Math 319 - Differential Equations II
Assignment # 4
due by NOON on Fri Oct 31st
in the Document Holder beside SCI 386

Special Note: Please put your assignment in the document holder on the wall outside my office, and NOT under my office door!

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

The assignment may be done with up to 4 other classmates (i.e. total group size: no more than 5). If you collaborate with classmates, the group should hand in one document with all contributing names at the top.

1. Using variation of parameters, solve the ODE

$$\frac{d^2u_n(t)}{dt^2} + \left(\frac{n\pi\alpha}{L}\right)^2 u_n(t) = h_n(t),$$

for arbitrary coefficient functions $h_n(t)$.

2. Consider the vibrating string problem

$$\frac{\partial u^2}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \quad t > 0 \tag{1}$$

$$u(0,t) = u(1,t) = 0, \qquad t > 0, \tag{2}$$

$$u(x,0) = \sin(3\pi x) - 5\sin(8\pi x) + \frac{10}{3}\sin(12\pi x), \qquad 0 < x < 1, \tag{3}$$

$$\frac{\partial u}{\partial t}(x,0) = 0, \qquad 0 < x < 1. \tag{4}$$

   (a) Solve for $u(x,t)$. *Note: For the spatial BVP, you may assume that the eigenvalue is positive.*

   (b) What method did you use to solve for the Fourier coefficients? What property of the eigenfunctions allowed you to do this?

3. Solve the PDE problem given in section 10.6 # 2. Find the Fourier coefficients for the series solution and give the final answer for $u(x,t)$. *Note: You may start from equation (5) in the text (p 611).*

4. Consider the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}, \qquad -\infty < x < \infty, \quad t > 0, \tag{5}$$

$$u(x,0) = \begin{cases} \cos(\pi x), & \text{if } -1 < x < 1, \\ 0 & \text{else,} \end{cases} \qquad -\infty < x < \infty, \tag{6}$$

$$\frac{\partial u}{\partial t}(x,0) = 0, \qquad -\infty < x < \infty. \tag{7}$$

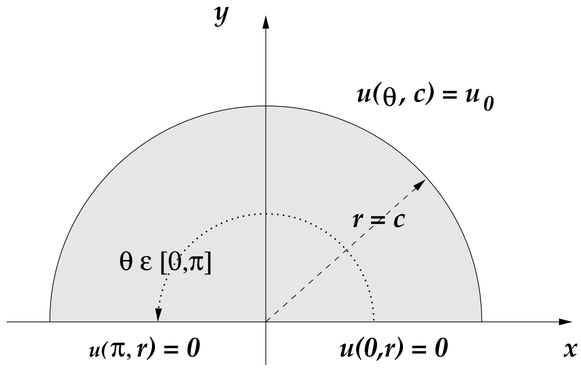

Figure 1: Semicircular plate for problem 6.

(a) Find the solution.

(b) Plot the solution at $t = 0$, 1 and 2.

5. Given

$$u(x,t) = \sum_{n=0}^{\infty} c_n \cos(nx) e^{-\alpha n^2 t},$$

where

$$u(x,0) = f(x) = 2\cos(5x) - \frac{7}{3}\cos(8x) + x^2,$$

find the Fourier coefficients c_n and write out the full solution for $u(x,t)$. *Note: Since the eigenvalues are always of the form* $\mu = n\pi x/L$, *you can deduce that* $0 < x < \pi$.

6. Find the steady-state temperature $u(\theta, r)$ in the semicircular plate shown in Figure 1.