PHYS 331 Sept. 19, 2024

<u>Last Time</u>: Lock-in detector can meas. small ac signals in the presence of noise by:

- multiplying V_s by a ref. signal V_r of know freq.
- passing the multiplied signal through a low-pass filter to remove high-freq term.

  Recall $V_{out}' \propto \cos[(\omega_s - \omega_r)t + (\phi - \theta)]$
- averaging the filtered signal to suppress freqs in V_s that differ from ω_r, the freq. of V_r
- Manipulating the phase θ of V_r to determine X & Y.

<u>Question</u>: How well can the lock-in detector discriminate between frequencies that deviate from ω_r?

Start by considering $V_{out}' \propto \cos[(\omega_s - \omega_r)t]$

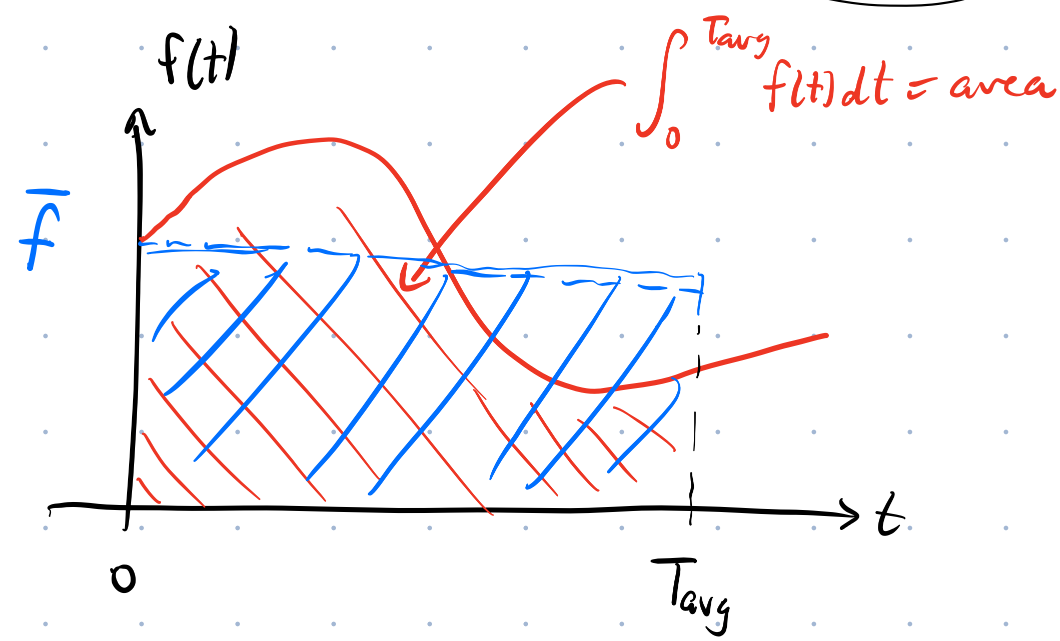
after low-pass filter, but before averaging

$$\bar{f} = \frac{1}{T_{avg}} \int_0^{T_{avg}} f(t)\,dt$$

T_{avg} is averaging time.

$\bar{f}$ is time avg. of $f(t)$ between 0 & T_{avg}.

Aside:



Blue area: $\bar{f}\,T_{avg} = \int_0^{T_{avg}} f(t)\,dt$

$$\Rightarrow \bar{f} = \frac{1}{T_{avg}} \int_0^{T_{avg}} f(t)\,dt$$

$\therefore$ To average v_{out}', need to consider

$$\langle v_{out}' \rangle \propto \frac{1}{T_{avg}} \int_0^{T_{avg}} v_{out}' \, dt$$

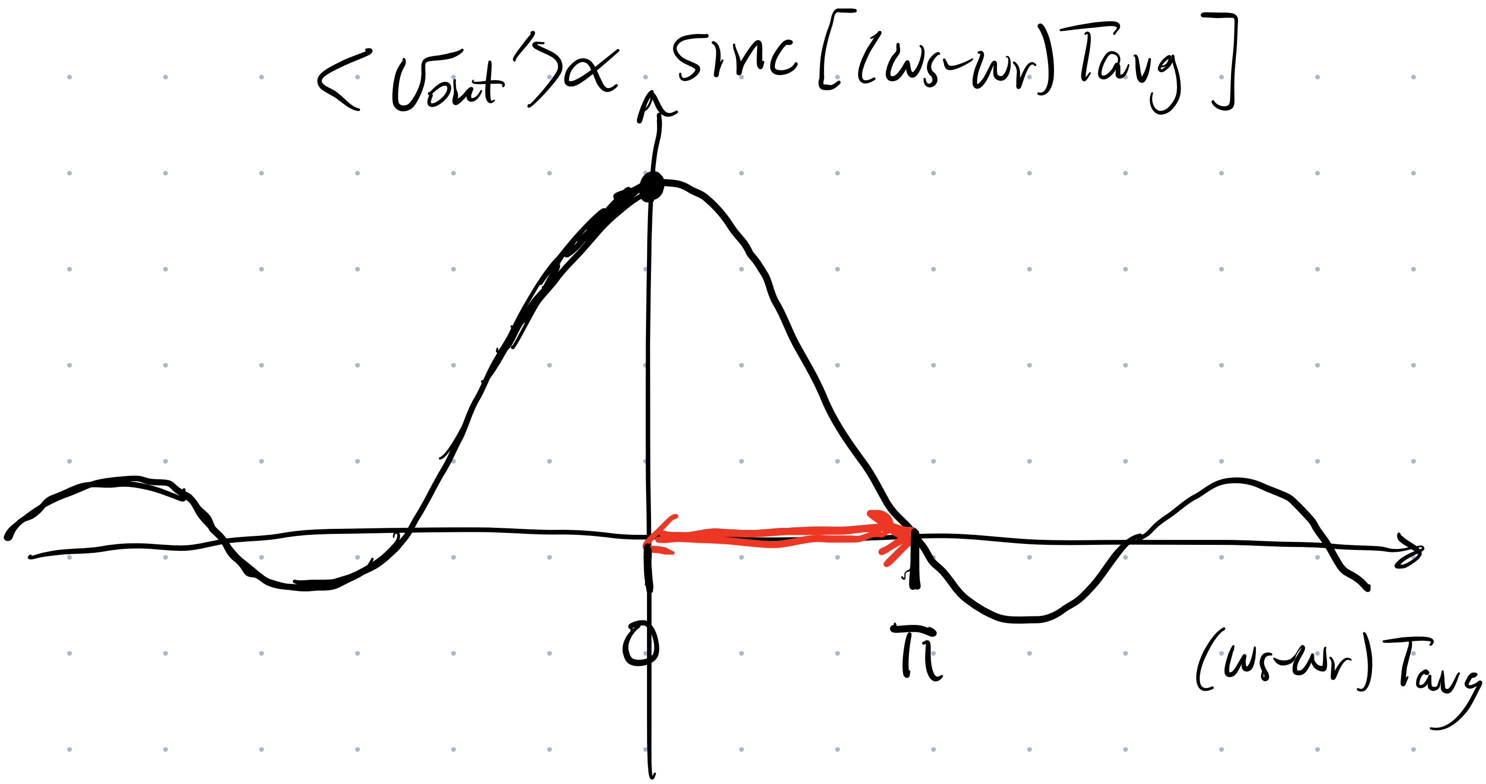
$$= \frac{1}{T_{avg}} \int_0^{T_{avg}} \cos\left[(\omega_s - \omega_r) t\right] dt$$

$$= \frac{1}{T_{avg}(\omega_s - \omega_r)} \sin\left[(\omega_s - \omega_r) t\right] \Big|_0^{T_{avg}}$$

$$\therefore \langle v_{out}' \rangle \propto \frac{\sin\left[(\omega_s - \omega_r) T_{avg}\right]}{(\omega_s - \omega_r) T_{avg}}$$

$$= \text{sinc}\left[(\omega_s - \omega_r) \underbrace{T_{avg}}_{\text{averaging time.}}\right]$$

sinc fcn.



In order to characterize the width of $\langle V_{out}' \rangle$, find the values of $(\omega_s-\omega_r)T_{avg}$ that produce the first zero in the sinc fcn.

The first zero crossing occurs when

$$(\omega_s-\omega_r)T_{avg} = \pi$$

$$\therefore T_{avg} = \frac{\pi}{\omega_s-\omega_r} = \frac{\cancel{\pi}}{2\cancel{\pi}\underbrace{(f_s-f_r)}_{\Delta f}}$$

$$\therefore T_{avg} = \frac{1}{2\Delta f} \Rightarrow \Delta f = \boxed{\frac{1}{2T_{avg}}}$$

range of freq. accepted
by lock-in dector after
averaging for a time T_{avg}.

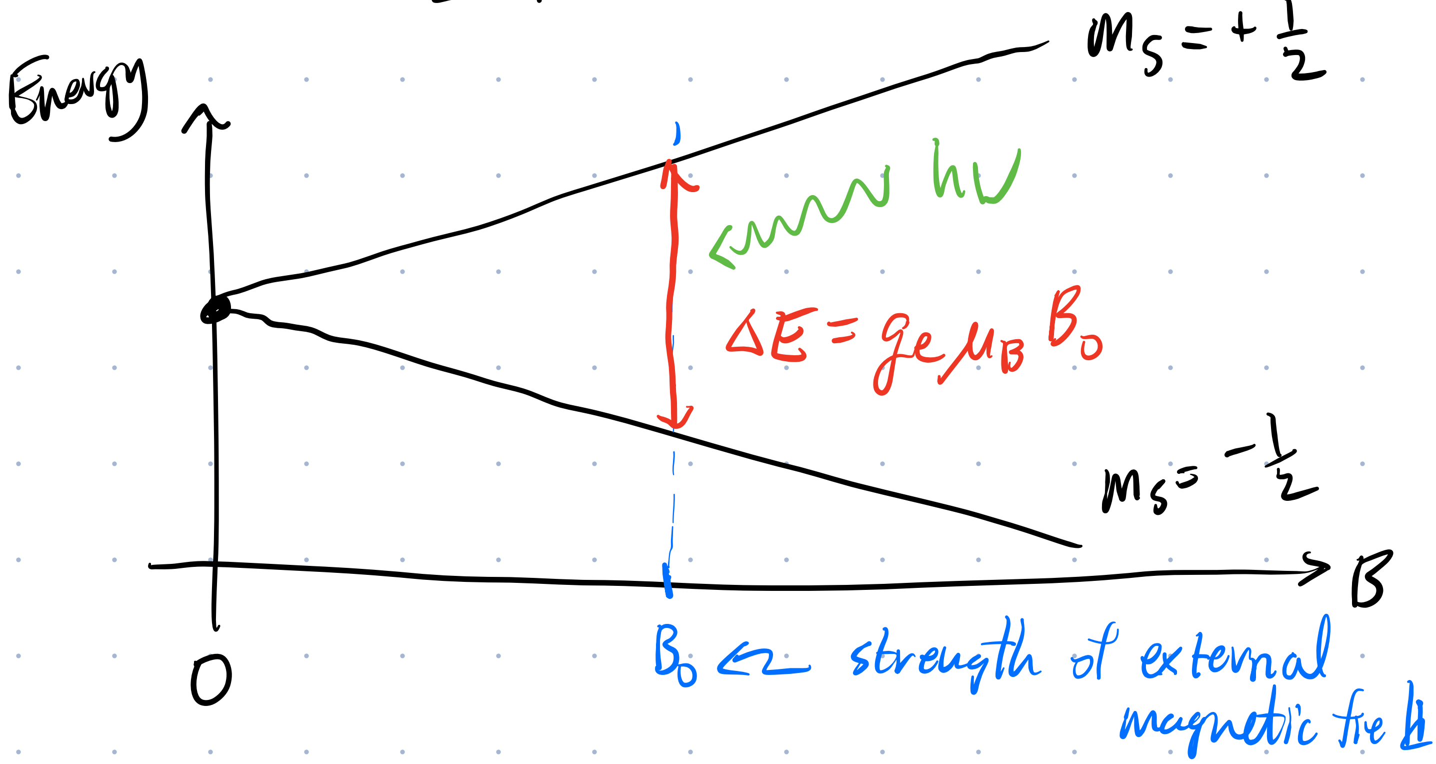
To suppress signal leakage from freq. diff. from ω_r, should increase T_{avg}.

In our example of an RC circuit, we supplied a sinusoidal input & detected a sinusoidal output.

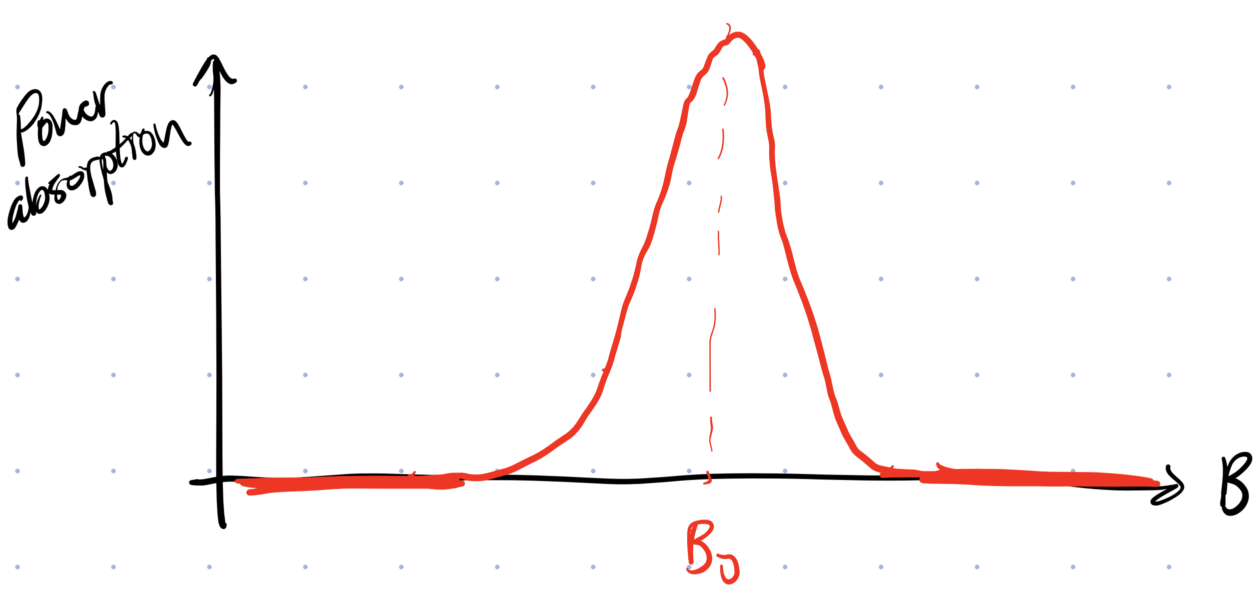
Not all experiments can be driven by a pure sine wave. Consider, for example, an Electron Spin Resonance (ESR) experiment.

Consider a system of spin-$\frac{1}{2}$ electrons w/ spin angular momentum $m_s = \pm \frac{1}{2}$.

In an external magnetic field applied along the z-axis, the energy levels of $m_s = -\frac{1}{2}$ and $m_s = +\frac{1}{2}$ split.



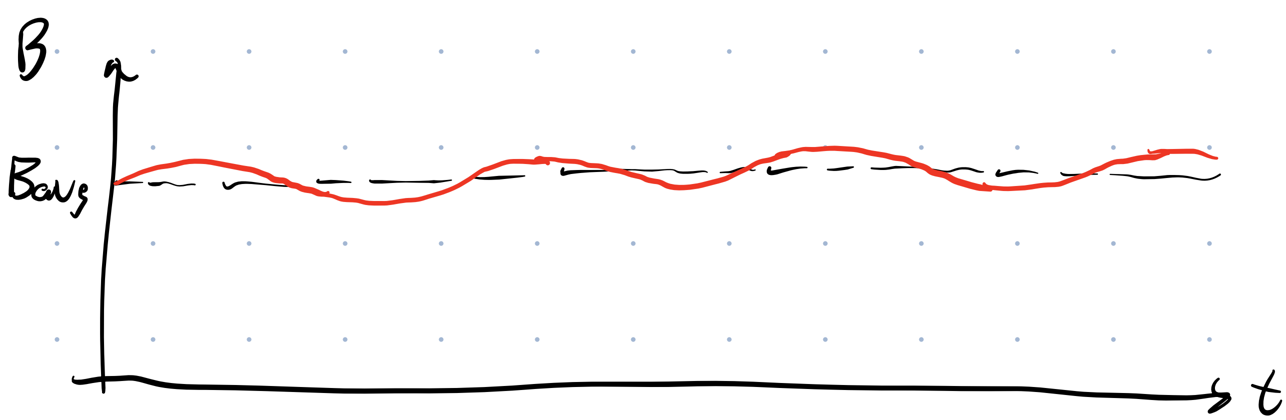
If external field is set to B_0, then a sample irradiated w/ EM waves w/ energy $h\nu = \Delta E$, can cause transitions between the two states.



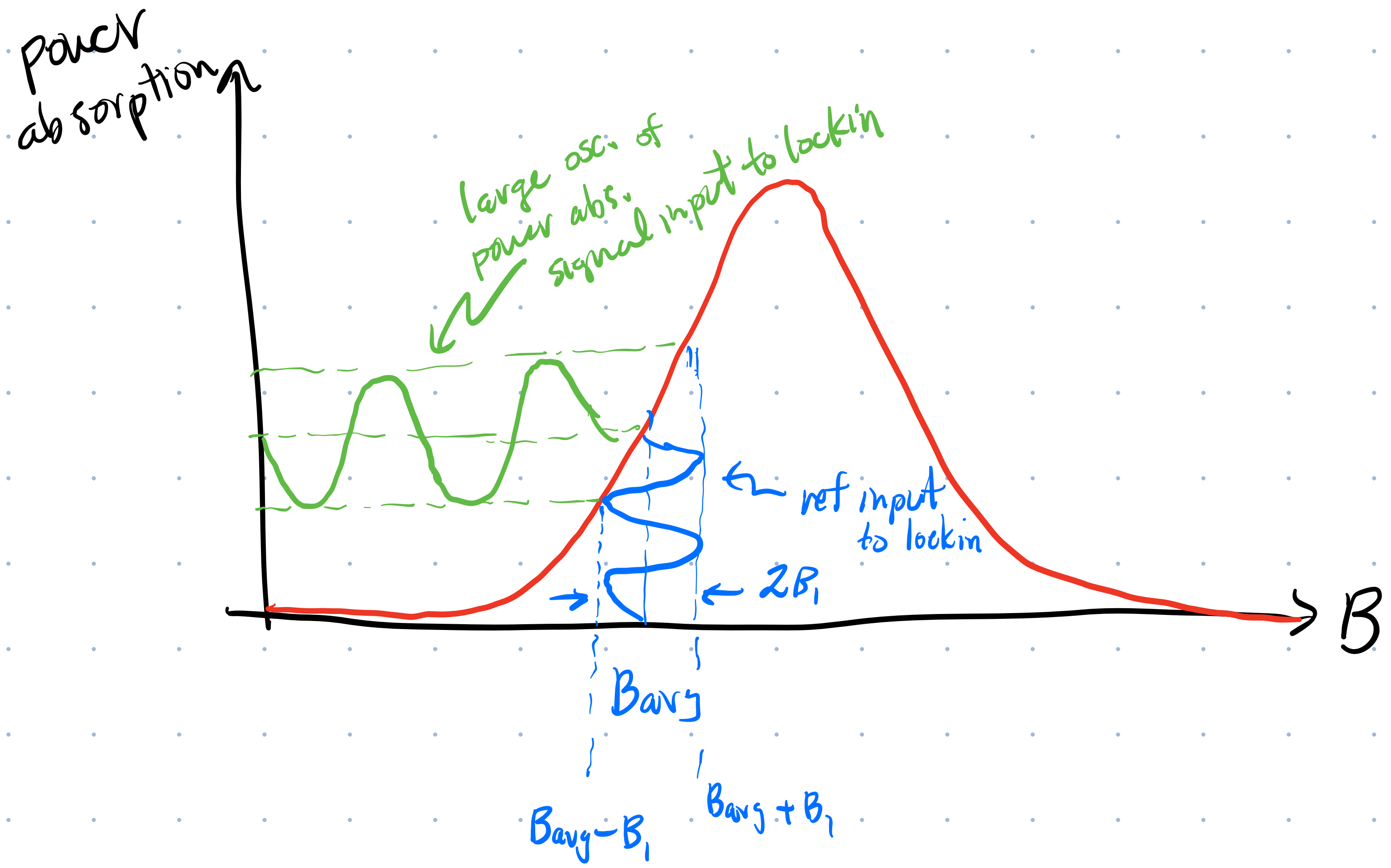
Challenge: The absorption peak can be weak & hard to meas. Need sensitive exptl setup → use lock-in detection.

We need to introduce a sinusoidal signal into expt so that we can meas. a sinusoidal response w/ lock-in.

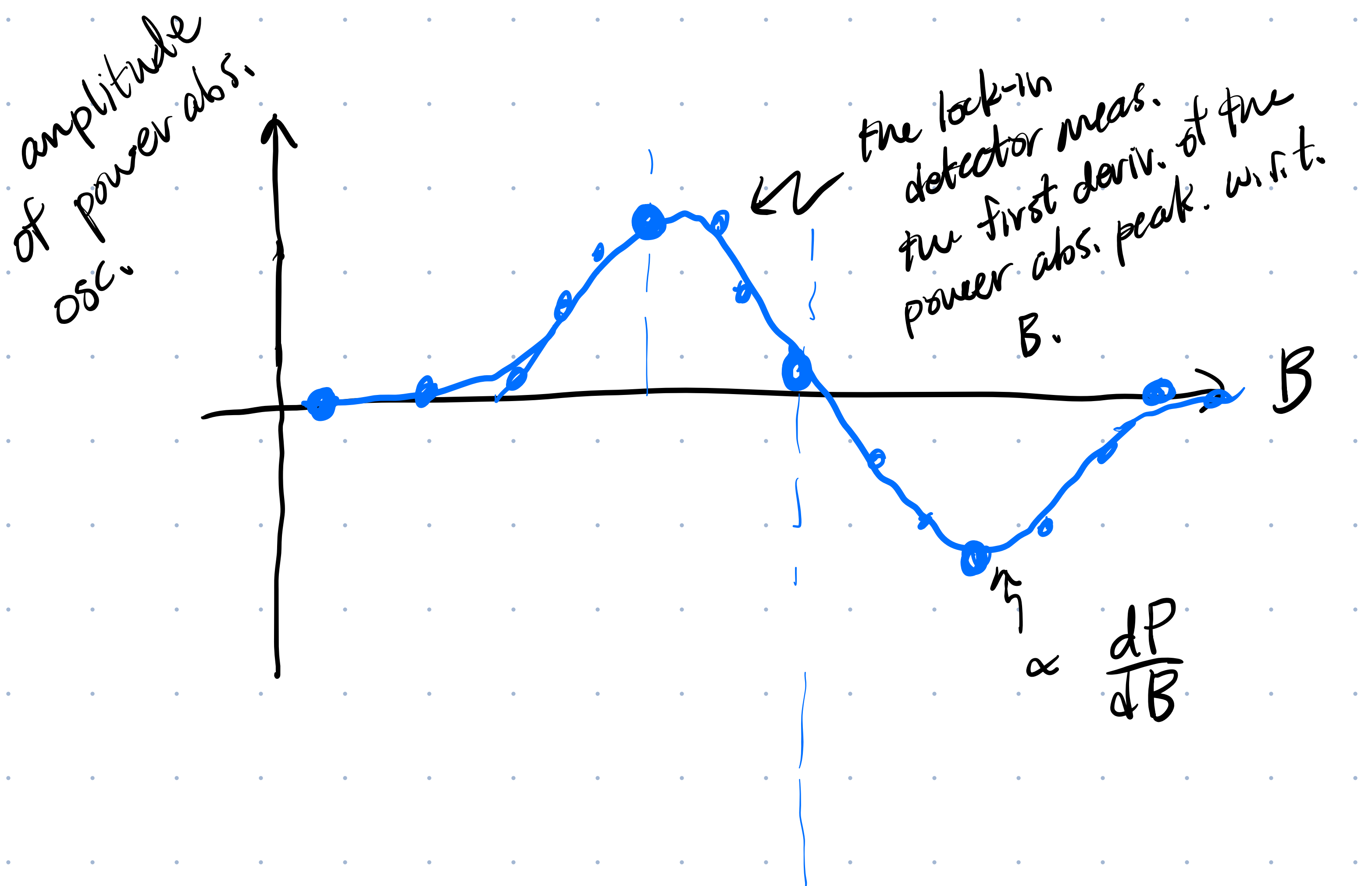
Idea is to add a small ac component on top of avg. or DC magnetic field B.



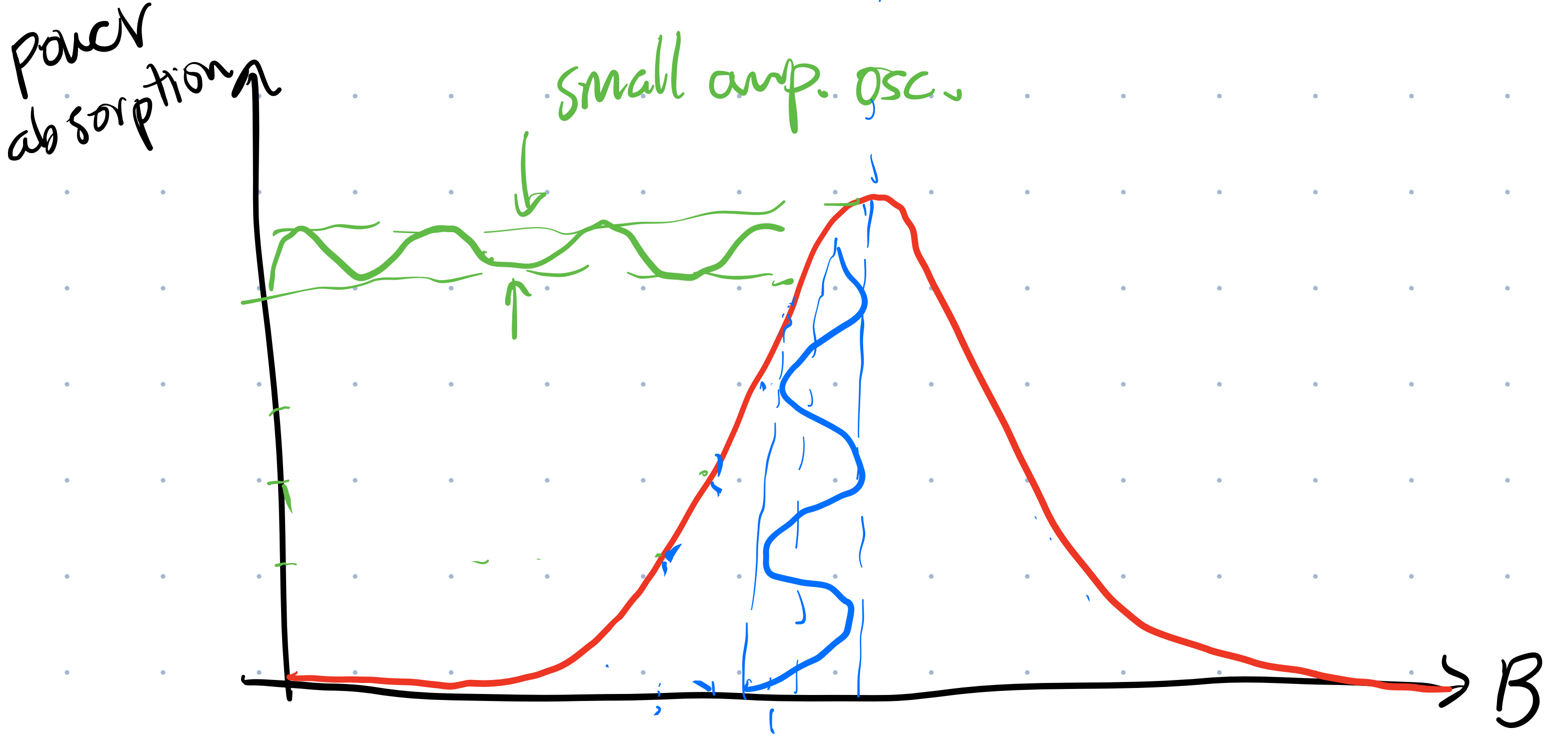
$$B = B_{avg} + \underbrace{B_1 \sin(\omega t)}_{\text{reference signal}} \qquad B_1 \ll B_{avg}$$



power absorption
large osc. of power abs.
signal input to lockin
ref input to lockin
$2B_1$
B
B_{avg}
$B_{avg}-B_1$
$B_{avg}+B_1$



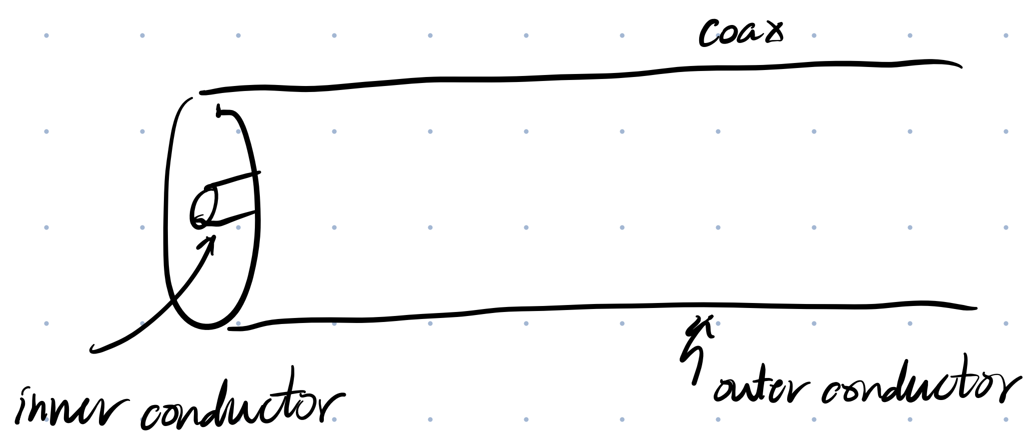
amplitude of power abs. osc.
the lock-in detector meas. the first deriv. of the power abs. peak. w.r.t. B.
B
$\propto \frac{dP}{dB}$

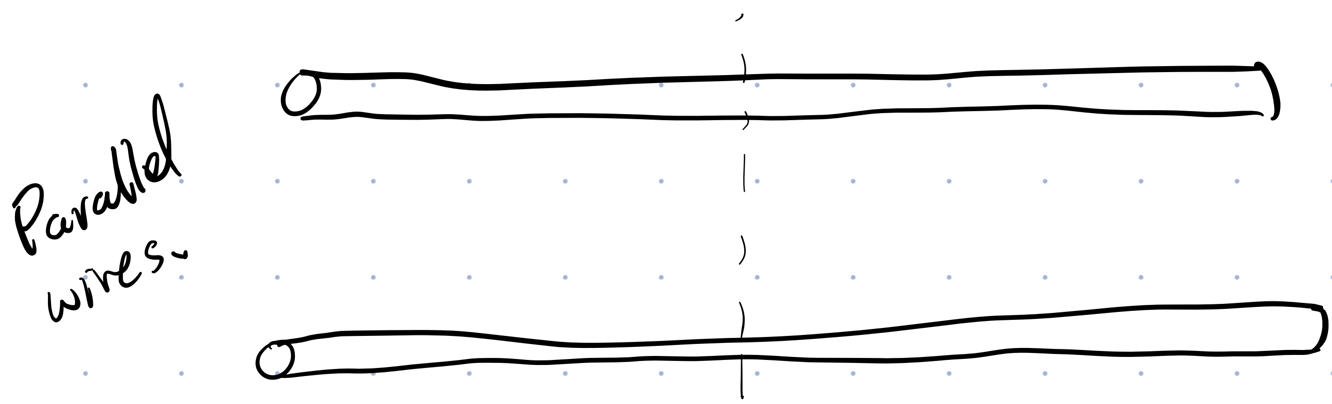


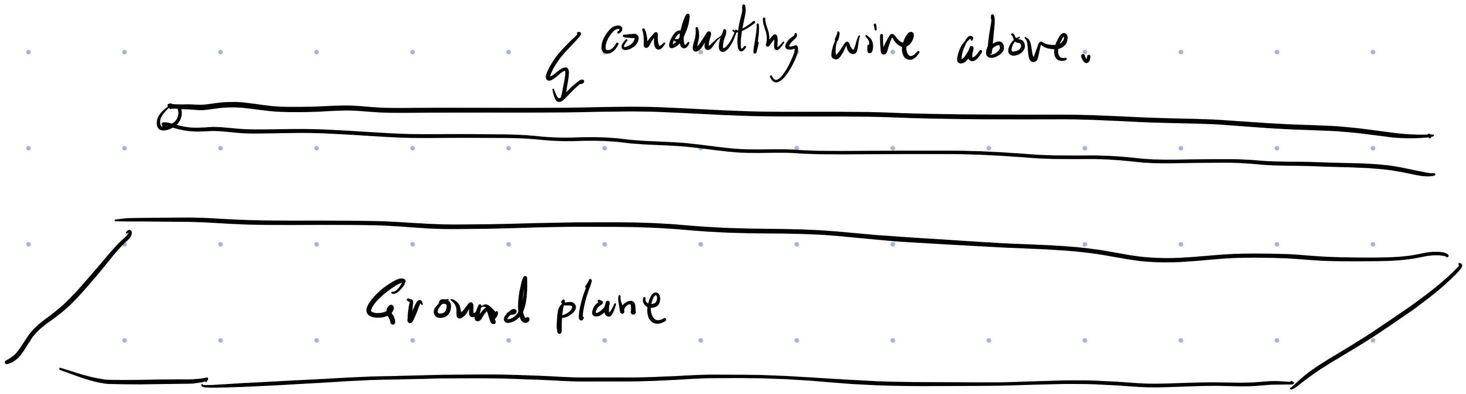
Power absorption
small amp. osc.
B

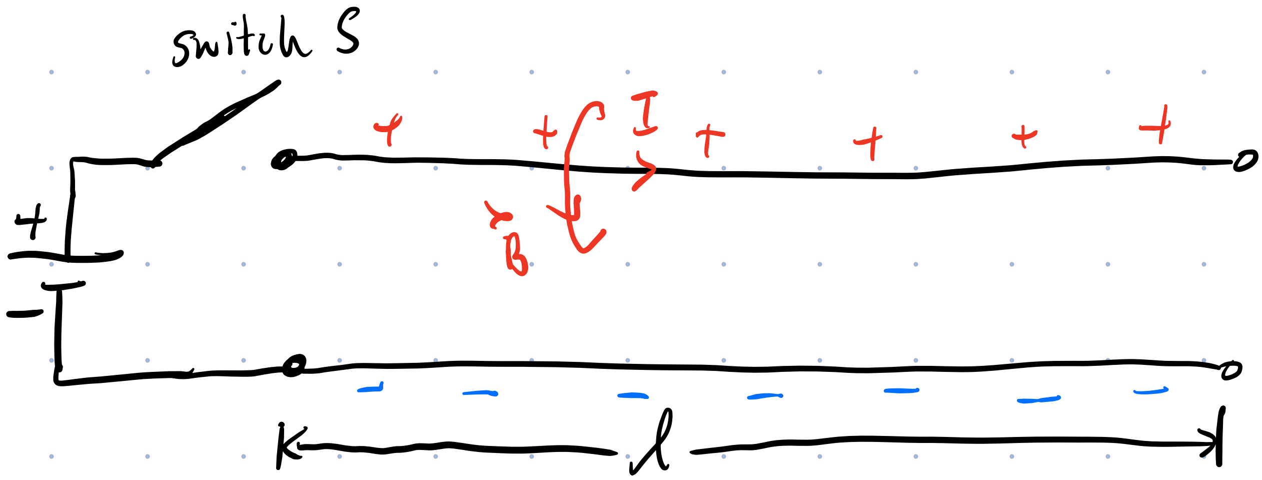
Transmission Lines

Pair of parallel conductors that maintain their geometry along their length.









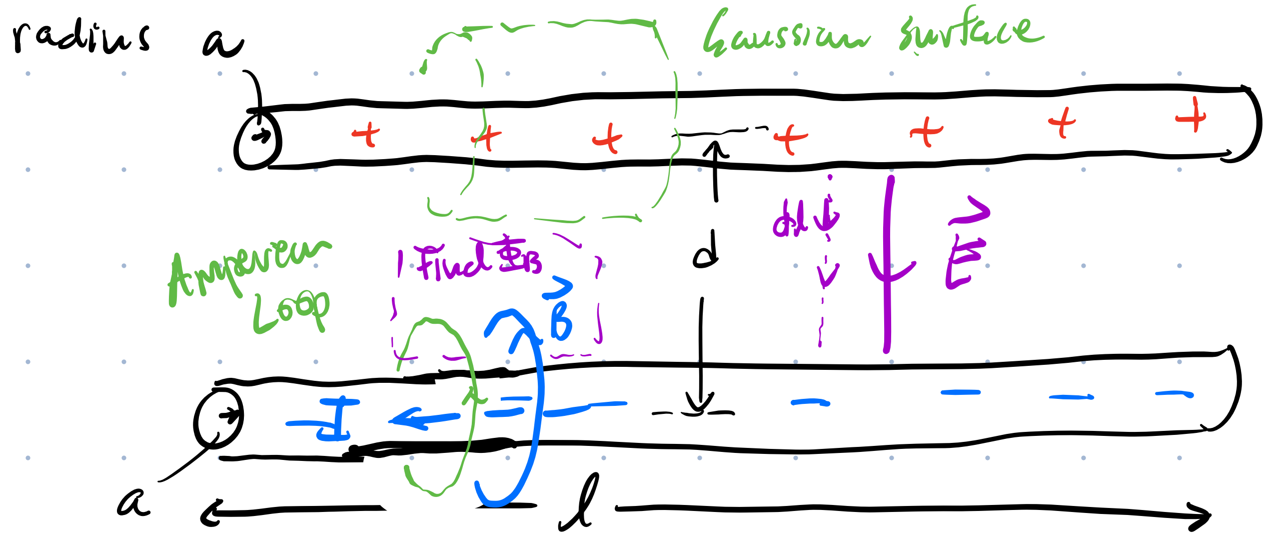
When switch is closed, get a pot. diff. ΔV across wires. → Wires store charge q → Capacitance

$$C = \frac{q}{\Delta V}$$

When switch is closed, must have a transient current that moves charge from one wire to the other & moves charge to far ends of the transmission line. → inductance

$$V_L = -L\frac{dI}{dt}$$

⇒ transmission has both capacitance & inductance.



Gauss's Law $\quad \oint \vec{E} \cdot d\vec{a} = \dfrac{q_{encl}}{\varepsilon_0}$

$\Delta V = -\int \vec{E} \cdot d\vec{l}$

Find cap. per unit length

$$\frac{C}{l} = C_l = \frac{\pi \varepsilon_0}{\ln\left(\frac{d}{a}\right)}$$

Ampère's Law $\quad \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow$ find $\vec{B}$

$$\Phi_B = \int \vec{B} \cdot d\vec{a}$$

Faraday's Law $\quad V_L = -\dfrac{d\Phi_B}{dt} = -L\dfrac{dI}{dt}$

Find an inductance per unit length

$$\frac{L}{l} = L_l = \frac{\mu_0}{\pi} \ln\left(\frac{d}{a}\right)$$