Math 350 **Winter, 2013**

Review # 2

1. (a) Give the definition of the Principal branch Log(z) of the lorarithm.
(b) Describe the domain and range of Log(z)? Where is Log(z) analytic?
(c) Is it true that Log(x^2)=2 Log(z) for all $z \in \mathbb{C}$?
(d) Sketch and describe carefully the domain of analyticity of Log($2z-1$).

2. Express the following quantities in the $u + iv$ form.
(a) $\sinh(1+\pi i)$
(b) $\mathcal{L}_{\pi/2}(-\sqrt{3}+i)$

3. Find all values of z for which $\text{Log}(z^2+1) = \frac{i\pi}{2}$.

4. Find the solutions for $\cos z = 2i$.

5. Evaluate the following integrals **without** performing an explicit computation.
(a) $\displaystyle\int_\gamma \frac{1}{z}\, dz$, where $\gamma(t) = \cos t + 2i \sin t$ and $t \in [0, \pi]$.
(b) $\displaystyle\int_\gamma \frac{1}{z^2}\, dz$, where $\gamma(t) = \cos t + 2i \sin t$ and $t \in [0, \pi]$.
(c) $\displaystyle\int_\gamma \frac{e^z}{z}\, dz$, where $\gamma(t) = 3 + 2e^{it}$ and $t \in [0, 2\pi]$.

6. Compute by two different methods:

$$\int_i^{2i} (z^2 - 2e^{2z})\, dz.$$

7. Let γ be the boundary of the circle of radius 2 centered at the origin. Compute
(a) $\displaystyle\int_\gamma \frac{dz}{z^2-1}$ (b) $\displaystyle\int_\gamma \frac{dz}{z^2-8}$ (c) $\displaystyle\int_\gamma \frac{dz}{z^2+2z-3}$

8. Let z_0 denote a fixed complex number, and let γ be a simple closed contour with positive orientation such that z_0 lies in the interior of γ. Derive the following formula:

$$\oint_\gamma \frac{dz}{(z-z_0)^n} = \begin{cases} 0 & n \neq 1, \\ 2\pi i & n = 1. \end{cases}$$

9. (a) State Cauchy's Integral Theorem.
(b) State the Deformation Invariance Theorem
(c) Show that Cauchy's Integral Theorem implies the Deformation Invariance Theorem.

10. Prove (2) $\Leftrightarrow$ (3) in the Path Independence Lemma.