**Math 350** **Winter, 2016**

## Assignment #2

**Due:** Thursday, October 6, 4:00 pm

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

**1.** Describe and sketch the following sets of complex numbers.

(a) The image of $\mathrm{Re}(s) \geq 1$ under the mapping $f(z) = iz - i$.

(b) $\{z \in \mathbb{C} \,|\mathrm{Re}(\mathrm{z}^2) > 0\}$

**2.** Use the formal definition of limits to show that $\displaystyle\lim_{z \to i} \frac{z^2+1}{z-i} = 2i$ .

**3.** Use the definition of the derivative to show that the function $f(z) = \mathrm{Im}\,(\,z)$ is nowhere analytic.

**4.** Let $f(z) = u(x,y) + iv(x,y)$, $z_0 = x_0 + iy_0$, and $w_0 = u_0 + iv_0$. Use the properties of limits, and the fact that $f(z) \to w_0$ if and only if $\overline{f(z)} \to \overline{w_0}$ as $z \to z_0$, to prove that

$$\lim_{z \to z_0} f(z) = w_0$$

iff and only iff

$$\lim_{\substack{x \to x_0 \\ y \to y_0}} u(x,y) = u_0 \qquad \text{and} \qquad \lim_{\substack{x \to x_0 \\ y \to y_0}} v(x,y) = v_0.$$

**5.** Use the Cauchy-Riemann equations to show that the function $f(z) = x^2 + iy^2$ is nowhere analytic. Where is $f$ differentiable?