## Test #1

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Show your work. Where appropriate, your solutions should include explanations and references to theorems.

1. Let 
$$f(z) = \frac{z^2}{|z^2|}$$
. Does f have a limit as  $z \to 0$ ?

**2.**(a) Solve for  $z: z^4 + i = 0$ . Give your answer in exponential form.

- (b) Evaluate  $\left(\frac{1}{1+i}\right)^{10}$ .
- (c) Find all distinct values of  $(1)^{3/4}$ . Simplify your answer.

**3.** Describe the set of points z in the complex plane that satisfy the following equations and determine which of these set is a domain.

(a) 
$$(Re \ z)^2 > 1$$

(b)  $\text{Re}(\bar{z} - i) = 2$ 

**4.** Use the formal definition of limits to show that the function  $f(z) = \frac{iz}{2}$  is continuous at z = i.

5. Derive the Cauchy Riemann equations, and explain why they give a necessary conditions for the existence of the derivative of f(z) at  $z = z_0$ .

6. Consider the polynomial function  $f(z) = z^3 + iz^2 - z$ .

- (a) Write f(z) in the form u(x, y) + iv(x, y).
- (b) Show that f(z) is entire.
- (c) Are the functions u and v of part (a) harmonic? Justify your answer.