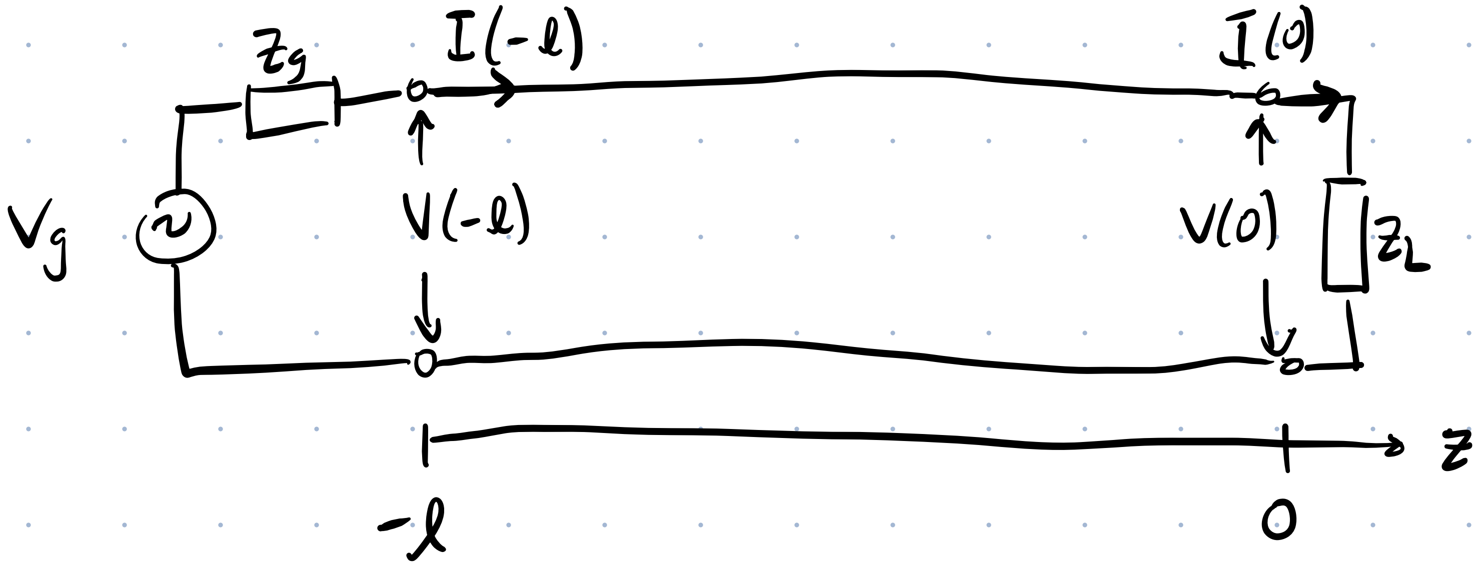
PHYS 331 - Nov. 20, 2023

Useful results from earlier in the term:

1. Transmission line results for amplitude of harmonic voltage & current



Frequency Domain

$$V(z,\omega) = V_+\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right] \equiv \hat{v}(z,\omega)$$ #

$$I(z,\omega) = \frac{V_+}{Z_0}\left[e^{-j\beta z} - \Gamma e^{j\beta z}\right] \equiv \hat{i}(z,\omega)$$ *

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \beta = \omega\sqrt{L_\ell C_\ell} = \frac{\omega}{s}$$

prop. speed.

② Inverse Fourier Transforms

(a) $f(t) = F^{-1}\left[\hat{f}(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega) e^{j\omega t} d\omega$

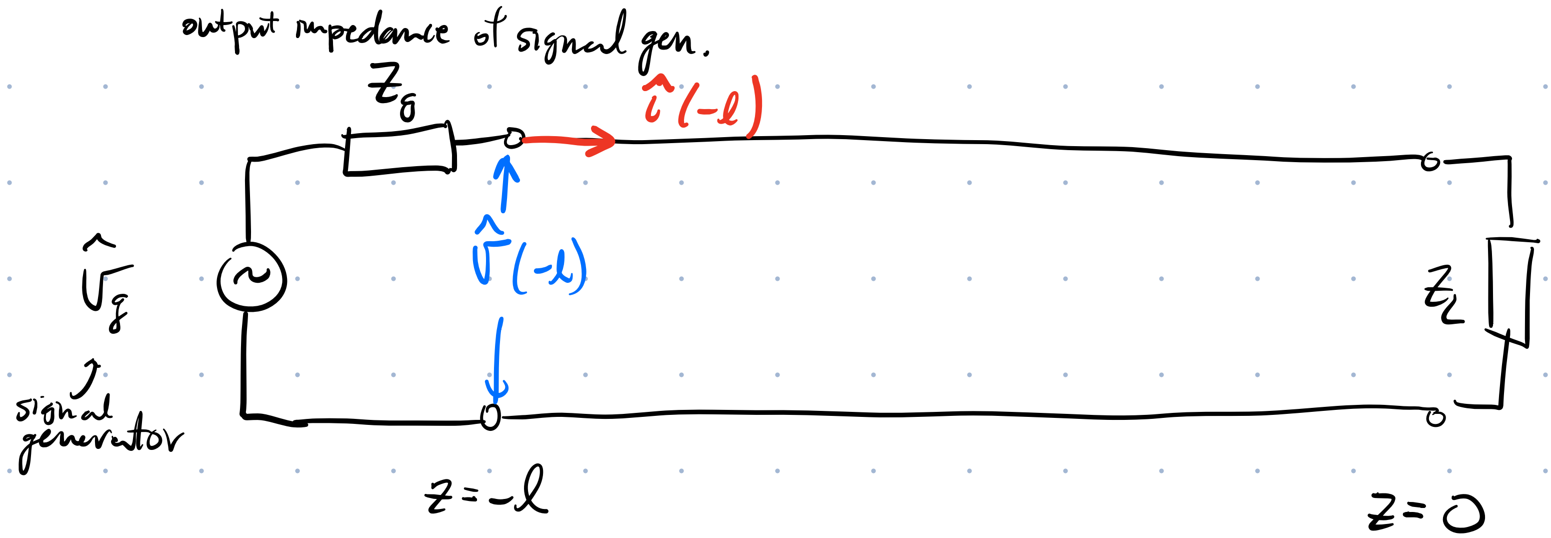
(b) Delta fcn: $\delta(t-t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega(t-t_0)} d\omega$

(c) Convolution Theorem

$$F^{-1}\left[\hat{x}_1(\omega)\,\hat{x}_2(\omega)\right] = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

Goal is to calc. transient response (time domain) of a transmission line.
Strategy: Employ the freq. domain results that we already know. Then take inverse Fourier transform to convert to time domain.



Track changes in volt. from $\hat{v}_g$ to $\hat{v}(-l)$:

$$\hat{v}_g - \underbrace{\hat{i}(-l)}_{\text{use } *} Z_g = \underbrace{\hat{v}(-l)}_{\text{use } \#}$$

$\beta = \frac{\omega}{s}$

speed of signal

$$\hat{v}_g - \frac{V_+}{Z_0}\left[e^{j\omega l/s} - \Gamma e^{-j\omega l/s}\right] Z_g$$

$$= V_+\left[e^{j\omega l/s} + \Gamma e^{-j\omega l/s}\right]$$

solve for V_+. We can control all other parameters using signal gen. and/or through our choise of trans. line.

$$\therefore V_+ = \frac{\hat{v}_g}{\left(e^{j\omega l/s} + \Gamma e^{-j\omega l/s}\right) + \cancelto{1}{\frac{Z_g}{Z_0}}\left(e^{j\omega l/s} - \Gamma e^{-j\omega l/s}\right)}$$

Assume that we chose $Z_g = Z_0$ s.t. $\frac{Z_g}{Z_0} = 1$ (typical)

$$V_+ = \frac{\hat{V}_g}{2e^{j\omega l/s}} = \boxed{\frac{\hat{v}_g}{2} e^{-j\omega l/s}}$$

$Z_0 = \sqrt{\frac{L_l}{C_l}}$

For this case ($Z_g = Z_0$), the volt. at the trans. line input in the freq. domain is

$\hat{v}_{in} = \hat{v}(-l)$ Eq. (#) w/ $z = -l$.

$$\hat{v}_{in} = V_+ \left[e^{j\omega l/s} + \Gamma e^{-j\omega l/s} \right]$$

$$\frac{\hat{v}_g}{2} \left[1 + \Gamma e^{-2j\omega l/s} \right] \quad (!)$$

Suppose we meas. the volt. at trans. line input $v_{in}(t)$ w/ an oscilloscope. We expect to meas. the inverse Fourier trans. of $\hat{v}_{in}$.

$$v_{in}(t) = F^{-1}\left[\hat{v}_{in}(\omega) \right]$$

$$v_{in}(t) = F^{-1}\left[\frac{\hat{v}_g}{2}\left(1+\Gamma e^{-2j\omega l/s}\right)\right]$$

$$= \frac{1}{2}\underbrace{F^{-1}\left[\hat{v}_g\right]}_{v_g(t)} + \underbrace{\frac{1}{2}F^{-1}\left[\hat{v}_g \Gamma e^{-2j\omega l/s}\right]}_{I}$$

signal generator output in time domain

Let's assume that Γ has no freq. dependence.
Equiv. to assuming that Z_L is just a resistor (not inductor or cap.)

$$I = \Gamma F^{-1}\left[\underbrace{(\hat{v}_g)}_{\hat{X}_1}\underbrace{(e^{-2j\omega l/s})}_{\hat{X}_2}\right]$$

using convolution theorem; find:

$$I = \Gamma\,(x_1 * x_2)(t)$$

$$= \Gamma\left(F^{-1}(\hat{x}_1) * F^{-1}(\hat{x}_2)\right)(t)$$

$$= \Gamma\left(\underbrace{F^{-1}(\hat{U}_g)}_{U_g(t)} * F^{-1}\left(e^{-2j\omega\ell/s}\right)\right)(t)$$

Deal with $F^{-1}\left(e^{-2j\omega\ell/s}\right)$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-2j\omega\ell/s}\, e^{j\omega t}\, dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega(t-\underbrace{2\ell/s}_{t_0})}\, dt = \delta(t-t_0)$$

$$\therefore I = \Gamma\left(U_g(t) * \delta(t-t_0)\right)(t)$$

$$= \Gamma \int_{-\infty}^{\infty} V_g(\tau)\, \delta\left[(t-t_0)-\tau\right] d\tau$$

$$\left\{ \text{Recall } \int f(t)\, \delta(t-a)\, dt = f(a) \right.$$

δ-fcn selects t s.t.

$$\left. t-a=0 \quad \therefore \; t=a \right\}$$

δ-fcn select τ s.t. $(t-t_0)-\tau = 0$

$$\therefore \; \tau = t-t_0$$

$$\therefore I = \Gamma V_g(t-t_0) = \Gamma \underbrace{V_g(t-2l/s)}$$

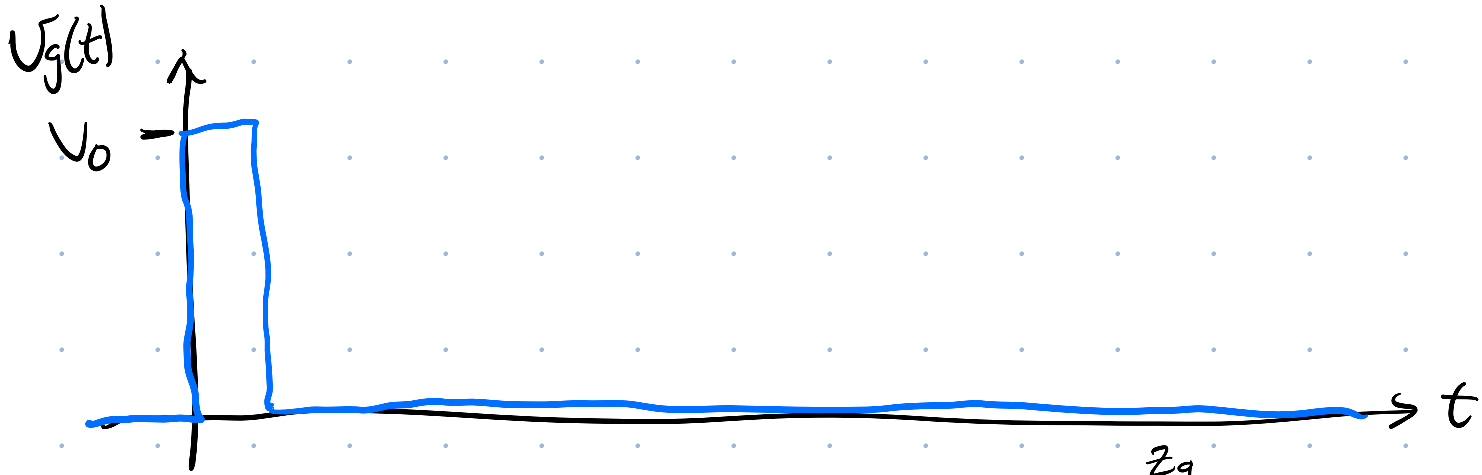
copy of fcn gen. output delayed in time by an amount $2l/s$.

Return to

$$\boxed{V_{in}(t) = \frac{1}{2} V_g(t) + \frac{1}{2} \Gamma V_g(t-2l/s)}$$

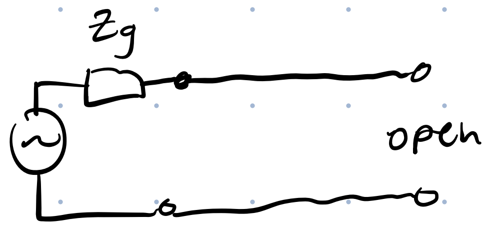
Intrepret this result:

Suppose $V_g(t)$ is a square pulse



Case 1

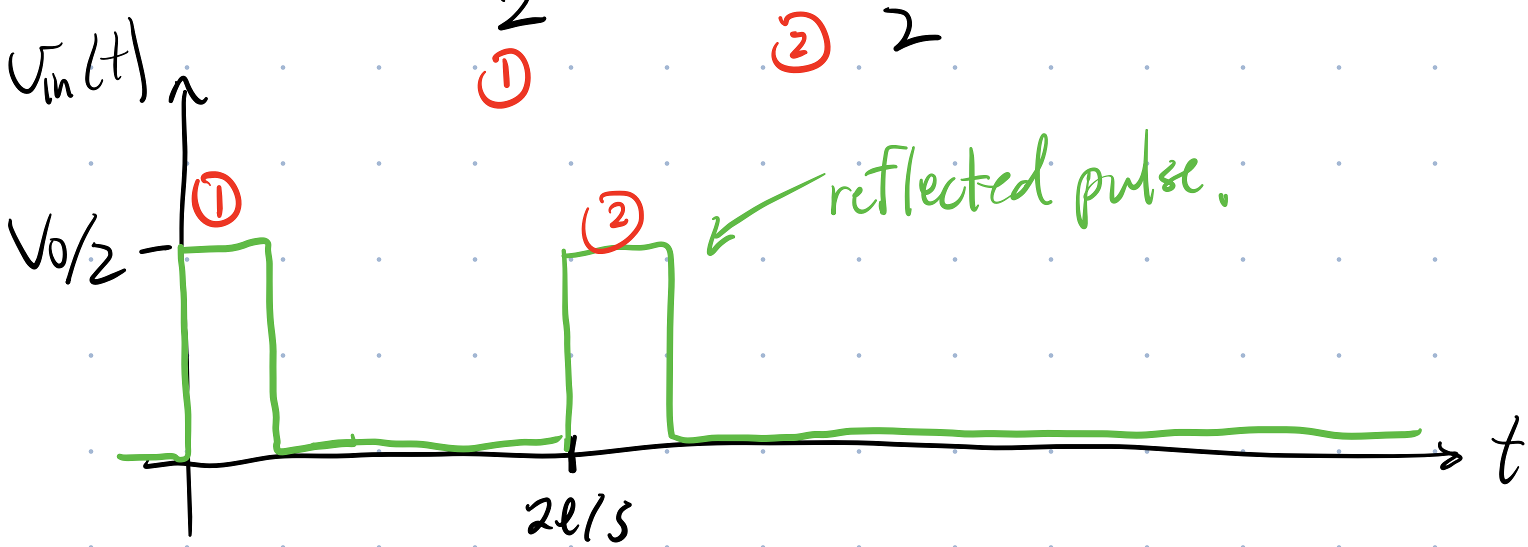
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$



$Z_L \to \infty$ (open circuit)

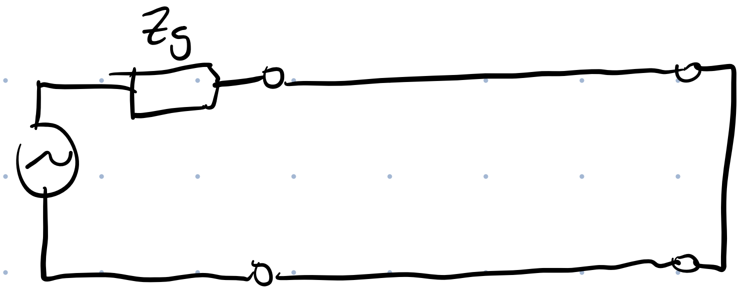
$\Gamma = 1$

$$V_{in}(t) = \underbrace{\frac{V_g(t)}{2}}_{1} + \underbrace{\frac{V_g(t - 2l/s)}{2}}_{2}$$

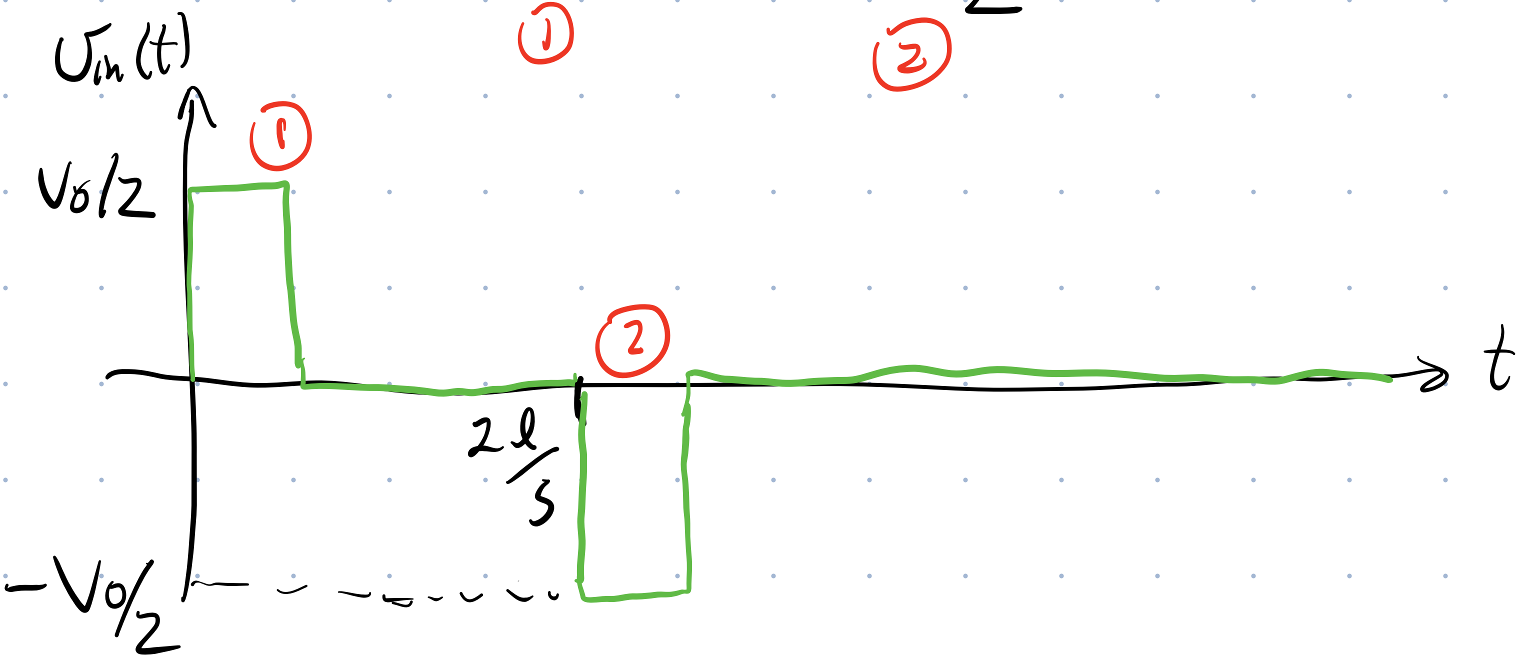


Case 2 $\quad Z_L = 0$ (short circuit)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

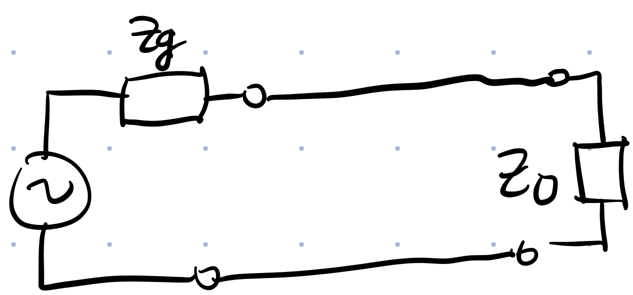


$$V_{in}(t) = \underbrace{\frac{V_g(t)}{2}}_{1} - \underbrace{\frac{V_g(t - 2l/s)}{2}}_{2}$$



Case 3 $\quad Z_L = Z_0$ (50 Ω)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$



Here $v_{in}(t) = \dfrac{v_g(t)}{2}$