Winter 2008 **Math 317**

Sample Test

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Show your work. Your solutions should include explanations and references to theorems where appropriate.

1. Sketch the curve given by the parametric equation

$$x = \ln t, \quad y = \sqrt{t}, \quad t \geq 1.$$

Your sketch should include the initial point, and the direction in which the curve is traced.

2. Consider the curve $\mathcal{C}$ given by the equations $x = 2 - t^3$, $y = 2t - 1$, $z = \ln t$.

(a) Find a parametric equation for the tangent line to $\mathcal{C}$ at the point $(1, 1, 0)$.

(b) Find an equation for the normal plane to $\mathcal{C}$ at the point $(1, 1, 0)$.

3. Let $\mathcal{C}$ be a smooth plane curve.

(a) What is the osculating circle of $\mathcal{C}$ at the point p? What does it tell you about the curve?

(b) Find an equation for the osculating circle of the curve $y = x^4 - x^2$ at the origin.

4. (a) State the Fundamental Theorem of Calculus for line integrals.

(b) Prove the following statement:

"If $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r} = 0$ for every closed path $\mathcal{C}$ in D then $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ is path independent."

5. (a) Find $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x + z)\vec{i} + z\vec{j} + y\vec{k}$ and $\mathcal{C}$ is the line from the point $(2, 4, 4)$ to the point $(1, 5, 2)$.

(b) Evaluate $\int_{\mathcal{C}} (3x - y)\, ds$, where $\mathcal{C}$ is the portion of the circle $x^2 + y^2 = 18$ traversed from $(3, 3)$ to $(3, -3)$ clockwise.

6. Let $\vec{F}(x, y) = (2xy + 3)\vec{i} + (x^2 + \cos y)\vec{j}$

(a) Show that $\vec{F}$ is a conservative vector field.

(b) Find a potential function for $\vec{F}$.

(c) Use part (b) to compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ where $\mathcal{C}$ is the curve beginning at the point $(1, 0)$ and ending at the point $(2, \pi)$.