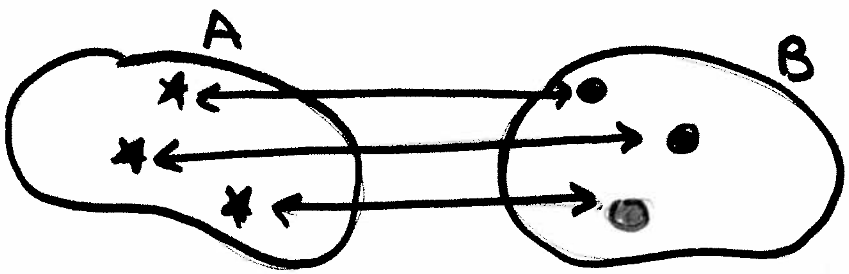
Test #1 - Solutions

1. Let A and B be sets. Explain what the following expressions represent.In each case, give an example to illustrate your answer.

(a) $A \sim B \quad \Rightarrow \quad A$ is equivalent to B means that there is a one-to-one correspondence between the elements of the set A and the elements of the set B.



(b) $A \subset B$

A is a proper subset of B. This means that every element of A is contained in B, and B has some elements that are not in A.

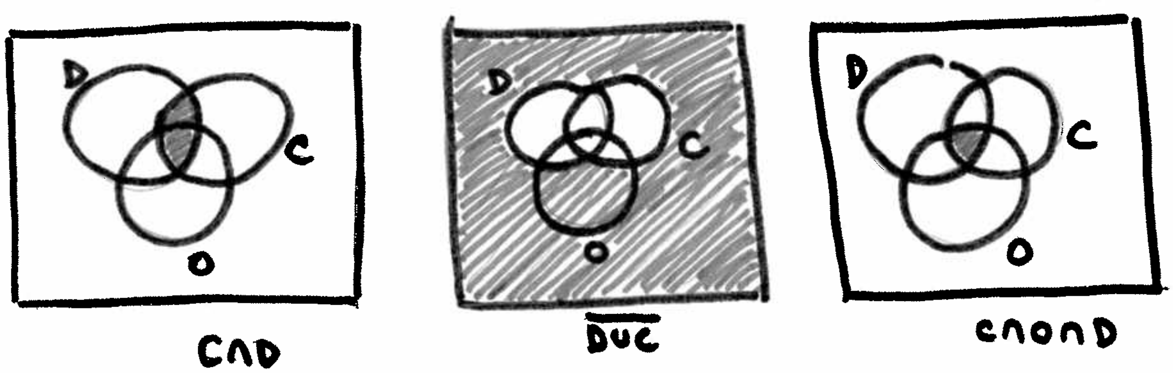
eg. Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then $A \subset B$.

2. An elementary teacher asks her class the following question: "Which pets live in your home?" Let D = the set of students who have at least one dog, C = the set of students who have at least one cat, and O = the set of students who have a pet that is not a dog or a cat. Draw the Venn diagram for each of the following sets, and describe in everyday english the elements in each of the sets.

(a) $C \cap D$ is the set of students who have at least one dog and one cat.

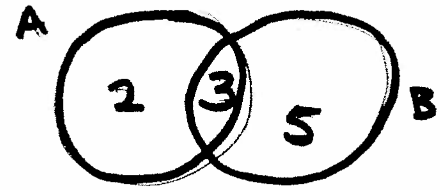
(b) $\overline{D \cup C}$ is the set of students who have no dogs and no cats.

(c) $C \cap O \cap D$ is the set of students who have at least one dog, one cat, and one other pet that is neither a cat nor a dog.



3. Let $n(A) = 5$, $n(B) = 8$, and $n(A \cup B) = 10$. What can you say about $n(A \cap B)$? Explain how you obtain your solution.

Since $n(A)+n(B) = 5+8 = 13 > 10$, the set A and B have some elements in common. The difference, $13 - 10 = 3$, is the number of elements in their intersection. It follows that $n(A \cap B) = 3$.



4. Explain why $3 < 4$ using the definition of whole-number inequality given in terms of addition.

By definition, if a and b are whole numbers, we say that $a < b$ if and only if there exists $k \in \mathbb{N}$ such that $a + k = b$. Here, $a = 3$, $b = 4$, and since $3 + 1 = 4$, $k = 1$. It follows that $3 < 4$.

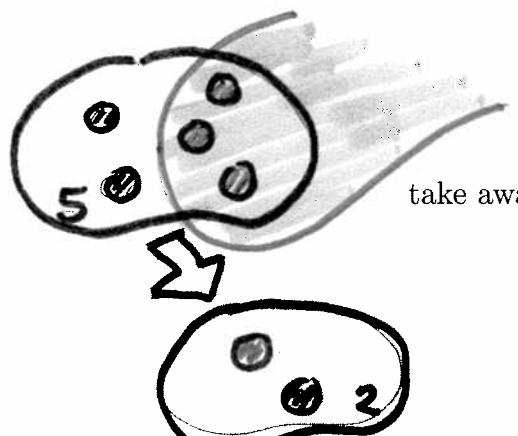
5.(a) State the definition of whole-number subtraction.

Given a, $b \in$W, such that $a > b$, $a - b$ is the element $c \in$W such that $a + b = c$.

(b) Illustrate $5 - 3 = 2$, using two different models.

1. take-away model

start with 5 elements

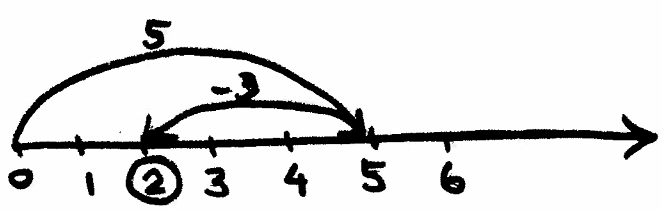


take away 3

2 remain so $5 - 3 = 2$.

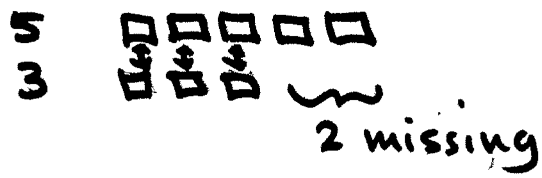
2. measurement model

We start with 5, which means move 5 units to the right, then 3 units to the right. We find ourselves at 2, so $5 - 3 = 2$.



3. comparison model

When I compare a set of 5 objects to a set of 3 objects, the difference is 2. Therefore $5-3=2$.



6. In the following pattern, one hexagon takes 6 toothpicks to build, two hexagons take 11 toothpicks to build, and so on. How many toothpicks would it take to build n hexagons? Use the four-step process to find a solution.

Step1: Understand the question.

At each step, I add an hexagon to the existing chain of hexagons. I need to find an expression for the number of toothpicks required to build a chain with n hexagons.

Step 2: Devise a plan

I will record the information in a table and look for a pattern.

Step 3: Carry out the plan

n	# of toothpicks
1	6
	+5
2	11
	+5
3	16
	+5
4	21

The sequence forms an arithmetic progression with initial number $a=6$, and a difference between consecutive terms of $d=5$. The formula for the n$^{\text{th}}$ term is $6+(n-1)5$.

Step 4: Lookback

Looking back at the process, at each step, I needed 5 toothpicks to construct the additional hexagon.

n	# of toothpicks	
1	6	$=6+0\cdot 5$
2	6+5	$=6+1\cdot 5$
3	6+5+5	$=6+2\cdot 5$
4	6+5+5+5	$=6+3\cdot 5$

So the formula is $6+(n-1)5$ as before.

7. Maria and Karl work at different jobs. Maria earns \$50 per hour and Karl earns \$40 per hour. They each earn the same amount per week but Karl works 2 more hours. How many hours a week does Karl work? Use algebraic reasoning to find a solution.

We know that Karl works 2 hours more than Maria.

Let $H = \#$ of hours worked by Maria.

then $K = H + 2$ is the # of hours Karl works during the week.

Maria and Karls earn the same amount per week

weekly amount for Maria $\rightarrow$ $50H$

weekly amount for Karl $\rightarrow$ $40K = 40(H + 2)$

Since these are the same, we set them equal and solve the equation for H:

$$\begin{aligned} 50H &= 40(H+2) \\ 50H &= 40H + 80 \\ 50H - 40H &= 40H + 80 - 40H \\ 10H &= 80 \\ \frac{10}{10} &= \frac{80}{10} \\ H &= 8 \end{aligned}$$

The number of hours of work for Karl is $K = H + 2 = 8 + 2 = 10$ hours.

check back:

Maria: $50H = 50 \cdot 8 = \$400$.

Karl: $40K = 40 \cdot 10 = \$400$.

Our answer is correct. Karl works 10 hours a week.