Assignment #3, Solutions - Maple part

1. We plot the initial condition, extended as an even function *fe(x)*, and the corresponding Fourier cosine series, *g(x)*.

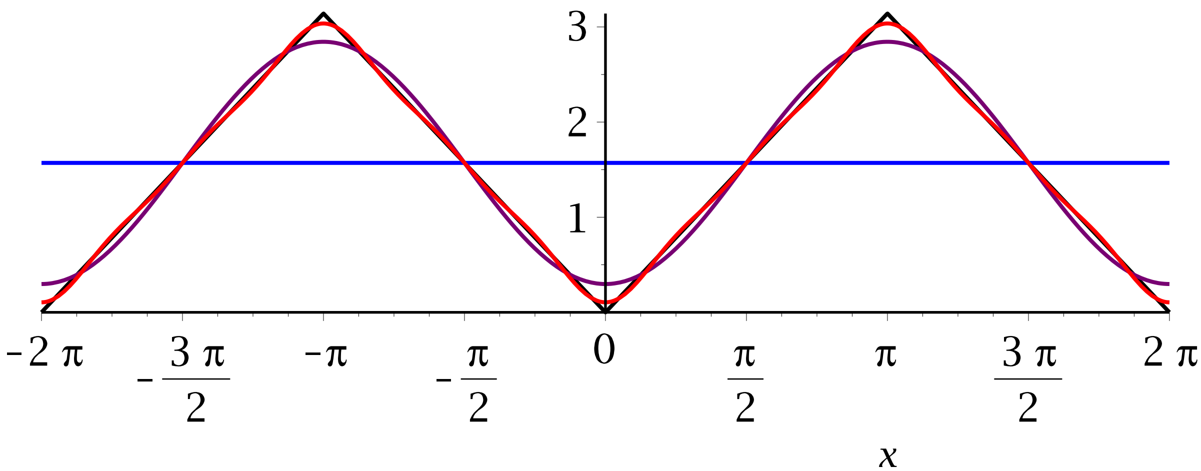
> $fe := x \to piecewise(-2\cdot\text{Pi} < x < 0, -|x+\text{Pi}| + \text{Pi}, 0 < x < 2\cdot\text{Pi}, -|x-\text{Pi}| + \text{Pi});$

$$fe := x \to piecewise(-2\pi < x \text{ and } x < 0, -|x+\pi| + \pi, 0 < x \text{ and } x < 2\pi, -|x-\pi| + \pi) \quad (1)$$

> $g := (x, nmax) \to \frac{\text{Pi}}{2} + sum\left(\frac{4}{n^2\cdot\text{Pi}}\cdot\left(2\cdot\cos\left(\frac{n\cdot\text{Pi}}{2}\right) - (-1)^n - 1\right)\cdot\cos\left(\frac{n\cdot x}{2}\right), n = 1..nmax\right);$

$$g := (x, nmax) \to \frac{1}{2}\pi + \sum_{n=1}^{nmax} \frac{4\left(2\cos\left(\frac{1}{2}n\pi\right) - (-1)^n - 1\right)\cos\left(\frac{1}{2}nx\right)}{n^2\pi} \quad (2)$$

> $plot([fe(x), g(x, 1), g(x, 3), g(x, 10)], x = -2\cdot\text{Pi}..2\cdot\text{Pi}, colour = [black, blue, purple, red]);$



The plot above shows that the Fourier coefficients have been calculated correctly. Now we plot the full solution *u(x,t)*, below.

> $u := (x, t, nmax) \to \frac{\text{Pi}}{2} + sum\left(\frac{4}{n^2\cdot\text{Pi}}\cdot\left(2\cdot\cos\left(\frac{n\cdot\text{Pi}}{2}\right) - (-1)^n - 1\right)\cdot\cos\left(\frac{n\cdot x}{2}\right)\cdot\exp\left(-\frac{n^2\cdot t}{20}\right), n = 1..nmax\right);$

$$u := (x, t, nmax) \to \frac{1}{2}\pi + \sum_{n=1}^{nmax} \frac{4\left(2\cos\left(\frac{1}{2}n\pi\right) - (-1)^n - 1\right)\cos\left(\frac{1}{2}nx\right)e^{-\frac{1}{20}n^2 t}}{n^2\pi} \quad (3)$$

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> plot3d(u(x, t, 10), x = 0..2·Pi, t = 0..10);
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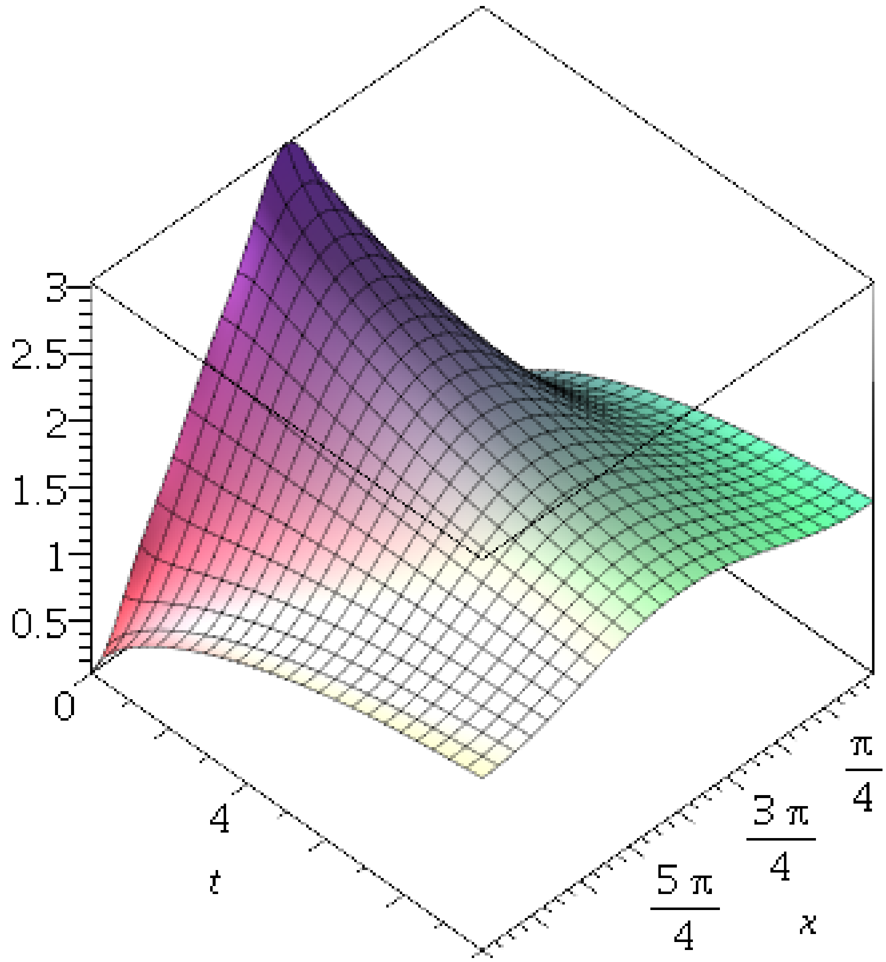

We see that the solution starts with the inverted V shape of the initial condition, and then gradually smooths out, tending toward a uniform distribution. This solution is consistent with physical intuition: The equations define a rod with insulated ends and containing an initial amount of heat distributed in an inverted V shape (the hottest point is in the middle of the rod). Since the ends are insulated, no heat leaves the rod, and so the heat should eventually be uniformly distributed throughout the rod.