Math 350 **Winter 2013**

Test #2

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Show your work. Where appropriate, your solutions should include explanations and references to theorems.

1. Let Ω be the region of the complex plane defined by

$$\Omega = \{\ z \mid \mathrm{Re} z \geq 1,\ -\tfrac{\pi}{4} \leq \mathrm{Im} z \leq \tfrac{\pi}{4}\ \}.$$

Sketch the image of Ω under the mapping $w = 3e^{z+1}$.

2. Find all complex values z such that $\sin z = \frac{i}{2}$.

3. Consider $\mathrm{Log}(z)$, the principal branch of the logarithm.

(a) Sketch and describe the domain of analyticity of $f(z) = \mathrm{Log}(2i + z)$.

(b) Evaluate $\mathcal{L}_{\pi/4}(1 - i)$.

4. (a) State the Fundamental Theorem of Calculus for contour.

(b) Let γ be the part of the unit circle that joins the points $z = i$ to $z = -i$ traversed with positive orientation. Compute $\int_\gamma \frac{1}{z}\, dz$.

5. (a) State Cauchy's Integral Theorem.

(b) State the Deformation Invariance Theorem.

(c) Show that Cauchy's Integral Theorem implies the Deformation Invariance Theorem.

6. Evaluate the following integrals:

(a) Γ is the boundary of the square whose sides lie along the lines $x = -2$, $y = \pm 1$, and the y-axis, taken with a positive orientation.

$$\oint_\Gamma \frac{1}{1 - z^2}\, dz$$

(b) γ has the parametrization $z(t) = 3 + 2e^{it}$ for $0 \leq t \leq 2\pi$.

$$\int_\gamma \frac{e^z}{z}\, dz$$