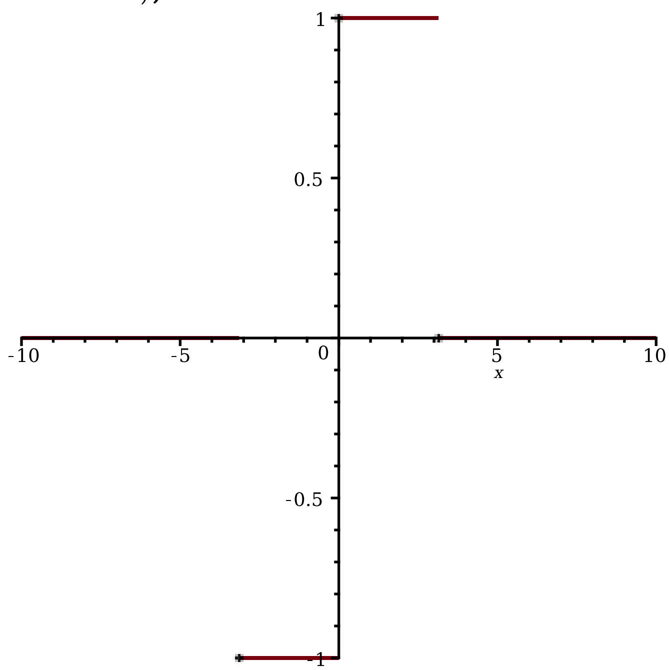
> $with(plots):$

Example 1

> $f1a := x \rightarrow piecewise(\text{And}(-\text{Pi} \leq x, x < 0), -1, \text{And}(0 \leq x, x < \text{Pi}), 1, 0);$

$$f1a := x \mapsto \begin{cases} -1 & -\pi \leq x \wedge x < 0 \\ 1 & 0 \leq x \wedge x < \pi \\ 0 & otherwise \end{cases} \tag{1}$$

> $plot(f1a(x), discont = true);$



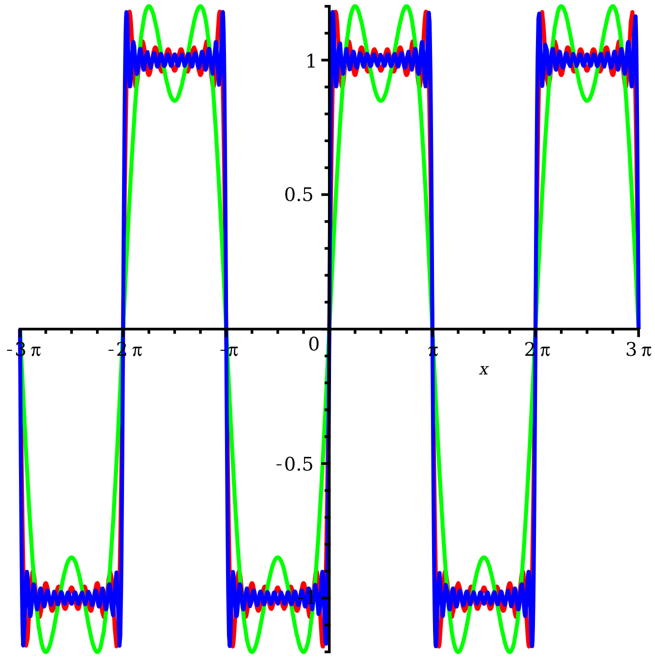
> $f1 := (x, nmax) \rightarrow add\left(\frac{2}{n \cdot \text{Pi}} \cdot (1 - (-1)^n) \cdot \sin(n \cdot x), n = 1 ..nmax\right);$

Warning, (in f1) `n` is implicitly declared local

$$f1 := (x, nmax) \mapsto add\left(\frac{2 \cdot (1 - (-1)^n) \cdot \sin(n \cdot x)}{n \cdot \pi}, n = 1 ..nmax\right) \tag{2}$$

Below is a plot of three instances of the fourier series solution, one using 15 terms, one using 3 terms, and one using 30 terms. Observe the ringing at the discontinuitiies.

> $plot([f1(x, 15), f1(x, 3), f1(x, 30)], x = -3 \cdot \text{Pi}..3 \cdot \text{Pi}, colour = [red, green, blue]);$

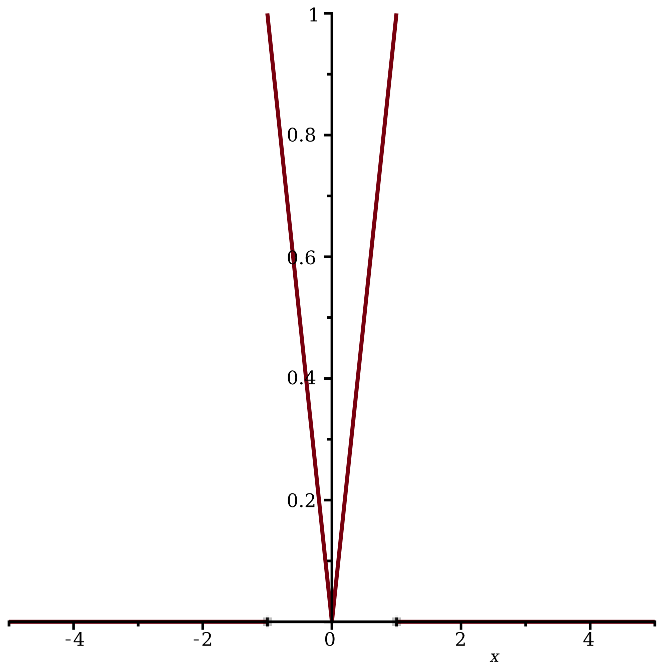


Example 2

> $f2a := x \rightarrow piecewise(\mathrm{And}(-1 < x, x < 1), \mathrm{abs}(x), 0);$

$$f2a := x \mapsto \begin{cases} |x| & -1 < x \wedge x < 1 \\ 0 & otherwise \end{cases} \quad (3)$$

> $plot(f2a(x), x = -5..5, discont = true);$



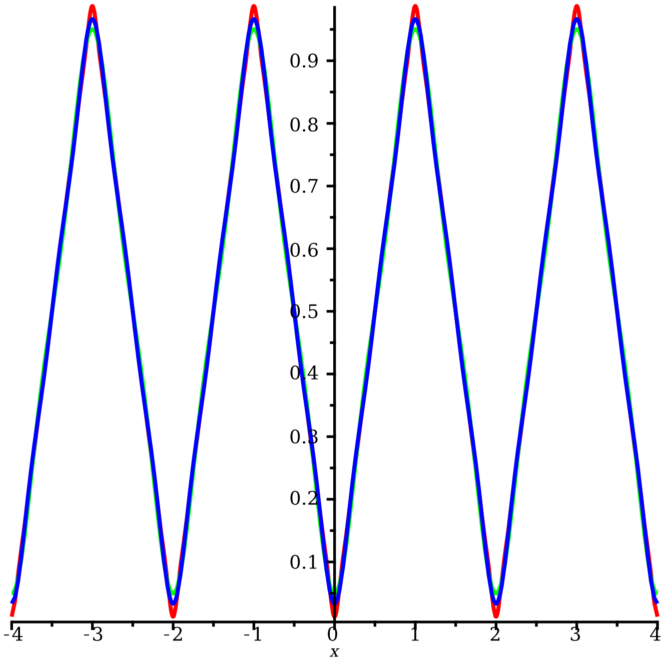
> $f2 := (x, nmax) \rightarrow \frac{1}{2} + add\left(\frac{2}{(\mathrm{Pi}\cdot n)^2} \cdot ((-1)^n - 1) \cdot \cos(n \cdot \mathrm{Pi} \cdot x), n = 1..nmax\right);$

Warning, (in f2) \`n\` is implicitly declared local

$$f2 := (x, nmax) \mapsto \frac{1}{2} + add\left(\frac{2 \cdot ((-1)^n - 1) \cdot \cos(n \cdot \pi \cdot x)}{n^2 \cdot \pi^2}, n = 1..nmax\right) \quad (4)$$

Below is a plot of three instances of the fourier series solution f2(x). Note that there is no ringing in this case, as there are no discontinuities, and that convergence is much faster (i.e., the solution with just 3 terms is very close to the true solution). Also note that convergence is much smoother: The solution doesn't oscillate at high frequency around the function f2a(x).

> $plot([f2(x,15), f2(x,3), f2(x,5)], x = -4..4, colour = [red, green, blue]);$



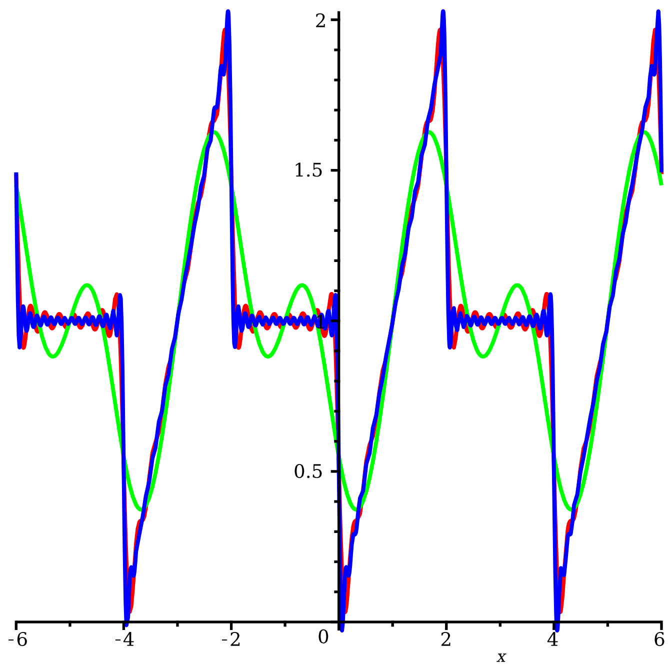
> $f3 := (x, nmax) \rightarrow 1 + add\left(\frac{2}{(n \cdot \mathrm{Pi})^2} \cdot \left((-1)^n - 1\right) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{2}\right), n = 1..nmax\right) + add\left(\frac{1}{n \cdot \mathrm{Pi}} \cdot \left(-1 - (-1)^n\right) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{2}\right), n = 1..nmax\right);$

Warning, (in f3) `n` is implicitly declared local

$$f3 := (x, nmax) \mapsto 1 + add\left(\frac{2 \cdot \left((-1)^n - 1\right) \cdot \cos\left(\frac{n \cdot \pi \cdot x}{2}\right)}{n^2 \cdot \pi^2}, n = 1..nmax\right) + add\left(\frac{\left(-1 - (-1)^n\right) \cdot \sin\left(\frac{n \cdot \pi \cdot x}{2}\right)}{n \cdot \pi}, n = 1..nmax\right) \quad (5)$$

Belowis a plot of three instances of the fourier series solution f3(x). Note the ringing at the discontinuities.

> $plot([f3(x,15), f3(x,3), f3(x,30)], x = -6..6, colour = [red, green, blue]);$


2
1.5
0.5
-6
-4
-2
0
2
4
6
x

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