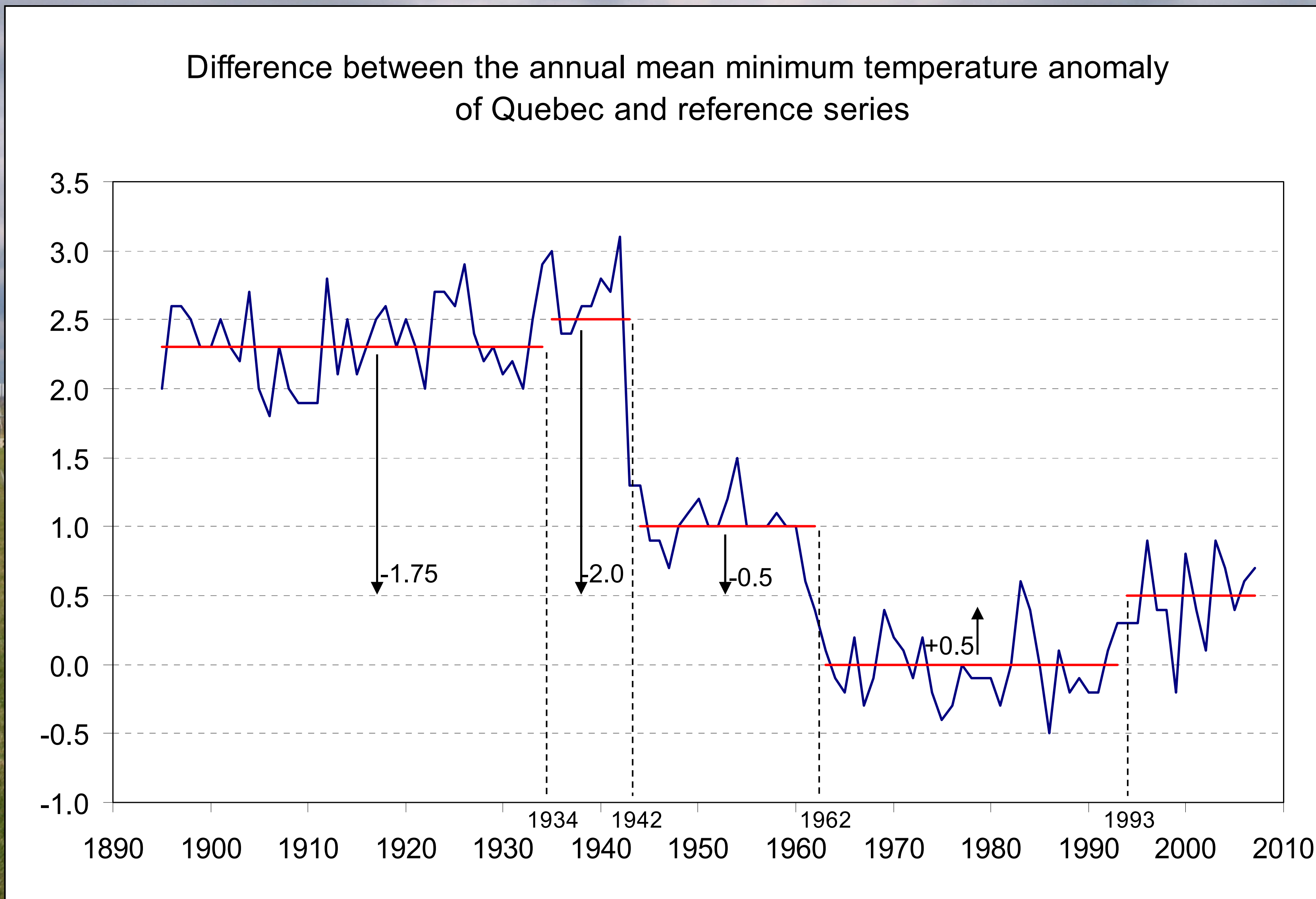
Instruments located near the parking lot

Quebec City Airport

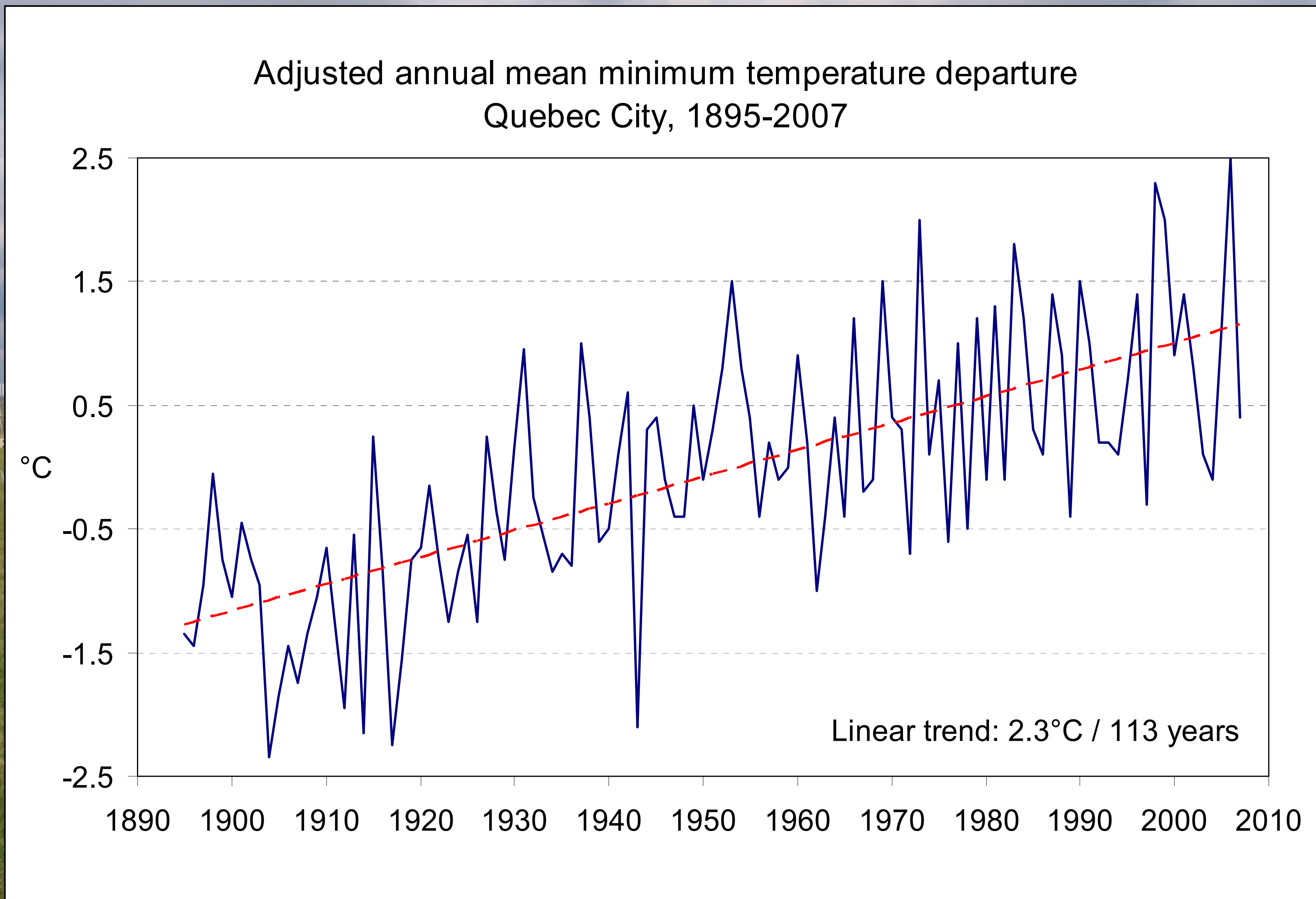
Picture taken in 1964

National Climate Data Archive, Environment Canada

Adjusting the time series

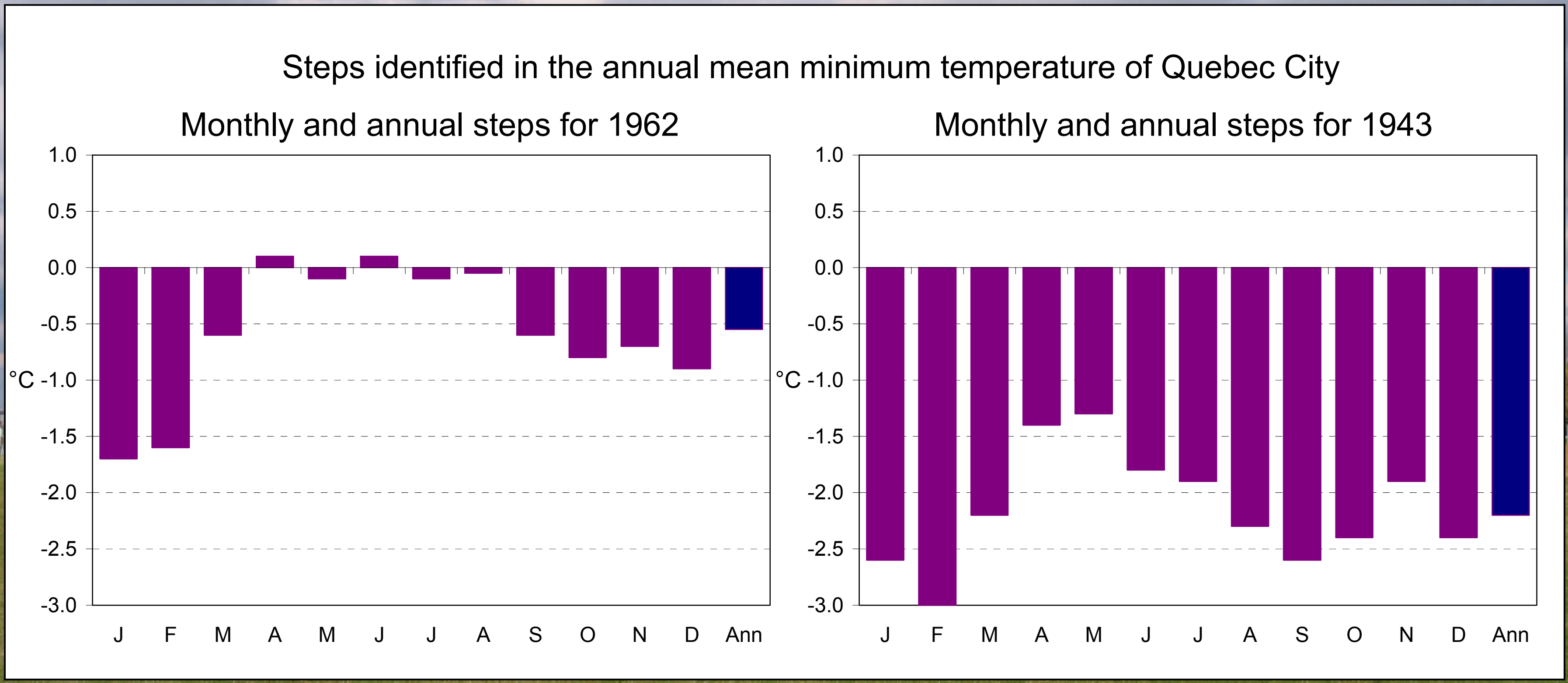


Final dataset




More research is needed …

How to adjust monthly and daily observations?



If the instruments relocation (or changing procedures) has created steps of various magnitude on the monthly values, how should we adjust the daily observations?


Thank you!