Math 160 **Fall 2008**

Assignment # 1 - Solutions

#1. Start with a piece of paper. Cut it into five pieces. Take just one piece and cut it again into five pieces, and so on. What is the number of pieces after the n^{th} experiment? Use the four-step process to solve the problem, and obtain the solution using two different strategies.

SOLUTION:

Step 1: We need to obtain a formula that will tell us how many pieces of paper we obtain if we carry out the experiment n times.

Step 2: We will store the information in a table, and look for a pattern.

Step 3: At every step, cutting one piece into five pieces produces an additional four pieces.

term	value	pattern
1	5	$5+0\cdot 4$
2	$5+4$	$5+1\cdot 4$
3	$5+4+4$	$5+2\cdot 4$
4	$5+4+4+4$	$5+3\cdot 4$
.	$\cdots$	$\cdots$
.	$\cdots$	$\cdots$
n	$5+4+4+\cdots+4$	$5+(n-1)4$

The formula for the n^{th} term is $5+4(n-1)$.

Step 4: We verify that this is correct by finding the formula using a different approach. If the two agree then the formula should be correct.

This time, we record the number of pieces after each step and look for a pattern. We record the information as a sequence of numbers:

$$1 \quad 5 \quad 9 \quad 13 \quad 17 \quad 21 \quad ...$$

Notice that the difference between each term in the sequence is 4. The sequence of terms representing the total number of pieces is an arithmetic sequence. The formula for the n^{th} term is $1+4n$. We compare with the previous formula:

$$5+4(n-1) = 5+4n-4 = 1+4n.$$

The previous formula is right since it reproduces the same sequence of numbers.

#2. Gerry spends \$60 of her paycheque on clothes and then spends one-half of her remaining money on food. If she has \$80 left, what was the amount of her paycheque? Use algebraic reasoning and the four-step process to obtain a solution.

SOLUTION:

Step 1: We need to determine the amount of the paycheque.

Step 2: We will use algebraic reasoning. That is, assign a variable to the unknown, construct an equation using the given information, and solve for the unknown variable.

Step 3: Let $P =$ amount of Gerry's paycheque.

$P - 60$ is the amount of money left after Gerry bought some clothes.

Gerry spent half of this amount on food. Therefore, the remaining half is \$80. This gives the following equation:

$$\frac{1}{2}(P - 60) = 80\,.$$

Solving for P, we obtain

$$\begin{aligned} 2 \cdot \frac{1}{2}(P - 60) &= 2 \cdot 80 \\ P - 60 &= 160 \\ P - 60 + 60 &= 160 + 60 \\ P &= 220\,. \end{aligned}$$

The amount of Gerry's paycheque is \$220.

Step 4: We could check if this is right using a different approach. Let's try to reason backward. Gerry had \$80 after spending half of what remained on food. So he must have had $2 \times 80 = 160$. We only need to add the money spent on clothes to get the amount of the paycheque: $160 + 60 = 220$. So the amount of Gerry's paycheque is \$220.

#3. Consider the following figures made with tiles. (see assignment sheet)

If this pattern is continued, it will result in one of the figures having 290 tiles. Which figure will have this many tiles?

SOLUTION:

Step 1:

the first figure has 5 tiles

the 2nd " " 8 tiles

the 3rd " " 11 tiles

We need to find a pattern for the remaining figures. Once we have an expression for the n^{th} term, we can find which figure has 290 tiles.

Step 2: We collect the information in a table and look for a pattern.

For each figure, we record the number of tiles in each legs, and the number of uncounted

tiles joining the two legs at the top.

term	# tiles in each legs	# tiles on top	pattern
1	2	1	$2 \cdot 2 + 1 = 5$
2	3	2	$2 \cdot 3 + 2 = 8$
3	4	3	$2 \cdot 4 + 3 = 11$
4	5	4	$2 \cdot 5 + 4 = 14$
.	$\cdots$	$\cdots$	$\cdots$
.	$\cdots$	$\cdots$	$\cdots$
n	$n+1$	n	$2(n+1)+n$

The n^{th} term is given by the expression $2(n+1)+n = 2n+2+n = 3n+2$.

Step 4: Alternatively, we can look at the sequence of numbers obtained in the last column of the table:

$$5,\ 8,\ 11,\ 14\ , \cdots$$

We observe that the difference between each term is 3. We have an arithmetic sequence. We use the formula for an arithmetic sequence with initial term $a = 5$, and difference $d = 3$, to get the expression for the n^{th} term.

$$a + (n-1)d = 5 + (n-1)3 = 5 + 3n - 3 = 3n - 2.$$

This is the same as before.

To find which figure has 290 tiles, we set the expression we found in part 3 equal to 290 and solve for n.

$$\begin{aligned} 290 &= 3n + 2 \\ 290 - 2 &= 3n \\ \frac{288}{3} &= \frac{3n}{3} \\ 96 &= n. \end{aligned}$$

The 96^{th} figure will have 290 tiles.