Assignment #5

Due: Tuesday, April 8, 4:00 pm

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Find the Taylor series for $f(z) = \frac{1}{z}$ about $z = 1$ and find the radius of convergence for the series.

2. For what values of z does the following series converges?

$$\sum_{n=0}^{\infty} \left(\frac{z}{1+z} \right)^n$$

3. Describe the region of convergence for the series:

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{2^k (z+i)^k}{(2+3i)^k}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(3-i)^k}{k^2} (z+2)^k$

4. Consider the function
$$f(z) = \frac{z}{z^2 + 4z - 12}.$$

(a) Expand f in a Laurent series for $|z| < 1$.

(b) Expand f in a Laurent series for $3 < |z| < 4$.

(c) Expand f in a Laurent series for $8 < |z|$.