Math 350 **Winter 2016**

Assignment #4

Due: Thursday, November 3, 2:00 pm

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark. Also indicate which theorem you are using.

1. Evaluate the following integrals **without** performing an explicit computation.

(a) $\displaystyle\int_\gamma \frac{1}{z}\,dz$, where $\gamma(t) = \cos t + 2i\sin t$ and $t \in [0, \pi]$.

(b) $\displaystyle\int_\gamma \frac{1}{z^2}\,dz$, where $\gamma(t) = \cos t + 2i\sin t$ and $t \in [0, \pi]$.

(c) $\displaystyle\int_\gamma \frac{e^z}{z}\,dz$, where $\gamma(t) = 3 + 2e^{it}$ and $t \in [0, 2\pi]$.

2. Evaluate the following integral along the line segment from $-1 + i$ to i

$$\int_\Gamma \frac{dz}{z^2 + 4}\,dz.$$

3. Let Γ be a path from $z = 0$ to $z = 1 + 2i$ consisting of the line segment from 0 to 1, and from 1 to $1 + 2i$.

(a) Evaluate $\int_\Gamma (z + 2\bar{z})\,dz$.

(b) Is this path integral independent? Explain.

4. Let γ be the boundary of the circle of radius 2 centered at the origin. Compute

(a) $\displaystyle\int_\gamma \frac{dz}{z^2 - 1}$ (b) $\displaystyle\int_\gamma \frac{dz}{z^2 - 8}$ (c) $\displaystyle\int_\gamma \frac{dz}{z^2 + 2z - 3}$