Quiz #5- Solutions

1. Find a parametric representation for the lower half of the ellipsoid $2x^2 + 4y^2 + z^2 = 1$. Your answer should include the parameter domain.

There are many possibilities:

A - $x = x,\ y = y,\ z = -\sqrt{1 - 2x^2 - 4y^2}, \quad D = \{(x, y) \mid -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}},\ -\frac{1}{2} \le y \le \frac{1}{2}\ \}$

B - $x = \frac{1}{\sqrt{2}} u\cos(v),\ y = \frac{1}{2} u\sin(v),\ z = -\sqrt{1 - u^2}, \quad D = \{(u, v) \mid 0 \le u \le 1,\ 0 \le v \le 2\pi\ \}$

C - $x = \frac{1}{\sqrt{2}} \sin(\phi)\cos(\theta),\ y = \frac{1}{2}\sin(\phi)\sin(\theta),\ z = \cos(\phi), \quad D = \{(\phi, \theta) \mid \frac{\pi}{2} \le \phi \le \pi,\ 0 \le \theta \le 2\pi\ \}$

2. Find the area of the surface for the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 9$.

The surface corresponds to the graph $z = g(x, y) = xy$, with domain determined by $x^2 + y^2 \le 9$. We use the formula for area:

$$A = \int\int_D \sqrt{1 + (g_x)^2 + (g_y)^2}\ dA.$$

We need to parametrize the area. We use circular symmetry and set $x = r\cos\theta$ and $y = r\sin\theta$. Then

$$D = \{(r, \theta) \mid 0 \le r \le 3,\ 0 \le \theta \le 2\pi\ \} \quad \text{and} \quad f_x^2 + f_y^2 = y^2 + x^2 = r^2$$

It follows that

$$\begin{aligned} A &= \int_0^{2\pi} \sqrt{1 + r^2}\ r\ dr\ d\theta \\ &= \frac{1}{2}\int_0^{2\pi}\int_1^{10} u^{1/2}\ du\ d\theta \\ &= \frac{1}{2}\int_0^{2\pi} \frac{2}{3}u^{3/2}\Big|_1^{10}\ d\theta \\ &= \frac{2\pi}{3}\left(10^{3/2} - 1\right). \end{aligned}$$

Where we have use the substitution: $u = 1 + r^2,\ du = 2r\ dr,\ r = 0 \to u = 1,\ r = 3 \to u = 10.$