Math 350 **Winter 2013**

Practice Test #1

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Show your work. Where appropriate, your solutions should include explanations and references to theorems.

1. For what values of z is the function $f(z) = z\bar{z}$ differentiable? State the theorem you used to obtain your result, and determine where f is analytic.

2.(a) Evaluate $(\sqrt{3} - i)^7$.

(b) Find all distinct values of $i^{1/2}$.

3. Describe the set of points z in the complex plane that satisfy the following equations, and determine which of these is a domain.

(a) $|\ Re\ (z + 3 + i)\ | > 1$

(b) $\text{Im}(z) < \text{Re}(z)$

4. Verify that the function $u(x, y) = e^x \sin y$ is harmonic, and find the harmonic conjugate of u.

5. Use the rigorous definition of limits to prove that

$$\lim_{z \to 1+i} (6z - 4) = 2 + 6i.$$

6. Answer any ONE of the following questions:

(i) Use the definition of the derivative to show that $f(z) = \bar{z}$ is nowhere differentiable.

(ii) Prove that if $f(z)$ is differentiable at z_0, then $f(z)$ is continuous at z_0.

(iii) Explain why an analytic function satisfies the Cauchy Riemann equations. Your answer should include a derivation of the Cauchy Riemann equations.