Math 319 - Differential Equations II
Pre-Reading Assignment # 15
due 10am Tue Nov 4th, via email

Note: Hopefully this assignment is arriving early enough that you have time to finish it, and late enough that you aren't in any danger of forgetting to do it!

Reading Class notes from last Thursday.

Questions In class last Thursday we introduced Sturm-Liouville Theory and started solving the regular Sturm-Liouville problem

$$(xy')' + \frac{\lambda}{x}y = 0, \quad 1 < x < e \tag{1}$$

$$y'(1) = 0, \quad y(e) = 0. \tag{2}$$

We expanded the ODE, found the characteristic equation, and determined that there were three cases to be considered: $\lambda < 0$, $\lambda = 0$, and $\lambda > 0$. We found that the first two cases yielded only trivial solutions.

For the last case, we set $\lambda = \mu^2$, and found that the roots of the characteristic equation were $r = \pm i\mu$. Recalling that the solutions of the ODE are x^r, finish solving the problem as follows:

1. Show that the two linearly independent solutions of the ODE corresponding to the roots for $\lambda = \mu^2$ are $y_1 = x^{i\mu}$ and $y_2 = x^{-i\mu}$.
2. Show that $x^{i\mu} = \cos(\mu \ln(x)) + i \sin(\mu \ln(x))$. *Hint: If you convert your base x to e, then you can use Euler's formula (the one that we used in chapter 4 when finding the real-valued solutions when the roots of the characteristic equation were complex).*
3. Form the general solution to the ODE.