PHYS 301 Dec. 2, 2024

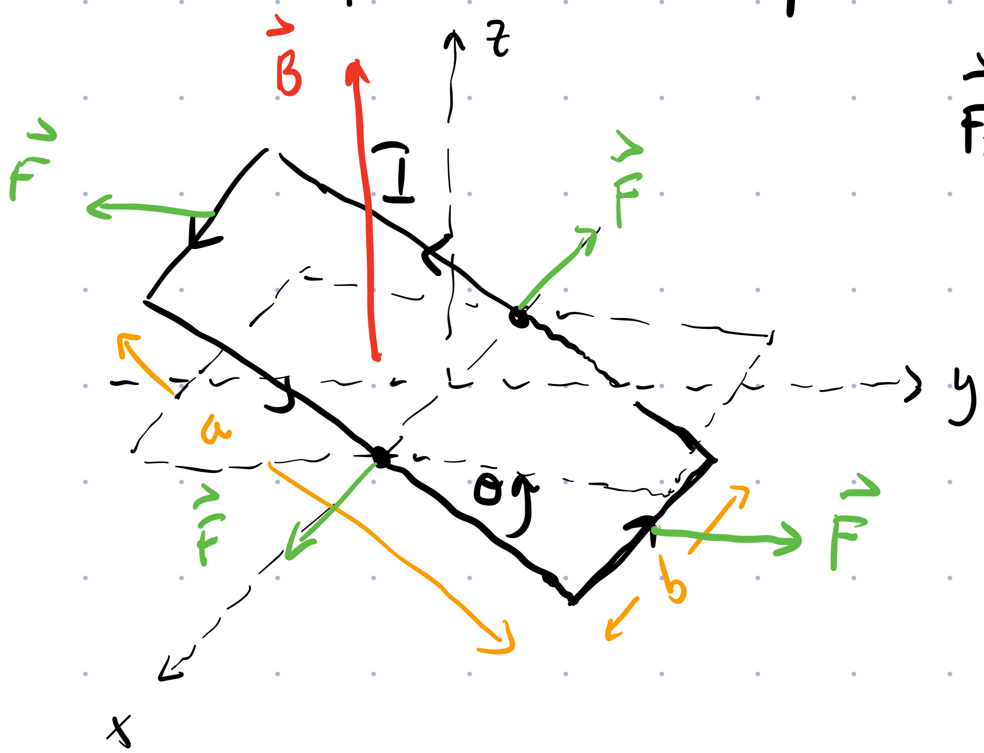
Griffiths Ch. 6: Magnetic Fields in Matter

Magnetism in materials is due to:

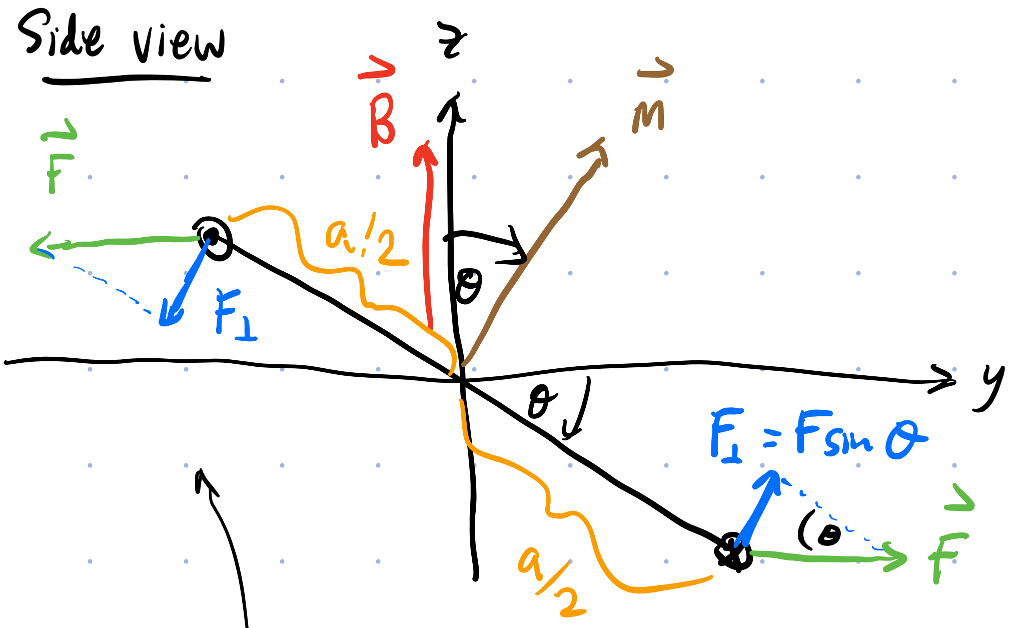
- Alignment of unpaired electron "spins". Spin is a small magnetic dipole.
- A collective change to the orbital angular momentum of electrons around the nucleus.

Torques on magnetic dipoles.

Current loop in an external magnetic field.



$$\vec{F}_{mag} = \vec{I} \times \vec{B}\, l$$

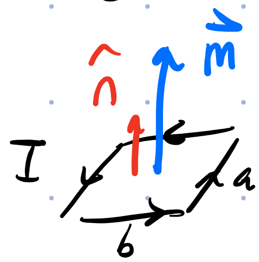


torque: $\vec{N} = 2\left(\frac{a}{2}F\sin\theta\right)\hat{x} = aF\sin\theta\,\hat{x}$

$F = IBb$

$\vec{N} = \underbrace{ab}_{\text{loop area}}\, IB\sin\theta\,\hat{x}$

usually the quantity $abI\hat{n} \equiv \vec{m}$



$\vec{N} = mB\sin\theta\,\hat{x}$

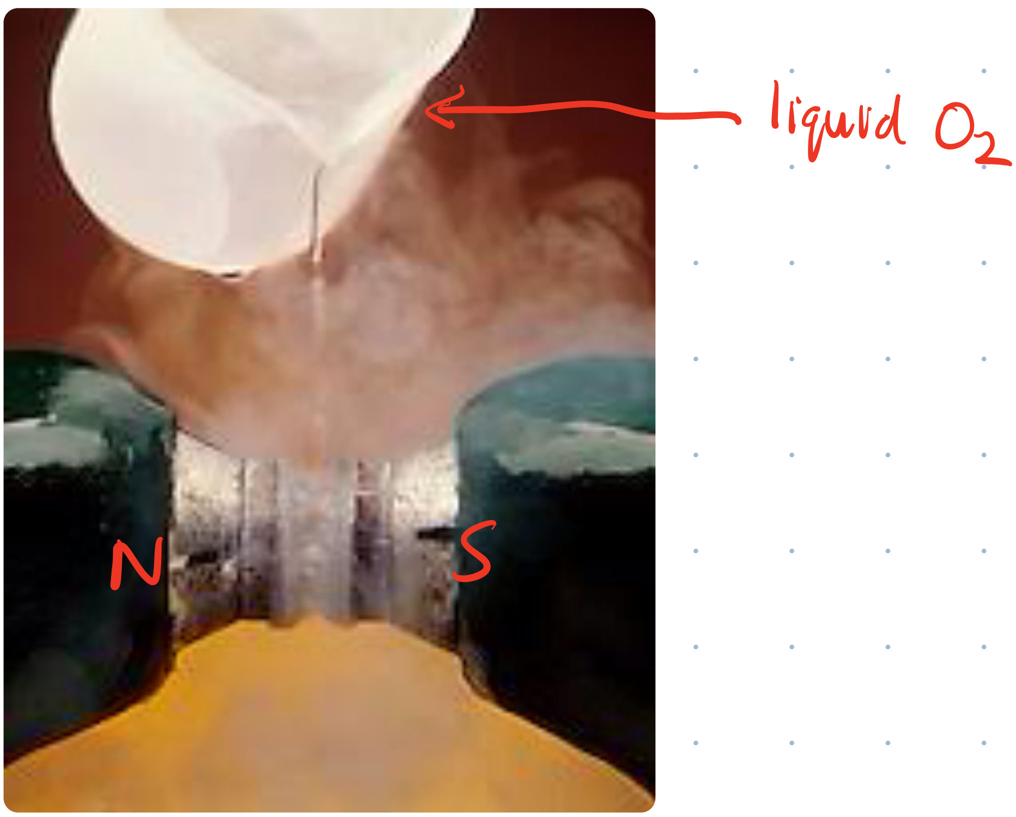
or $\boxed{\vec{N} = \vec{m}\times\vec{B}}$ Torque on a magnetic dipole in a magnetic field.

Thus, in a material w/ lots of unpaired electrons in an external magnetic field tend to align their spin/magnetic moment w/ $\vec{B}$ thereby magnetizing the material.

This effect is called paramagnetism.

The effect is observed in materials made up of atoms/molecules w/ an odd no. of e^-.

Eg. Molecular oxygen (O_2) has unpaired electrons and is paramagnetic.

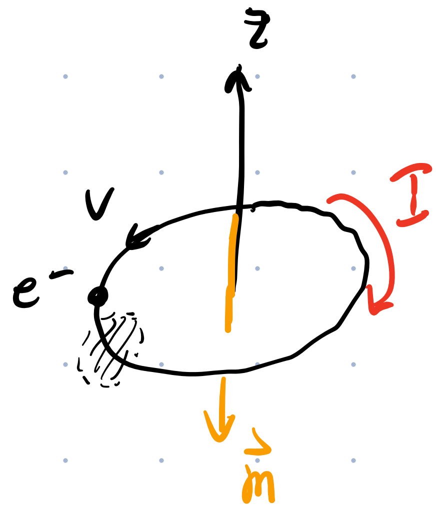


6.1.3 Effect of Magnetic Fields on Atomic Orbits.

→ Diamagnetism

Electrons have a magnetic moment associated w/ their spin. There is also a magnetic moment associated with their atomic orbits.

To see this effect, develop a simple cartoon model. Assume that atomic orbit is circular w/ radius R.



$$I = \frac{-e}{T}$$

$$T = \frac{2\pi R}{v}$$

$$\therefore \; I = \frac{-ev}{2\pi R}$$

loop area

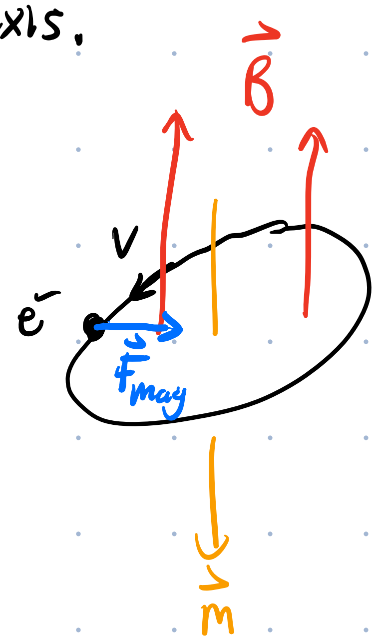
$$m = aI = \pi R^2 I = \cancel{\pi} R^{\cancel{2}} \left(\frac{-ev}{2\cancel{\pi} \cancel{R}} \right)$$

$$\boxed{\therefore \; \vec{m} = -\frac{1}{2} evR\,\hat{z}}$$

In the absence of an external field, the only force on e^- is Coulomb attraction to nucleus.

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2} = m\frac{V^2}{R} \quad (!)$$

Now, apply an external magnetic field $\vec{B}$, along z-axis.



$$\vec{F}_{mag} = -e\,\vec{V}\times\vec{B}$$

$\therefore F_{mag} = eVB$ (towards the centre.)

Now : $$\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}}_{\frac{mV^2}{R}} + e\bar{V}B = m\frac{\bar{V}^2}{R}$$

where V is speed w/o $\vec{B}$.

$\bar{V}$: speed of e^- after $\vec{B}$ is turn on.

$\bar{V} = V + \Delta V$

$$\therefore \quad e\bar{v}B = \frac{m}{R}\left(\bar{v}^2 - v^2\right)$$

$$= \frac{m}{R}\underbrace{(\bar{v}-v)}_{\Delta v}\underbrace{(\bar{v}+v)}_{\text{if } \Delta v \text{ is small}}$$

$\bar{v}+v \approx 2\bar{v}$

$$e\cancel{\bar{v}}B = \frac{m}{R}\Delta v \, 2\cancel{\bar{v}}$$

$$\boxed{\therefore \Delta v = \frac{eRB}{2m} > 0}$$

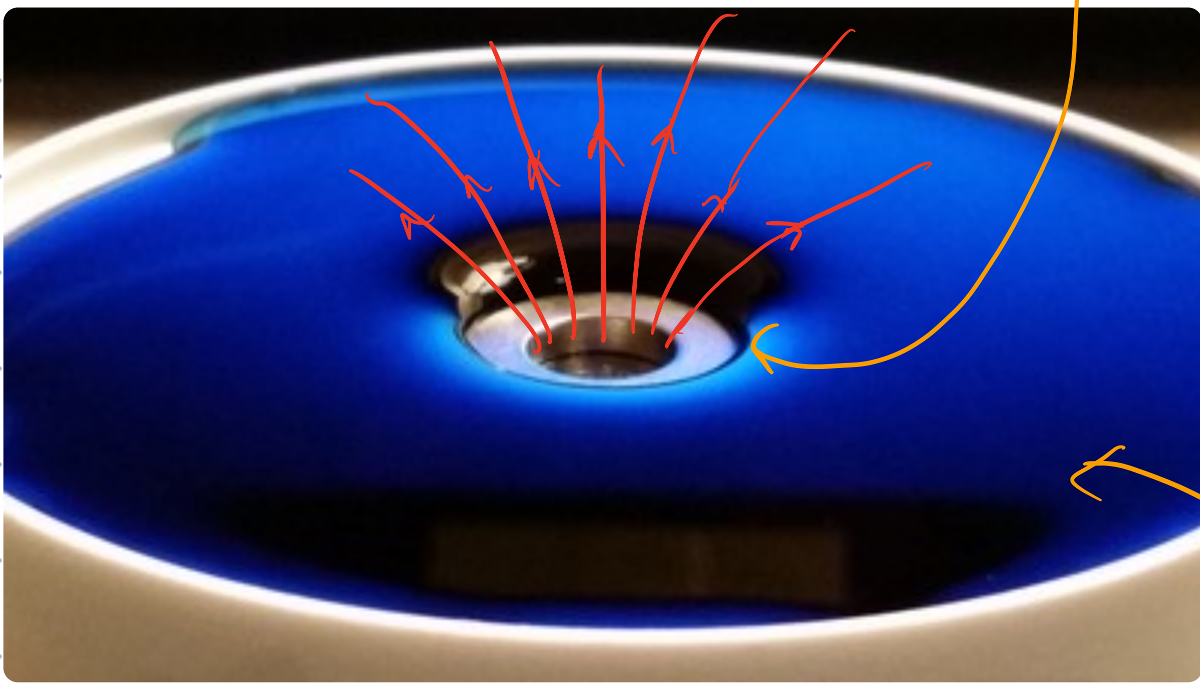
$\therefore$ external mag. field causes the e^- to speed up.

Since $\vec{m} = -\frac{1}{2}evR\hat{z}$, the magnetic moment in this case is enhanced in the dir'n opp. of $\vec{B}$.

$\rightarrow$ Diamagnetism.

Eg. Water is diamagnetic. $\vec{B}$

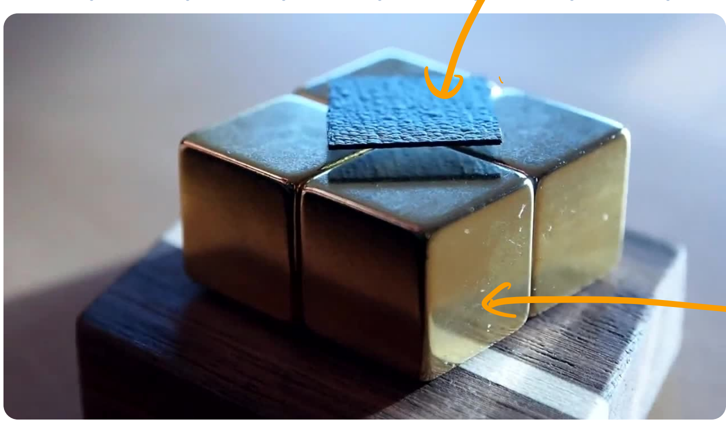
strong ring-shaped magnets



water (coloured blue for contrast)

pyrolitic carbon

→ an unusually strong diamagnet



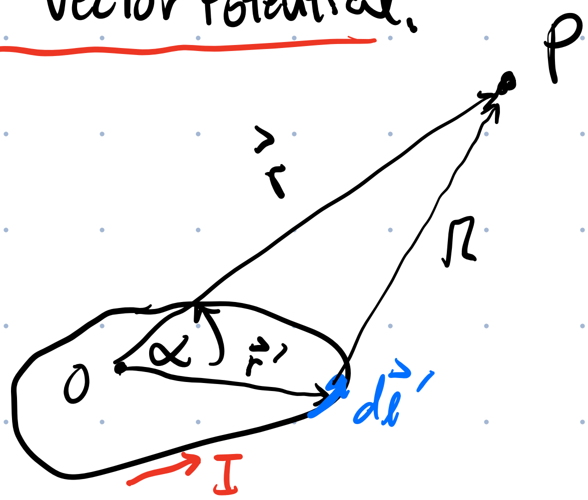
strong magnets

6.1.4 Magnetization

In the presence of an external magnetic field magnetized the material on interest.

magnetization $\vec{M}$ = magnetic moment $\vec{m}$ per unit volume.

Dipole Vector Potential.



$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \oint \frac{1}{\mathscr{r}} d\vec{l}'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{\sqrt{r^2 + (r')^2 - 2rr'\cos\alpha}}$$

$$= \frac{1}{r\sqrt{1 + \left(\frac{r'}{r}\right)^2 - 2\frac{r'}{r}\cos\alpha}} \quad \text{if } r \gg r'$$

$$\approx \frac{1}{r}\frac{1}{\sqrt{1-2\frac{r'}{r}\cos\alpha}} = \frac{1}{r}\left(1-2\frac{r'}{r}\cos\alpha\right)^{-1/2}$$

$$\approx \frac{1}{r}\left[1-\frac{1}{\cancel{2}}\left(-\cancel{2}\frac{r'}{r}\cos\alpha\right)\right]$$

$$\therefore \frac{1}{\mathscr{r}} \approx \frac{1}{r}\left[1+\frac{r'}{r}\cos\alpha\right]$$

$$\therefore \vec{A}(\vec{r}) \approx \frac{\mu_0 I}{4\pi}\frac{1}{r}\oint\left(1+\frac{r'}{r}\cos\alpha\right)d\vec{l}'$$

dipole contribution to $\vec{A}$.

$$= \boxed{\frac{\mu_0 I}{4\pi}\frac{1}{r}\oint\frac{r'}{r}\cos\alpha\, d\vec{l}' = \vec{A}_{dip}(\vec{r})}$$

note that $\vec{r}'\cdot\hat{r} = r'\cos\alpha$

$$\therefore \vec{A}_{dip} = \frac{\mu_0 I}{4\pi}\frac{1}{r^2}\oint(\vec{r}'\cdot\hat{r})\,d\vec{l}'$$

Aside: Show that $\oint (\vec{r}' \cdot \hat{r})\, d\vec{l}' = \vec{a} \times \hat{r}$

loop area.

$$\oint (\underbrace{\vec{r}' \cdot \hat{r}}_{\equiv T})\, d\vec{l}' = \oint T\, d\vec{l}'$$

Use Stoke's Th: $\displaystyle \int \vec{\nabla}' \times \vec{v} \cdot d\vec{a}' = \oint \vec{v} \cdot d\vec{l}'$

If we let $\vec{v} = \vec{c}\, T$ where $\vec{c}$ is a const.

$$\underbrace{\oint \vec{c}\, T \cdot d\vec{l}}_{\vec{c} \cdot \oint T\, d\vec{l}} = \int \underbrace{\vec{\nabla}' \times (\vec{c}\, T)}_{T\cancelto{0}{(\vec{\nabla}' \times \vec{c})} - \vec{c} \times \vec{\nabla}' T} \cdot d\vec{a}'$$

$$\vec{c} \cdot \oint T\, d\vec{l} = -\int \underbrace{\vec{c} \times \vec{\nabla}' T \cdot d\vec{a}'}_{d\vec{a}' \times \vec{c} \cdot \vec{\nabla}' T = \vec{\nabla}' T \times d\vec{a}' \cdot \vec{c}}$$

$$\therefore\ \underbrace{\vec{c}\cdot\oint T\,d\vec{l}}\ = \underbrace{-\vec{c}}\cdot\underbrace{\int \vec{\nabla}' T\times d\vec{a}'}$$

$$=$$

$$\therefore \oint T\,d\vec{l}' = -\int \vec{\nabla}' T \times d\vec{a}' = -\int \vec{\nabla}'(\vec{r}'\cdot\hat{r})\times d\vec{a}'$$

$$\vec{\nabla}'(\vec{r}'\cdot\hat{r}) = \vec{r}'\times\cancelto{0}{(\vec{\nabla}'\times\hat{r})} + \hat{r}\times\underbrace{(\vec{\nabla}'\times\vec{r}')}_{0}$$

$$+ \cancelto{0}{(\vec{r}'\cdot\vec{\nabla}')\hat{r}} + \underbrace{(\hat{r}\cdot\vec{\nabla}')\vec{r}'}_{\hat{r}}$$

$$(\hat{r}\cdot\vec{\nabla}')\vec{r}' = \frac{\partial}{\partial r'}\left(r'\hat{r}\right) = \hat{r}$$

$$\therefore \oint T\,d\vec{l}' = -\int \hat{r}\times d\vec{a}' = -\hat{r}\times\underbrace{\int d\vec{a}'}_{\vec{a}} = -\hat{r}\times\vec{a}$$

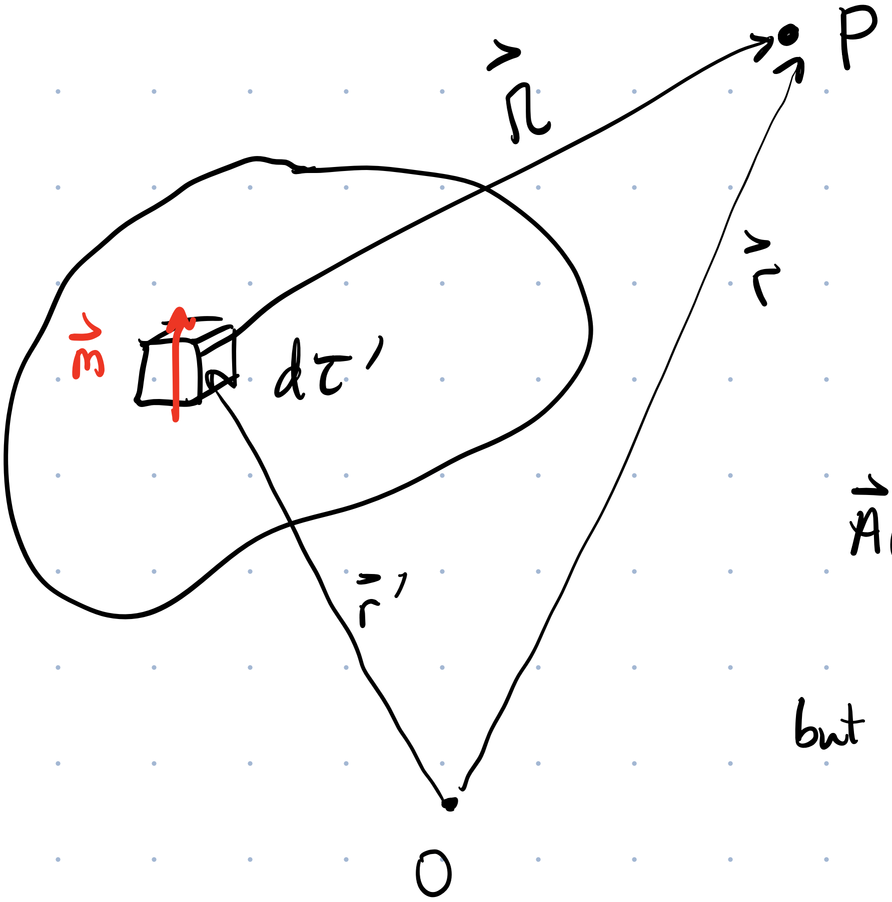
Finally $\vec{A}_{dip} = \dfrac{\mu_0 I}{4\pi}\,\dfrac{\vec{a}\times\hat{r}}{r^2}$

or defining $I\vec{a} = \vec{m}$

$$\boxed{\vec{A}_{dip} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}}$$

6.2 The fields of Magnetized Objects

Consider a magnetized object



vector potential @ P due to just $d\tau'$ is:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

but $\vec{m} = \vec{M}\,d\tau'$

$\therefore$ $\vec{A}$ @ P due to entire magnetized object is :

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}\,') \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, d\tau'} \quad \textcircled{$\ast$}$$