Math 319 - Differential Equations II
Pre-Reading Assignment # 18
due 10am Thu Nov 20th, via email

Reading In section 11.3 we learned that Sturm-Liouville operators have a special property, shown in equation (13), that we call being "selfadjoint". If the operator L is selfadjoint, then L and its adjoint are the same operator. We also learned that every Sturm-Liouville operator is selfadjoint.

On the other hand, even though every linear second order homogeneous ODE can be transformed into Sturm-Liouville form, not every second order linear differential operator is selfadjoint. In class last Tuesday (Nov 18th), we worked through an example showing that the operator $L[y] := y'' + 2y' + (1 + \lambda^2)y$ is not selfadjoint.

So how do we extend this idea of an adjoint operator to other linear second order differential operators? In tutorial today, you saw that for any general second order linear differential operator (i.e. equation (1) in section 11.4) we can define a formal adjoint operator L^+ (equation (3) in section 11.4).

Questions Note that with this new definition of an adjoint operator, we now have two versions of Lagrange's Identity: the one given by Theorem 1 in section 11.3, and the one given by Theorem 9 in section 11.4. The second version should reduce to the first version in the case where L is selfadjoint. In Pre-Reading Assignment #17 you showed that L is selfadjoint if $A_1 = A_2'$. Use this same substitution to show that the right hand side of equation (9) in section 11.4 is the same as the right hand side of equation (9) in section 11.3 (i.e. show that $P(u, v) = pW[u, v]$ when $A_1 = A_2'$).