UBC Okanagan
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PRACTICE QUESTIONS FOR FINAL EXAM
Math 350 - Complex Analysis

1.(a) State the Cauchy-Riemann theorem for analytic functions.

(b) Sketch the points, if any, at which the function defined by $f(z) = x^5 + iy^3$ has a derivative. Where is f analytic? Explain.

(c) Show that $x^3 - 3xy^2$ is harmonic and find an harmonic conjugate.

(d) Use the definition of the derivative to show that the function $f(z) =\text{Re}(z)$ is nowhere analytic.

2. (a) Use the formal definition of limits to show that the function $f(z) = \bar{z}$ is continuous on the whole complex plane.

(b) Evaluate $\int_\gamma |z|^2 \, dz$ where γ is the line segment from 1 to $3 - i$.

(c) Is this integral path independent? Explain.

3. (a) Find all values of $1^{3/4}$.

(b) Find all values of $1^{\sqrt{2}}$.

(c) Sketch and describe the domain of analycity of $f(z) = \text{Log}(2i + z)$.

4.(a) Fix z_0, and let γ be a circle of radius R about z_0, with positive orientation. Show that

$$\oint_\gamma (z - z_0)^n \, dz = \begin{cases} 0 & n \neq -1, \\ 2\pi i & n = -1. \end{cases}$$

(b) Let C_R denote the circle of radius R centered at the origin, with positive orientation. Let $g(z)$ be analytic inside and on C_R. Show that if f is of the form

$$f(z) = \frac{A_n}{z^n} + \frac{A_{n-1}}{z_{n-1}} + \cdots + \frac{A}{z} + g(z), \qquad n \geq 1,$$

then

$$\frac{1}{2\pi i} \oint_{C_R} f(z) \, dz = A.$$

5. Evaluate the following integrals. In each case, state the theorem you are using, and verify that the hypothesis are satisfied.

(a) Let γ be the curve $|z| = 2$ traversed in the clockwise direction.

$$\oint_\gamma \frac{z}{(z+3)(z-1)} \, dz.$$

(b) Let $\mathcal{C}$ be the boundary of the unit square with vertices $\pm 1,\ \pm i$, with positive orientation

$$\oint_{\mathcal{C}} \frac{\sin z}{(4z+\pi)^2}\ dz$$

(c) Let C be the unit circle traversed counterclockwise, and let m and n be intergers.

$$\int_C z^m \bar{z}^n\ dz$$

6. (a) Obtain the McLaurin expansion for $f(z) = \frac{e^z}{1-z}$.
(b) Show that the function f defined by

$$f(z) = \begin{cases} \frac{\sin z}{z} & \text{for } z \neq 0, \\ 1 & \text{for } z = 0. \end{cases}$$

is analytic at $z = 0$, and find it's derivative $f'(0)$.

7. Consider the function $f(z) = \frac{1}{z^2(z+2i)}$.
(a) Expand f in a Laurent series $0 < |z| < 2$.
(b) Expand f in a Laurent series for $|z| > 2$.

8. Find and classify the isolated singularities.

(a) $f(z) = \dfrac{e^z - 1}{z^2}$ (b) $g(z) = \dfrac{\sin z}{z^2 - z}$

9. Compute

(a) $\mathrm{Res}_{z=-1} \dfrac{z^2}{(z+1)^2}$ (b) $\mathrm{Res}_{z=i} \dfrac{(z^3+2z)}{(z-i)^3}$

10. Let $f(z) = \sum_{k=0}^{\infty} \frac{k^3}{3^k}\ z^k$.
(a) Find the circle of convergence for f.
(b) Compute $\oint_{|z|=1} \frac{f(z)\sin(z)}{z^6}\ dz$.

11. Evaluate the following integrals

(a) $\displaystyle\oint_{|z|=5} z^2 \sin\left(\tfrac{1}{z}\right)\ dz$ (b) $\displaystyle\oint_{|z|=1} \frac{e^{-z^2}}{z^2}\ dz$

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