UBC Okanagan
The Irving K. Barber School of Arts and Sciences

FINAL EXAMINATION
Math 317 - Calculus IV

April 16, 2007 **Time: 3 hours**

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way.

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[4] **1.** Describe the motion of a particle with position (x, y) where $x(t) = \cos^2 t$, $y(t) = \cos t$, for $0 \leq t \leq 2\pi$. Your sketch should include the initial point and the direction in which the curve is being traversed.

[6] **2.** Consider the curve with parametric equation

$$x(t) = t - \sin t, \quad y(t) = 1 - \cos t, \quad 0 \leq t \leq 4\pi.$$

(a) Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
(b) At what points is the tangent horizontal?
(c) Use part (a) and (b) to sketch the curve. Your sketch should include the initial point and the direction in which the curve is being traversed.

[2] **3.** Find a parametric representation for the curve of intersection of the paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$.

[8] **4.** Consider the curve

$$\vec{r}(t) = (e^t \cos t, e^t \sin t, e^t), \qquad t \geq 0.$$

(a) Find the vectors $\vec{T}$, $\vec{N}$, and $\vec{B}$ at the point $(1, 0, 1)$.
(b) Compute the curvature κ at the point $(1, 0, 1)$.
(c) Reparametrize $\vec{r}(t)$ with respect to arc length measured from the point $(1, 0, 1)$ in the direction of increasing t.

[7] **5.**(a) State the Fundamental Theorem for line integrals.

(b) Let $\vec{F}(x,y,z) = e^y\vec{\mathbf{i}} + (xe^y + e^z)\vec{\mathbf{j}} + ye^z\vec{\mathbf{k}}$. Show that $\vec{F}$ is conservative.

(c) **Use part (a)** to evaluate $\int_{\mathcal{C}} \vec{F} \cdot \, d\vec{r}$ where $\mathcal{C}$ is the line segment from $(0,2,0)$ to $(4,0,3)$.

[5] **6.**(a) State Green's Theorem.

(b) Use Green's Theorem to compute $\int_{\mathcal{C}} xy^2 \, dx - x^2 y \, dy$, where $\mathcal{C}$ consists of the parabola $y = x^2$ from $(1,1)$ to $(-1,1)$ and the line segment from $(-1,1)$ to $(1,1)$.

[6] **7.**(a) Evaluate $\int_{\mathcal{C}} (xy + \ln x) \, dy$, where $\mathcal{C}$ is the arc of the parabola $y = x^2$ from $(1,1)$ to $(3,9)$.

(b) Find the area of the part of the surface $z = x^2 + 2y$ that lies above the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$.

[7] **8.**(a) State Stokes' Theorem.

(b) **Use part (a)** to verify that Stokes' Theorem is true for the vector field $\vec{F}(x,y,z) = y\vec{\mathbf{i}} + z\vec{\mathbf{j}} + x\vec{\mathbf{k}}$ and the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$, taken with positive orientation.

[5] **9.**(a) State the Divergence Theorem.

(b) **Use part (a)** to calculate the flux of the vector field $\vec{F}(x,y,z) = (\cos z + xy^2)\vec{\mathbf{i}} + xe^{-z}\vec{\mathbf{j}} + (\sin y + x^2 z)\vec{\mathbf{k}}$ across the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

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