UBC Okanagan
The Irving K. Barber School of Arts and Sciences

PRACTICE QUESTIONS FOR FINAL EXAM
Math 350 - Complex Analysis

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way.

1.(a) State the Cauchy-Riemann theorem for analytic functions.

(b) Sketch the points, if any, at which the function defined by $f(z) = x^5 + iy^3$ has a derivative. Where is f analytic? Explain.

(c) Find an analytic function $f(z) = u(x, y) + iv(x, y)$ given that $v(x, y) = \sin x \cosh y$ maps the origin to the point $(1, 0)$.

2. (a) Find all values of $1^{3/4}$.

(b) Find all values of $1^{\sqrt{2}}$.

(c) Sketch and describe the domain of analycity of $f(z) = \mathrm{Log}(2i + z)$.

3.(a) Fix z_0, and let γ be a circle of radius R about z_0, with positive orientation. Show that

$$\oint_\gamma (z - z_0)^n \, dz = \begin{cases} 0 & n \neq -1, \\ 2\pi i & n = -1. \end{cases}$$

(b) Let C_R denote the circle of radius R centered at the origin, with positive orientation. Let $g(z)$ be analytic inside and on C_R. Show that if f is of the form

$$f(z) = \frac{A_n}{z^n} + \frac{A_{n-1}}{z_{n-1}} + \cdots + \frac{A}{z} + g(z), \qquad n \geq 1,$$

then

$$\frac{1}{2\pi i} \oint_{C_R} f(z) \, dz = A.$$

4. Evaluate the following integrals. In each case, state the theorem you are using, and verify that the hypothesis are satisfied.

(a) Let γ be the curve $|z| = 2$ traversed in the clockwise direction.

$$\oint_\gamma \frac{z}{(z+3)(z-1)} \, dz.$$

(b) Let $\mathcal{C}$ be the boundary of the unit square with vertices ± 1, $\pm i$, with positive orientation

$$\oint_{\mathcal{C}} \frac{\sin z}{(4z + \pi)^2} \, dz$$

5. (a) Obtain the McLaurin expansion for $f(z) = \frac{e^z}{1-z}$.
(b) Show that the function f defined by

$$f(z) = \begin{cases} \frac{\sin z}{z} & \text{for } z \neq 0, \\ 1 & \text{for } z = 0. \end{cases}$$

is analytic at $z = 0$, and find it's derivative $f'(0)$.

6. Consider the function $f(z) = \frac{1}{z^2(z+2i)}$.
(a) Expand f in a Laurent series $0 < |z| < 2$.
(b) Expand f in a Laurent series for $|z| > 2$.

7. Find and classify the isolated singularities.

(a) $f(z) = \dfrac{e^z - 1}{z^2}$

(b) $g(z) = \dfrac{\sin z}{z^2 - z}$

8. Compute

(a) $\text{Res}_{z=-1} \dfrac{z^2}{(z+1)^2}$

(b) $\text{Res}_{z=i} \dfrac{(z^3 + 2z)}{(z-i)^3}$

9. Let $f(z) = \sum_{k=0}^{\infty} \frac{k^3}{3^k}\ z^k$.
(a) Find the circle of convergence for f.
(b) Compute $\oint_{|z|=1} \frac{f(z)\sin(z)}{z^6}\ dz$.

10. Evaluate the following integrals

(a) $\displaystyle\oint_{|z|=5} z^2 \sin\left(\tfrac{1}{z}\right)\ dz$

(b) $\displaystyle\oint_{|z|=1} \frac{e^{-z^2}}{z^2}\ dz$

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