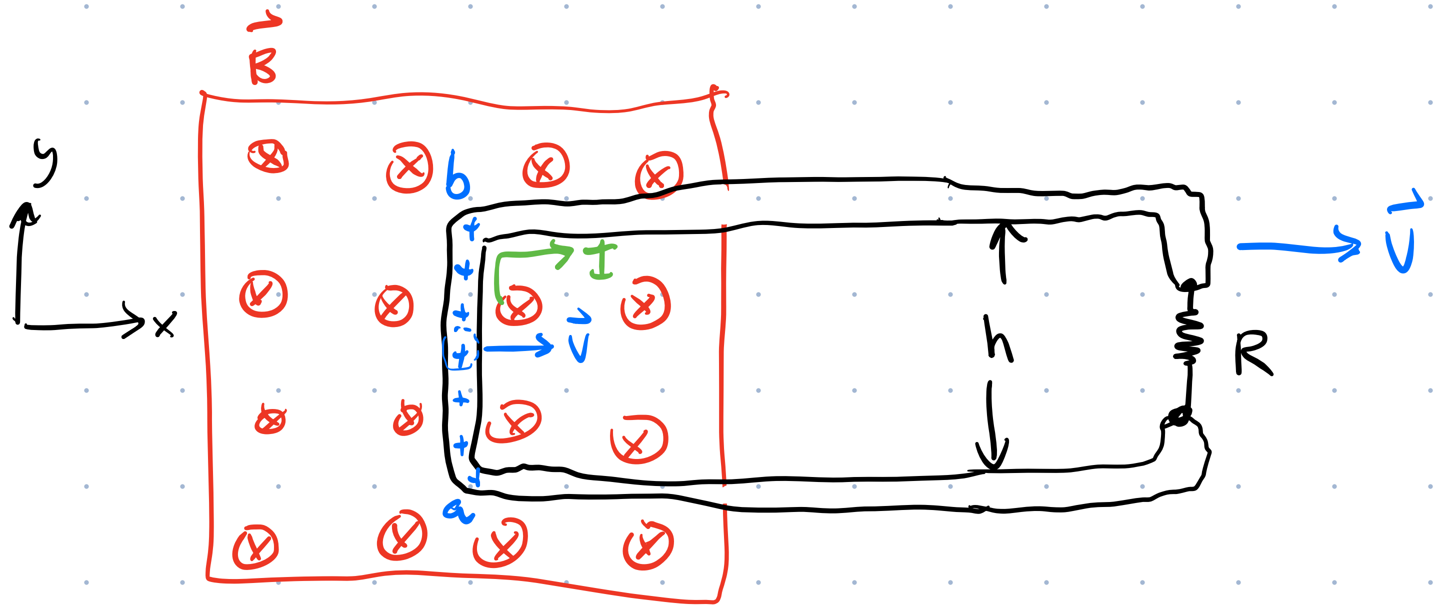
PHYS 301 Nov. 18, 2024

Griffiths Ch. 7

7.1.3 Motional emf (electromotive force)

Arises when a wire is moved through a magnetic field.



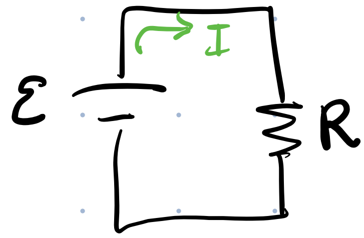
Loop of wire w/ resistor R is moved to right w/ speed v.

Force on mobile charges in section a-b of wire is

$$\vec{F}_s = \vec{F}_{mag} = e\vec{v} \times \vec{B} = evB\,\hat{y}$$

drives a clockwise current in loop.

Equiv. circuit



There are really two forces driving charge around loop.

1. $\vec{F}_s$: source force ($\vec{F}_{mag}$, in our example)

2. $q\vec{E}$: electric interaction between adjacent charges.

Net force per unit charge:

$$\frac{\vec{F}}{e} = \vec{f} = \vec{f}_s + \vec{E}$$

For a steady current, charges move around loop at a const avg. speed s.t. $\vec{f} = 0$

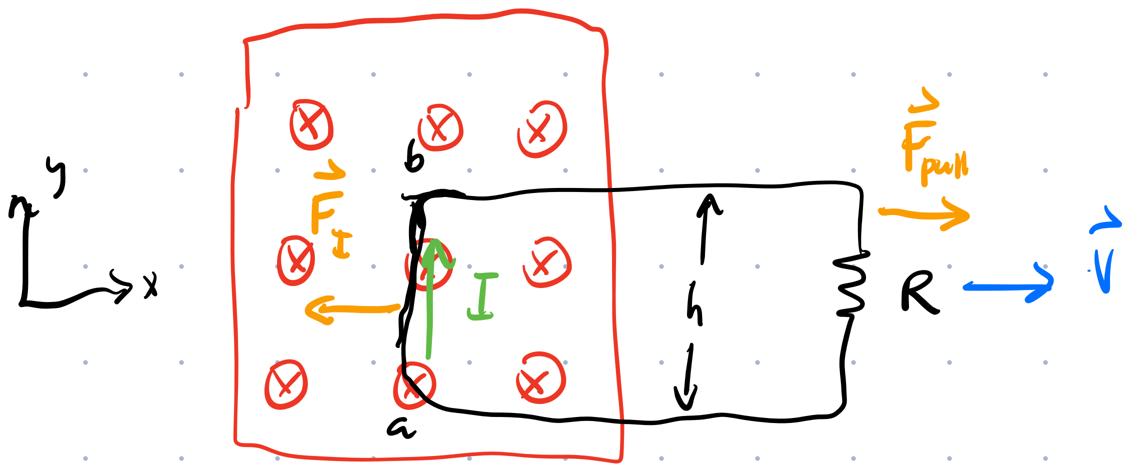
$$\Rightarrow \vec{E} = -\vec{f}_s = -\vec{f}_{mag} = -vB\,\hat{y}$$

$$\therefore \mathcal{E} = -\int_a^b \vec{E}\cdot d\vec{l} = \int_a^b \vec{f}_{mag}\cdot d\vec{l}$$

$$= \boxed{vBh = \mathcal{E}}$$

motional emf.

Recall that magnetic forces do no work. $(\vec{v} \perp \vec{F}_{mag})$
What is the source of energy/work that drives the current?



Force on current I is section ab of loop is

$$\vec{F}_I = \vec{I} \times \vec{B} h = -IBh\,\hat{x}$$

In order to keep loop moving right w/ const. speed, need to offset/balance $\vec{F}_I$ w/ $\vec{F}_{pull}$ of equal mag. in opp. dir'n. Whoever or whatever is supplying $\vec{F}_{pull}$ is doing the work to sustain the current.

The power supplied by pulling force is

$$P_{pull} = \vec{F}_{pull} \cdot \vec{v} = F_{pull} v = IBhv$$

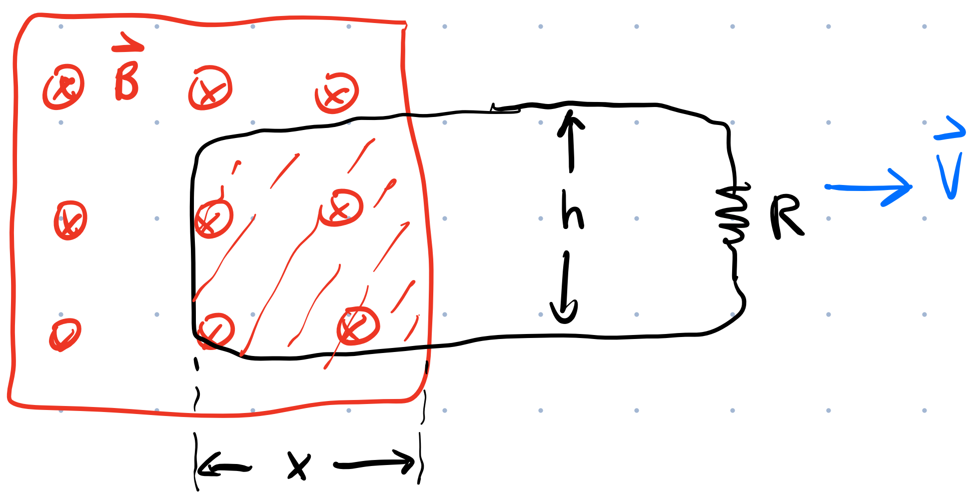
but $I = \frac{\mathcal{E}}{R} = \frac{vBh}{R}$

$\therefore \; P_{pull} = \frac{(vBh)^2}{R}$

Note that the power dissipated by the resistor is $P_R = I^2 R = \left(\frac{\mathcal{E}}{R}\right)^2 R = \frac{\mathcal{E}^2}{R} = \frac{(vBh)^2}{R}$

conservation of energy.

Consider the magnetic flux Φ through the loop at some instant of time.



$$\Phi \equiv \int \vec{B} \cdot d\vec{a} = BA = Bhx$$

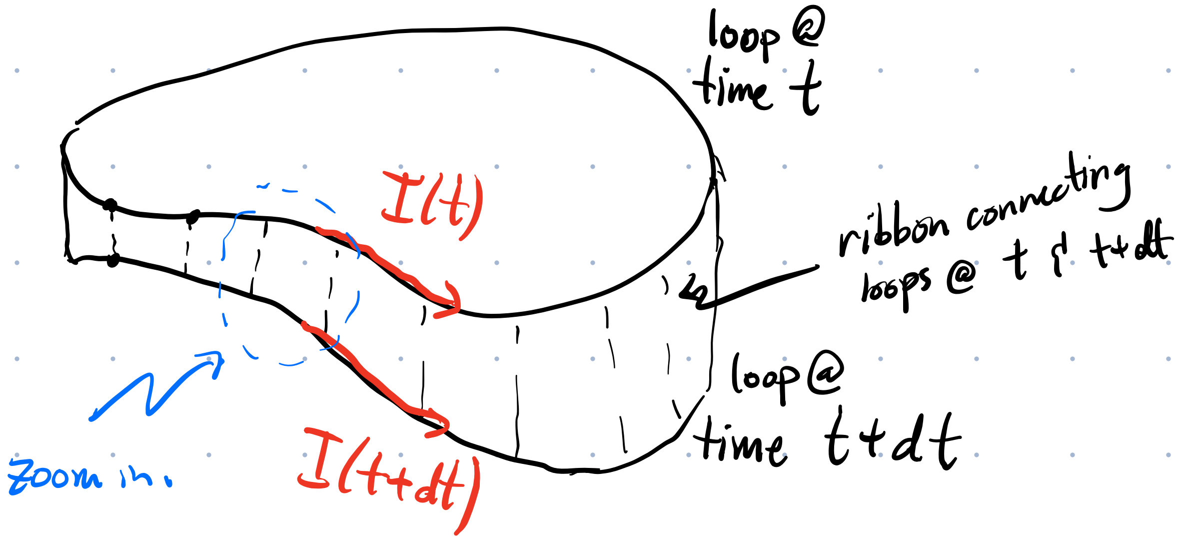
Consider $\frac{d\Phi}{dt} = \frac{d}{dt}(Bhx) = Bh\underbrace{\frac{dx}{dt}}_{-v} = -\mathcal{E}$

$\therefore$ At least in our case,

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$

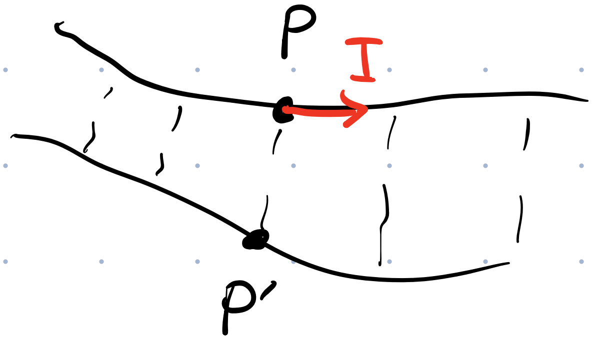
We now attempt to prove that $\mathcal{E} = -\frac{d\Phi}{dt}$ is generally true for any shape of loop moving in an arbitrary way in a non-uniform $\vec{B}$.

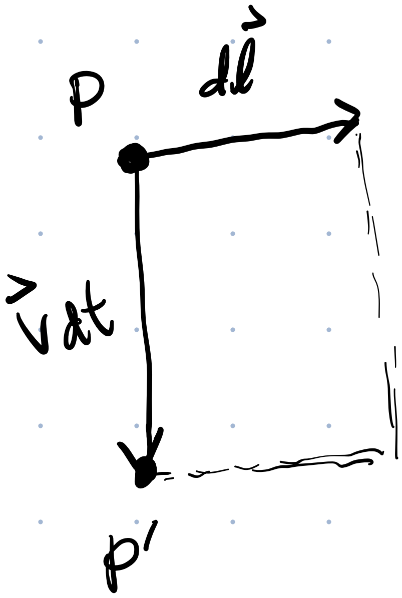
non-uniform magnetic field (not shown)



Change in flux:

$$d\Phi = \Phi(t+dt) - \Phi(t) = \Phi_{ribbon} = \int_{ribbon} \vec{B}\cdot d\vec{a}$$





area of this object is

$$d\vec{a} = (\vec{v}\,dt \times d\vec{l}\,)$$
$$= (\vec{v} \times d\vec{l}\,)\,dt$$

$$d\Phi = \Phi_{ribbon} = \int_{ribbon} \vec{B}\cdot d\vec{a} = \int_{ribbon} \vec{B}\cdot(\vec{v}\times d\vec{l}\,)\,dt$$

$$\therefore \frac{d\Phi}{dt} = \int_{ribbon} \vec{B}\cdot(\vec{v}\times d\vec{l}\,)$$

recall, however, that

$$\vec{B} \cdot \vec{V} \times d\vec{l} = d\vec{l} \cdot \vec{B} \times \vec{V}$$

$$= - \vec{V} \times \vec{B} \cdot d\vec{l}$$

Recall that $\vec{F}_{mag} = q\vec{v} \times \vec{B}$ $\therefore$ $\vec{V} \times \vec{B} = \frac{\vec{F}_{mag}}{q} = \vec{f}_{mag}$

$$\therefore \quad \frac{d\Phi}{dt} = -\oint \vec{V} \times \vec{B} \cdot d\vec{l}$$

$$= - \int_{\text{ribbon}} \vec{f}_{mag} \cdot d\vec{l} = -\mathcal{E}$$

From earlier.

$$\boxed{\therefore \quad \mathcal{E} = -\frac{d\Phi}{dt}}$$

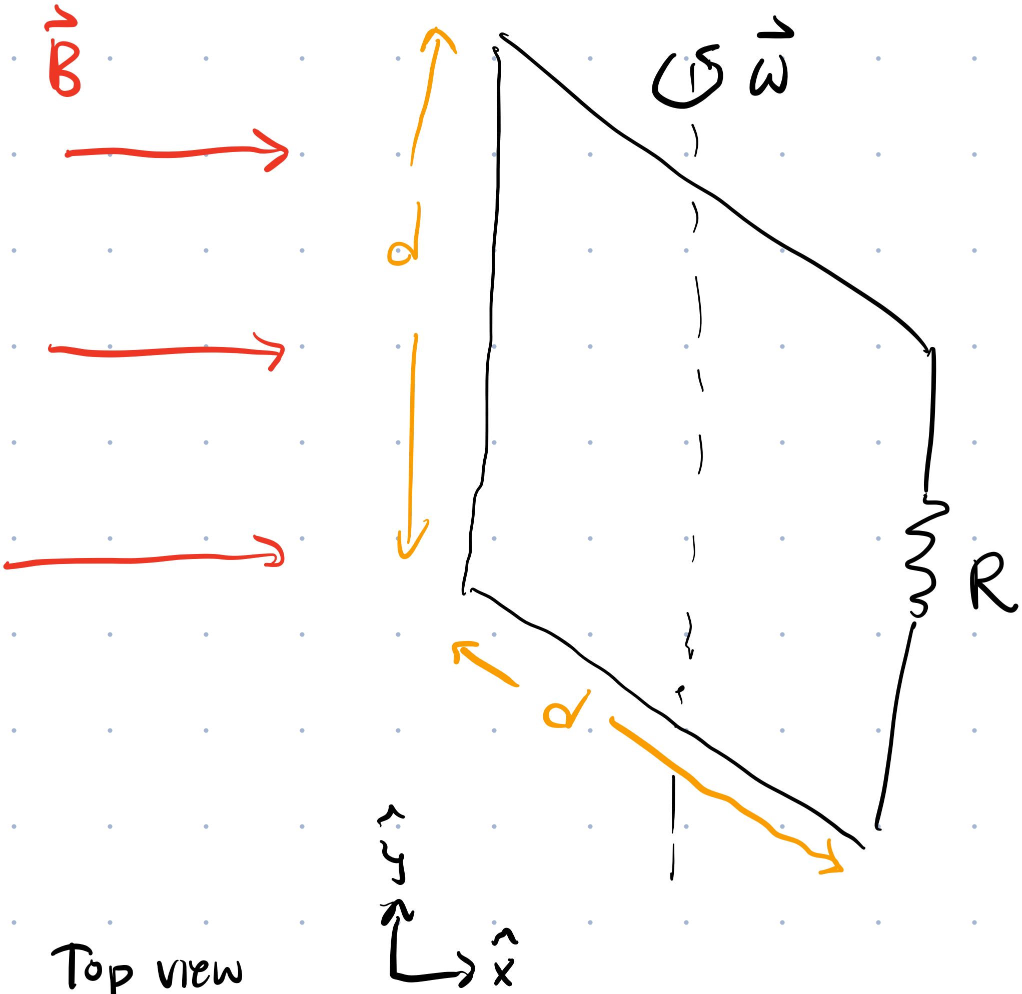
True for any loop moving in any way in any magnetic field.

In the above derivation, I've assumed that the motion of the charges is predominantly in the dir'n of displacement of the wire / loop.

However, since the charges are contributing to a current, there is a component of the motion a long the wire. This component of the charge's motion has an average speed given by the drift velocity v_d which is typically $\sim 0.1\,mm/s$. I'm effectively assuming that v of the wire is much greater that v_d. Griffith's treats the general case in Sec. 7.1.3.

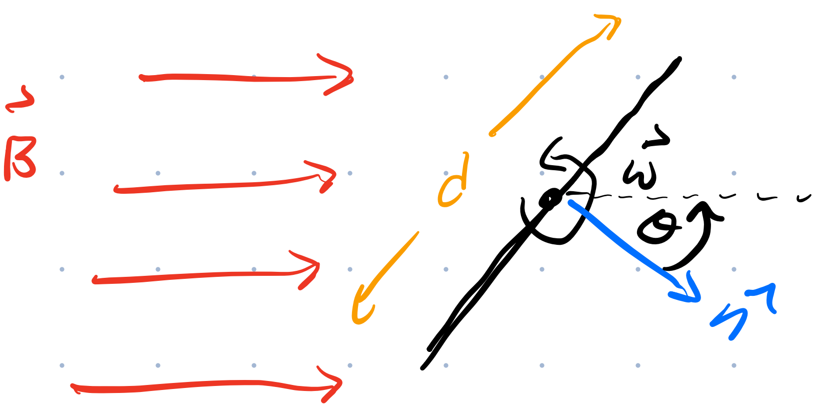
Eg. Prob. 7.10 AC generator

A square loop of wire w/ resistance R which spins w/ angular speed ω in a uniform mag. field $\vec{B}$. Find the current I in the loop.



$$\mathcal{E} = -\frac{d\Phi}{dt}$$

Top view



$$\vec{a} = d^2\,\hat{n}$$

$$\Phi = \vec{B}\cdot\vec{a}$$

$$= Bd^2\underbrace{(\hat{x}\cdot\hat{n})}_{\cos\theta}$$

$$\therefore \Phi = Bd^2 \cos\theta$$

$\hat{n}$ is constantly changing dir'n. $\rightarrow$ θ is always changing.

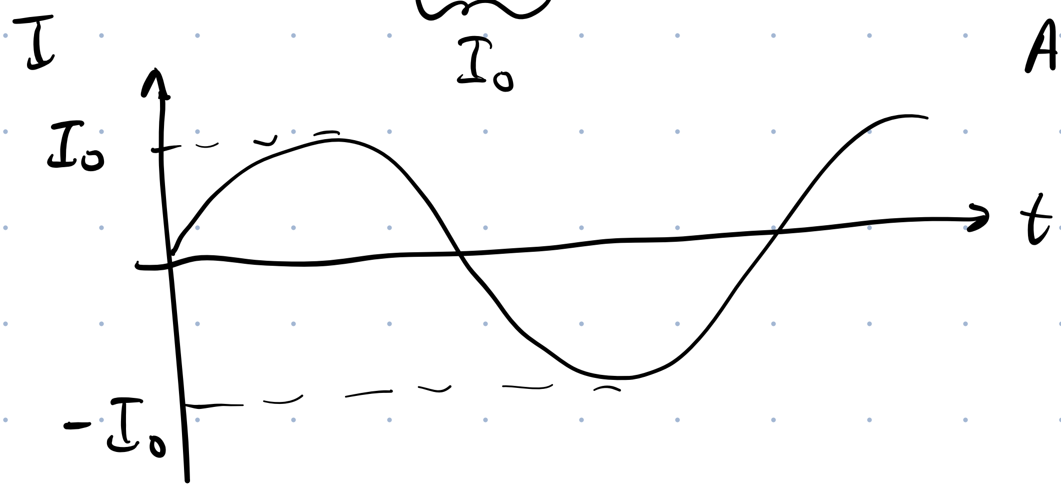
$$\theta = \omega t \quad (\text{assuming } \theta = 0 \text{ when } t = 0)$$

$$\Phi = Bd^2 \cos(\omega t)$$

$$\therefore \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(Bd^2 \cos \omega t\right)$$

$$\boxed{\mathcal{E} = \omega B d^2 \sin(\omega t)}$$

$$I = \frac{\mathcal{E}}{R} = \underbrace{\frac{\omega B d^2}{R}}_{I_0} \sin \omega t$$



AC current generator.

7.2 Faraday's Law (Induced eletric fields)

An induce emf is observed whenever there is a change if magnetic flux $\mathcal{E} = -\frac{d\Phi}{dt}$.

There are 3 ways to change mag. flux

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

understood from Lorentz force

1. Change the area of the loop (motional emf is an example)

2. Change the relative orientation of the loop & $\vec{B}$ (AC generator is an example)

3. Change the strength of $\vec{B}$
   (pull a bar magnet away from stationary loop for eg.)

In 3, the charges in loop of wire are stationary, and the $\vec{B}$ is changing.

Faraday deduce that changing magnetic field induces an electric field that can drive flow of charge in the loop.

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$

note that this is already unusual as it implies an induced $\vec{E}$ that forms a closed loop.

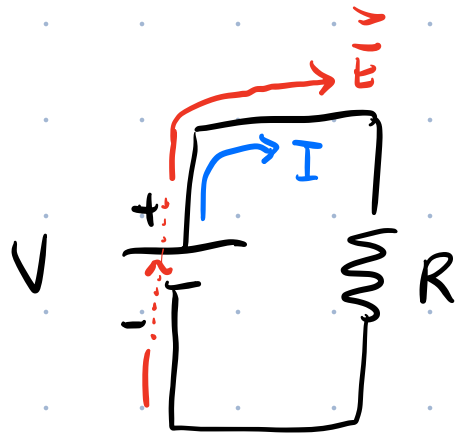
$$\underbrace{\oint \vec{E} \cdot d\vec{l}}_{\text{Stoke's Law}} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{a}$$

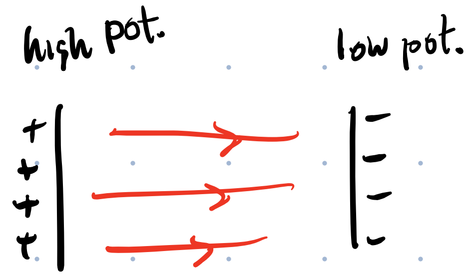
$$\int \vec{\nabla} \times \vec{E} \cdot d\vec{a} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

equal.

Faraday's Law : $\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$

Note: That if $\vec{B}$ is constant (magnetostatics) we recover $\vec{\nabla} \times \vec{E} = 0$.





$\vec{E}$ pts. from high to low pot.