Math 319 - Differential Equations II
Assignment # 3

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

1. Find a formal solution for $u(x,t)$ satisfying the initial boundary-value problem defined by

$$\begin{aligned} &\frac{\partial u}{\partial t} = \frac{1}{5}\frac{\partial^2 u}{\partial x^2}, && 0 < x < 2\pi, \quad t > 0, \\ &\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(2\pi,t) = 0, && t > 0, \\ &u(x,0) = f(x), && 0 < x < 2\pi \end{aligned}$$

   where

$$f(x) = -|x-\pi| + \pi, \quad 0 < x < 2\pi.$$

   Use Maple to confirm that your Fourier coefficients are correct, and plot $u(x,t)$ for $0 < t < 10$.

2. Find a formal solution for the initial boundary value problem

$$\begin{aligned} &\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, && 0 < x < \pi, \quad t > 0, \\ &u(0,t) = 0, \quad u(\pi,t) + \frac{\partial u}{\partial x}(\pi,t) = 0, && t > 0, \\ &u(x,0) = g(x), && 0 < x < \pi. \end{aligned}$$

   Your final solution will contain integral expressions for the coefficients in the Fourier series. Note that you need to extend $g(x)$ as appropriate in order to get the correct coefficients. You should arrive at formulas that look like the ones in section 10.4 of your text.

3. Consider the following initial boundary-value problem for the heat equation in a thin bar of length L:

$$\begin{aligned} &\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + P(x), && 0 < x < L, \quad t > 0, \\ &u(0,t) = K, \quad \frac{\partial u}{\partial x}(L,t) = u(L,t), && t > 0, \\ &u(x,0) = h(x), && 0 < x < L. \end{aligned}$$

   (a) Give physical interpretations for the boundary conditions at $x = 0$, $x = L$, and at $t = 0$.
   (b) Find a formal solution for $u(x,t)$.

4. Find a formal solution to the given boundary value problem.

$$\begin{aligned} &\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, && 0 < x < a, \quad 0 < y < b, \\ &u(x,0) = u(x,b) = 0, && 0 < x < a, \\ &u(a,y) = 0, \quad \frac{\partial u}{\partial x}(0,y) = -1, && 0 < y < b. \end{aligned}$$