PHYS 301 Dec. 4, 2024

- Final Exam is $\boxed{\begin{array}{l}\text{Wed. Dec. 18} \\ \text{15:30–18:00} \\ \text{LIB 306}\end{array}}$

You can bring a formula sheet to the final. One 8.5" x 11" (letter-sized) sheet of paper with anything written on it.

You can bring a calculator. Anything that can go online or wirelessly communicate with other devices is fine. Graphing calculators are fine.

Ferromagnetism : Magnetic domain boundaries video

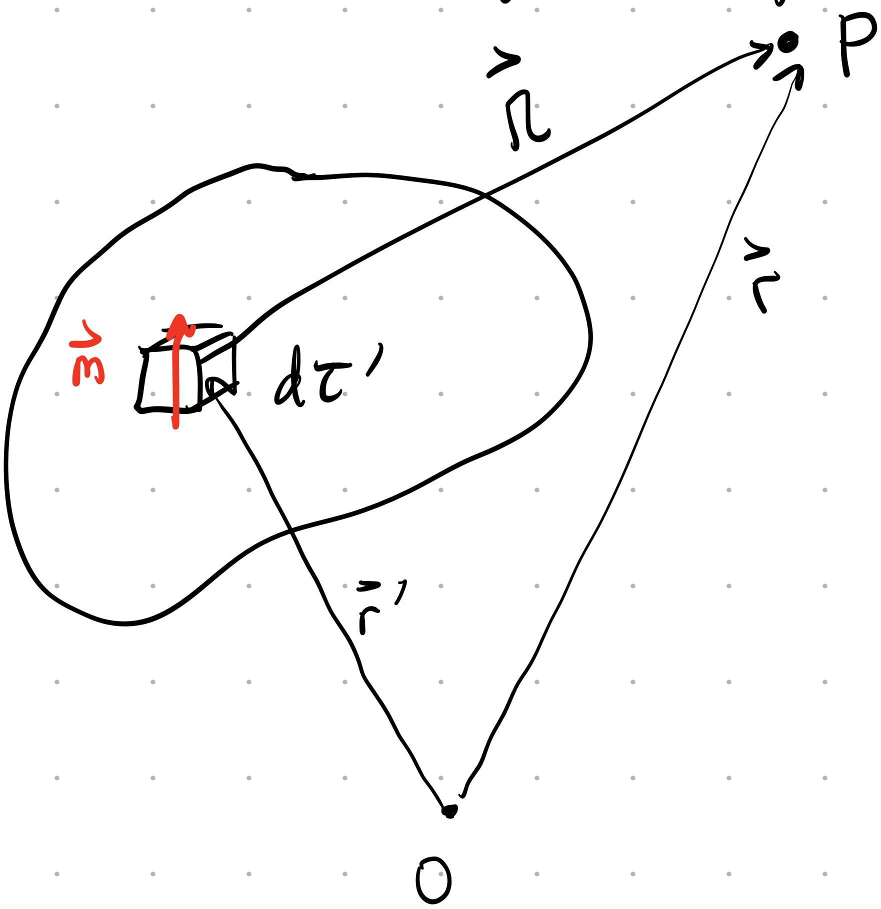
https://www.youtube.com/watch?v=qE_WoV2dYBY

See a video showing the movement of ferromagnetic domain boundaries when the material is immersed in a magnetic field of increasing strength.

Last Time: Paramagnetism - align unpaired electron spins w/ external $\vec{B}$. $\vec{M} \parallel \vec{B}$

Diamagnetism - External $\vec{B}$ alters orbital motion of electrons. $\vec{M}$ antiparallel $\vec{B}$

Consider a magnetized object



$\therefore \vec{A}$ @ P:

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, d\tau'} \quad (*)$$

Like we did for polarization, we aim to re-express ⊗ in a more convenient form. Start by replacing $\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$ with the equivalent $\vec{\nabla}'\frac{1}{\mathscr{r}}$

$$\therefore \vec{A} = \frac{\mu_0}{4\pi}\int \Big[\underbrace{\vec{M}(\vec{r}\,') \times \vec{\nabla}'\left(\tfrac{1}{\mathscr{r}}\right)}_{\text{using product rule (7)}} \Big] d\tau'$$

$$= \frac{1}{\mathscr{r}}\left[\vec{\nabla}' \times \vec{M}(\vec{r}\,')\right] - \vec{\nabla}' \times \left(\frac{\vec{M}(\vec{r}\,')}{\mathscr{r}}\right)$$

$$\vec{A} = \frac{\mu_0}{4\pi}\left\{ \int \frac{1}{\mathscr{r}}\left[\vec{\nabla}' \times \vec{M}(\vec{r}\,')\right] d\tau' - \int \vec{\nabla}' \times \left[\frac{\vec{M}(\vec{r}\,')}{\mathscr{r}}\right] d\tau' \right\}$$

Aside: Show that $\int (\vec{\nabla} \times \vec{v})\, d\tau = -\oint \vec{v} \times d\vec{a}$

If we can prove this result, we can use it to rewrite the second integral in $\vec{A}$ as

$$\oint \frac{\vec{M}(\vec{r}\,')}{\mathscr{r}} \times d\vec{a}\,'$$

Start w/ the divergence theorem $\int \vec{\nabla}\cdot\vec{v}\, d\tau = \oint \vec{v}\cdot d\vec{a}$

write $\vec{v} \to \vec{v}\times\vec{c}$ where $\vec{c}$ is a constant

$$\int \vec{\nabla}\cdot(\vec{v}\times\vec{c})\, d\tau = \oint \vec{v}\times\vec{c}\cdot d\vec{a}$$

by product rule (6)

$$\vec{c}\cdot(\vec{\nabla}\times\vec{v}) - \vec{v}\cdot(\cancel{\vec{\nabla}\times\vec{c}})$$

0

since $\vec{c}$ is const.

$$d\vec{a}\times\vec{v}\cdot\vec{c} = -\vec{c}\cdot\vec{v}\times d\vec{a}$$

$$\therefore\ \vec{c}\cdot\int \vec{\nabla}\times\vec{v}\, d\tau = -\vec{c}\cdot\oint \vec{v}\times d\vec{a}$$

=

$$\therefore\ \int \vec{\nabla}\times\vec{v}\, d\tau = -\oint \vec{v}\times d\vec{a} \quad \blacksquare$$

Using this result in our expression for the vector potential due to our magnetized material gives:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\left\{ \underbrace{\int \frac{1}{\mathscr{r}}\left[\vec{\nabla}' \times \vec{M}(\vec{r}')\right] d\tau'}_{\text{c.f. } \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{\mathscr{r}} d\tau'} + \underbrace{\oint \frac{1}{\mathscr{r}} \vec{M}(\vec{r}') \times d\vec{a}'}_{\text{c.f. } \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}}{\mathscr{r}} da'} \right\}$$

$$\boxed{\vec{J}_b = \vec{\nabla} \times \vec{M}}$$

bound current density

$$\boxed{\vec{K}_b = \vec{M} \times \hat{n}}$$

bound surface current.

Finally

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_b(\vec{r}')}{\mathscr{r}} d\tau' + \frac{\mu_0}{4\pi}\oint_S \frac{\vec{K}_b(\vec{r}')}{\mathscr{r}} da'}$$

Instead of applying ⊛, first find $\vec{J}_b$ & $\vec{K}_b$ and then deduce the vector potentials due to these bound currents. Finally evaluate $\vec{\nabla} \times \vec{A} = \vec{B}$.

6.3 The Auxilary Field $\vec{H}$.

The total field inside a material is due to bound currents plus any other "free" currents not associate $\vec{M}$.

$$\vec{J} = \vec{J}_b + \vec{J}_f$$

Ampère's Law becomes

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$= \mu_0 (\vec{J}_b + \vec{J}_f)$$

$$\therefore \frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times \vec{M} + \vec{J}_f$$

$$\therefore \vec{\nabla} \times \underbrace{\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right)}_{\vec{H}} = \vec{J}_f$$

$\vec{H}$: no "sensible" name according to Griffiths.

Ampère's Law becomes:

$$\vec{\nabla}\times\vec{H} = \vec{J}_f \quad \text{differential form}$$

$$\oint \vec{H}\cdot d\vec{l} = I_{f,\,encl} \quad \text{integral form.}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

6.4 Linear Media

Like we had for polarization, in linear media we should write:

$$\cancel{\vec{M} = \frac{1}{\mu_0}\chi_m \vec{B}}$$

but, by convention, that is not what is done. Instead, we write

$$\vec{M} = \chi_m \vec{H}$$ where χ_m is a dimensionless magnetic susceptibility.

$\chi_m > 0$: paramagnetic

$\chi_m < 0$: diamagnetic.

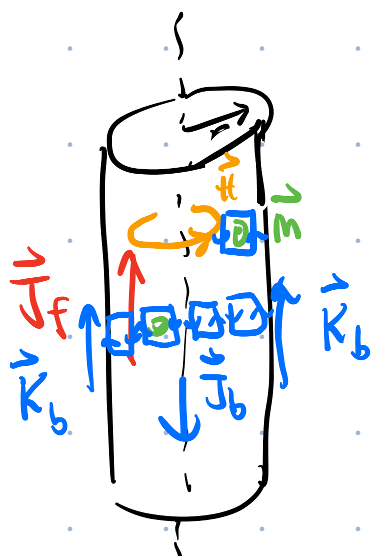
Since $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \Rightarrow \vec{B} = \mu_0(\vec{H} + \underbrace{\vec{M}}_{\chi_m \vec{H}})$

$$\therefore \vec{B} = \mu_0(1+\chi_m)\vec{H}$$
$$= \mu_0 \mu_r \vec{H}$$
$$= \mu \vec{H}$$

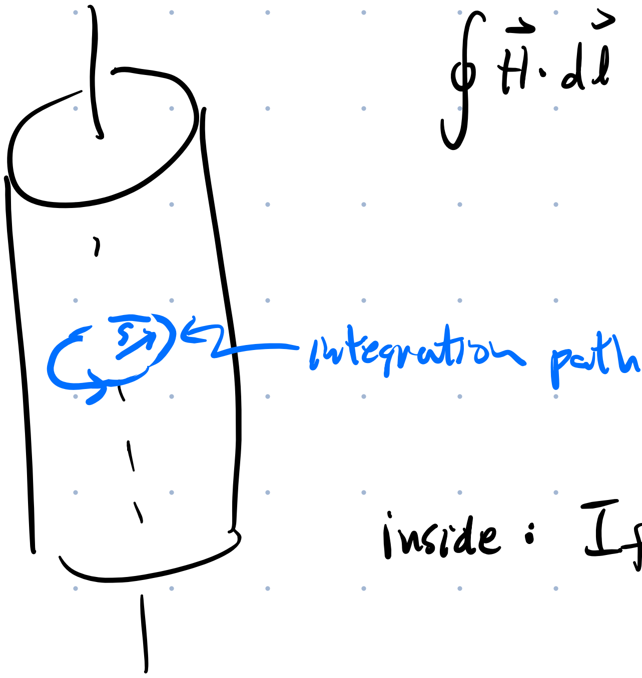
μ: permeability

μ_r : relative permeability.

Eg. A long copper rod carries a uniformly-distributed (free) current along its axis. Find $\vec{B}$ inside & outside the rod. Let's assume that we have a linear material.



use $\oint \vec{H} \cdot d\vec{l} = I_{f,encl}$



$$\oint \vec{H}\cdot d\vec{l} = H\, 2\pi s$$ valid $\forall\, s$.

inside: $$I_{f,encl} = I\,\frac{\pi s^2}{\pi R^2} = I\left(\frac{s}{R}\right)^2$$

outside: $$I_{f,encl} = I$$

$s > R$ (outside): $$H\, 2\pi s = I$$

$$\frac{1}{\mu_0}B - \cancel{M} = \frac{I}{2\pi s}$$

outside rod.

$$\boxed{\therefore\ \vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

$s < R$ (inside): $$\left(\frac{1}{\mu_0}B - M\right)2\pi s = I\left(\frac{s}{R}\right)^2$$

$$\frac{1}{\mu_0} B - M = \frac{I s}{2\pi R^2}$$

Now $M = \chi_m H$; $H = \frac{B}{\mu} = \frac{B}{\mu_0(1+\chi_m)}$

$$\therefore M = \chi_m \frac{B}{\mu_0(1+\chi_m)} = \frac{\chi_m}{1+\chi_m} \frac{B}{\mu_0}$$

$$\therefore \frac{B}{\mu_0} - \frac{\chi_m}{1+\chi_m} \frac{B}{\mu_0} = \frac{I s}{2\pi R^2}$$

$$B \underbrace{\left(1 - \frac{\chi_m}{1+\chi_m}\right)}_{\frac{1}{1+\chi_m} = \frac{1}{\mu_r}} = \frac{\mu_0 I s}{2\pi R^2}$$

$$\boxed{\therefore \vec{B} = \frac{\mu_0 \mu_r I s}{2\pi R^2} \hat{\phi} = \frac{\mu I s}{2\pi R^2} \hat{\phi}}$$

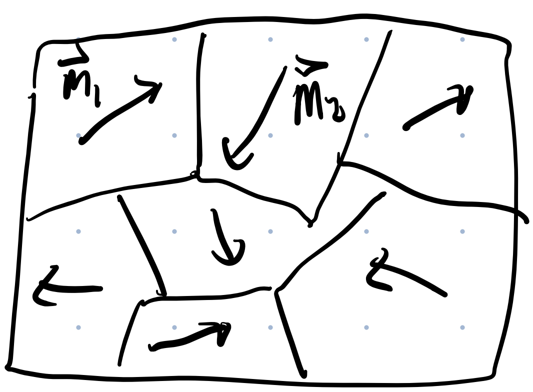
Since copper is diamagnetic $\chi_m < 0$

$\&$ $\mu = \mu_0 (1 + \chi_m) < \mu_0$.

6.4.2 Ferromagnetism

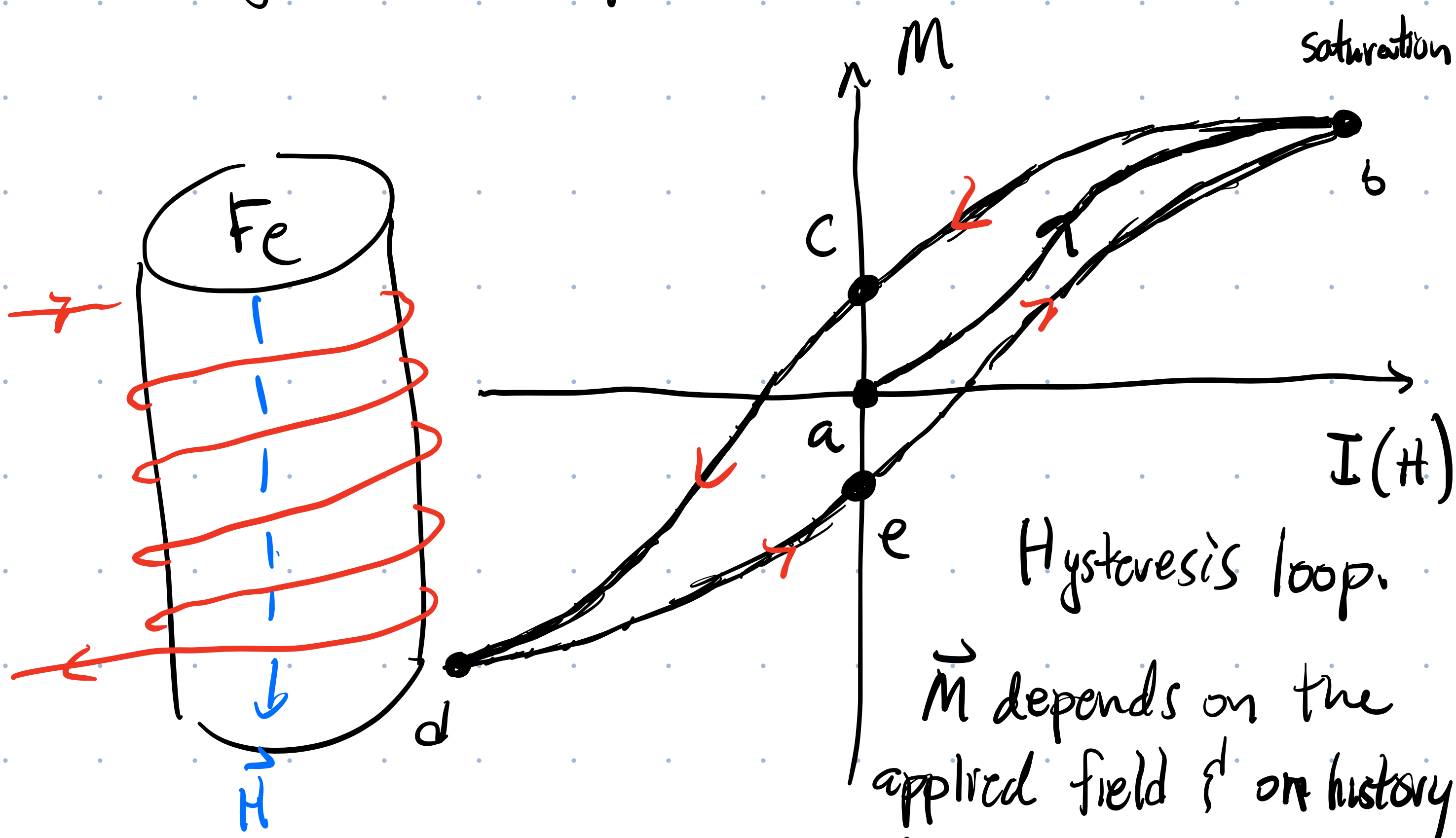
A quantum phenomena in which an interaction between nearby dipoles causes the unpair electron spins to point in the same dir'n, even in the absence of an external magnetic field.

Iron is the primary example of a ferromagnet. However, these aligned spins are broken up into domains that point in various dir'ns



In this case, there is no net magnetization & the ferromagnetic material does not act as a permanent magnet.

The various domains can be aligned by applying a strong external magnetic field.



Hysteresis loop.

$\vec{M}$ depends on the applied field & on history of the sample.

(a) unmagnetized Fe in zero field.

(b) All domains aligned w/ external field.

(c) Zero field applied, but some magnetization remains. Sample is a permagnet.

(d) Saturation in the other dir'n.

(e) permanent magnet w/ poles flipped.

The permanent magnetization in zero field (pts. c & e) can be destroyed by heating sample to high temp.

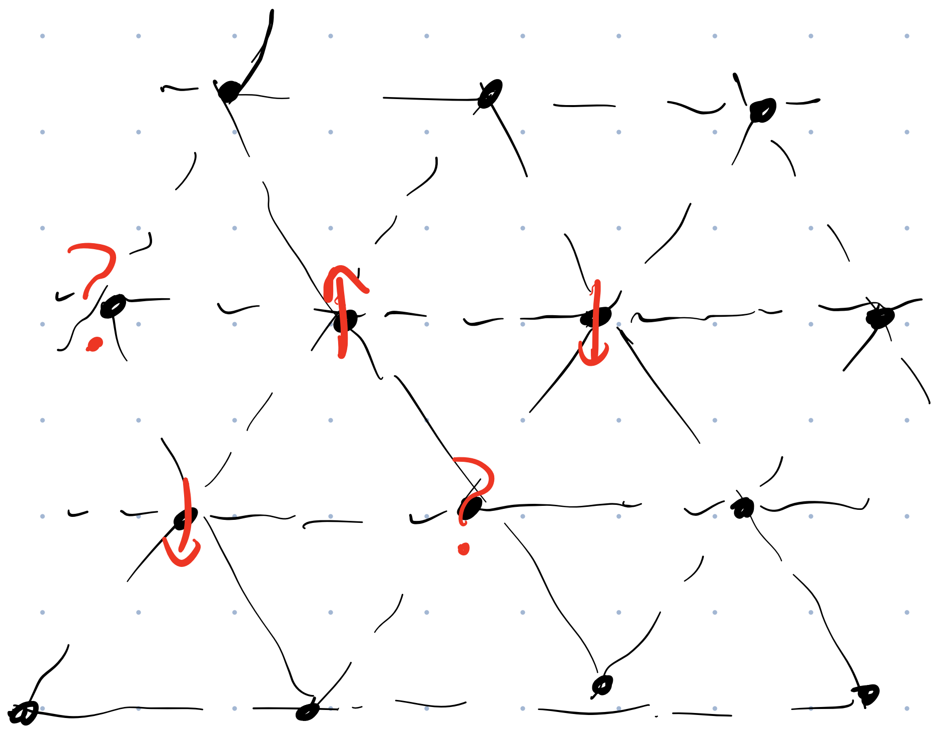
The temp. at which $\vec{M}$ vanishes is called the Curie temp. (for iron Curie temp is ~ 770 °C).

This is an example of a magnetic phase transition.

Below certain temperatures, some materials exhibit antiferromagnetism in which neighbouring spins align antiparallel. Hematite (Fe_2O_3) is an example of an antiferromagnet below 250 K.

$$\begin{array}{cccccc} \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \\ \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow \\ \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \end{array}$$

This can lead to so-called "frustrated" magnets. For example, consider an antiferromagnet on a triangular lattice.



Can't satisfy anti-aligned spins on this lattice.