Assignment #5

Due: Friday, November 23, 3:00 pm

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark. Show your work and simplify your solutions. Where appropriate, your solutions should include definitions and references to theorems.

1. Evaluate, if possible, the following integrals, where Γ is the positively oriented circle $|z| = 1$. Repeat for $|z| = 2$.

(a) $\displaystyle\int_\Gamma \frac{e^z}{z^2(z-i)}\, dz$ (b) $\displaystyle\int_\Gamma \frac{\sin z}{z(z^2+2)}\, dz$ (c) $\displaystyle\int_\Gamma \frac{\cosh z}{z^3}\, dz$

2. Find the Taylor series for $f(z) = \frac{1}{z}$ about $z = 1$ and find the radius of convergence for the series.

3. For what values of z does the following series converges?

$$\sum_{n=0}^{\infty} \left(\frac{z}{1+z}\right)^n$$

4. Describe the region of convergence for the series:

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{2^k(z+i)^k}{(2+3i)^k}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(3-i)^k}{k^2}(z+2)^k$