PHYS 301 Oct. 28, 2024

Assign. # 3 on course website
→ Separation of variables in cylindrical coords.

Last Time: Separation of variables in spherical coords assuming azimuthal symmetry

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

for a hollow sphere of radius R w/ $V_0(\theta) = k \sin^2(\frac{\theta}{2})$ on its surface, found

$$V_0(\theta) = \frac{k}{2}(P_0 - P_1) \quad \{$$

$$V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

all $A_l = 0$ except for $l = 0, 1$

$$A_0 = k/2 \qquad A_1 = -k/2R$$

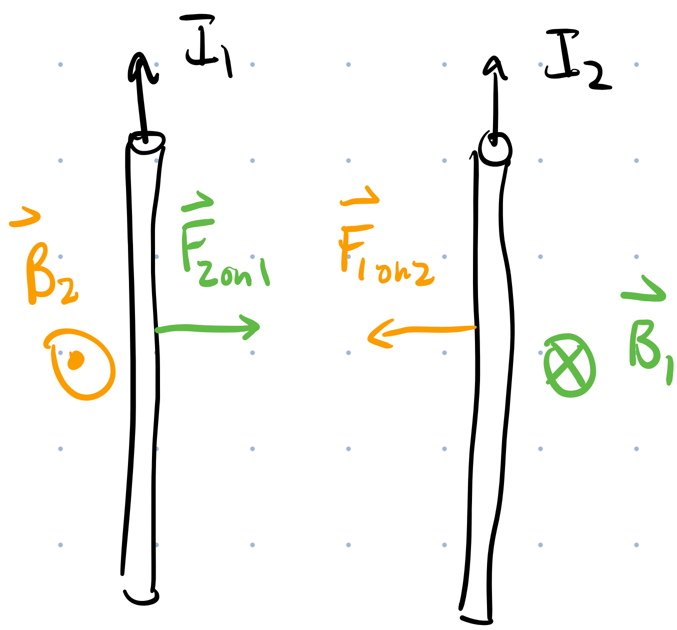
$$\therefore V(r,\theta) = \frac{k}{2}\left[1 - \left(\frac{r}{R}\right)\cos\theta\right]$$

Today: Griffiths Ch. 5 : The Lorentz Force

So far, we have limited ourselves to the case of stationary source charges that produce static electric fields. ⇒ electrostatics.

Moving charges/currents can interact with one another in a way that does not rely on electric fields / coulomb's Law.

Consider parallel currents:



Moving charges or currents are a source of magnetic fields.

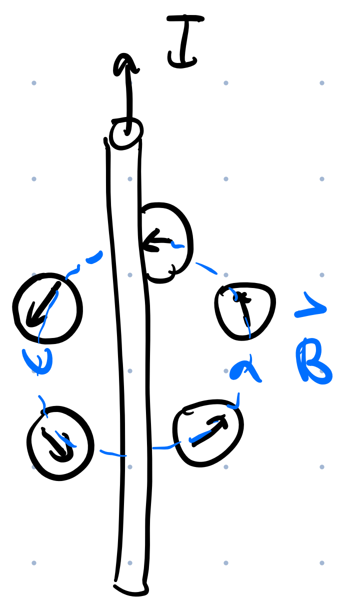
I_1 creates a magnetic field $\vec{B}_1$ at position of I_2
$\vec{B}_1$ exterts a force on I_2 that acts to left.

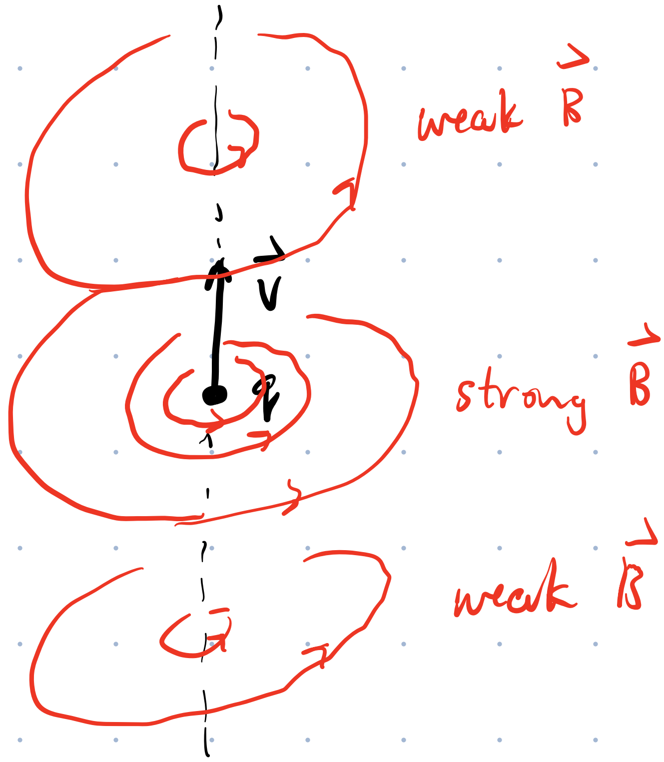
Likewise, I_2 creates magnetic field $\vec{B}_2$ @ position of I_1 & get equal but opp. force on I_1.

<u>Note that</u>: at all times both wires are electrically neutral, so the force is <u>not</u> due to electric fields. It is the motion of charge that creates magnetic fields that facilitate the interaction.

A compass can be used to confirm that moving charge is a source of $\vec{B}$.

Magnetic fields form closed loops around moving charges.



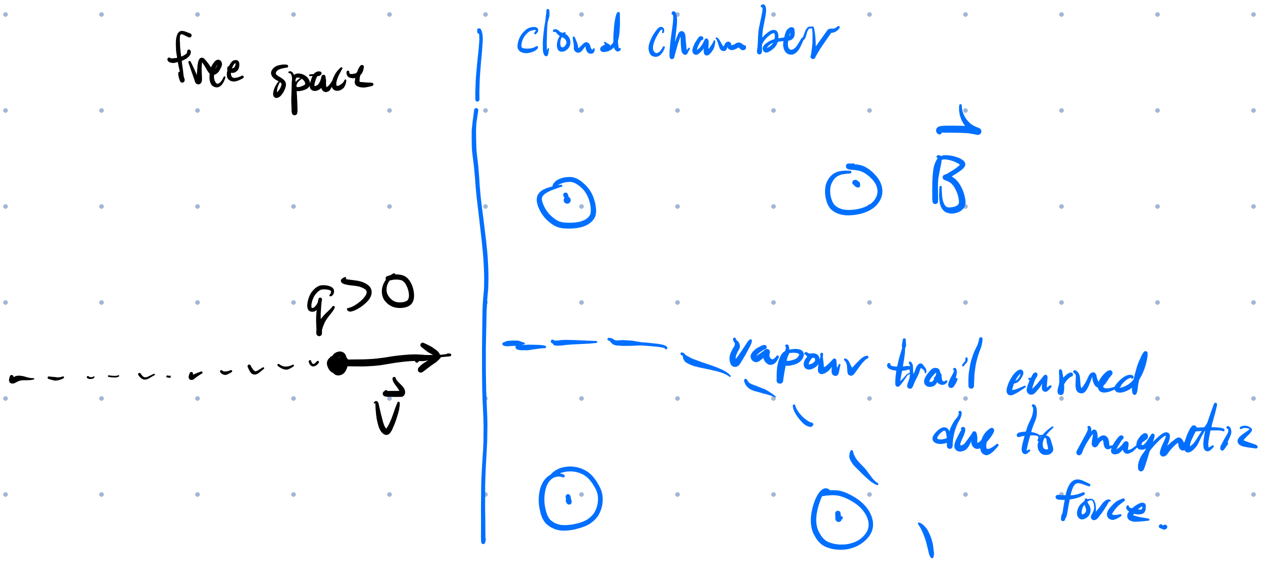


Right-Hand rule for $\vec{B}$ due to I.

"Grab" the current-carrying wire w/ right hand s.t. thumb is in dir'n of $\vec{B}$.

$\Rightarrow$ Finger curl in dir'n of $\vec{B}$.

Can also use something like a cloud chamber to confirm that moving charge experience of force due to magnetic fields.

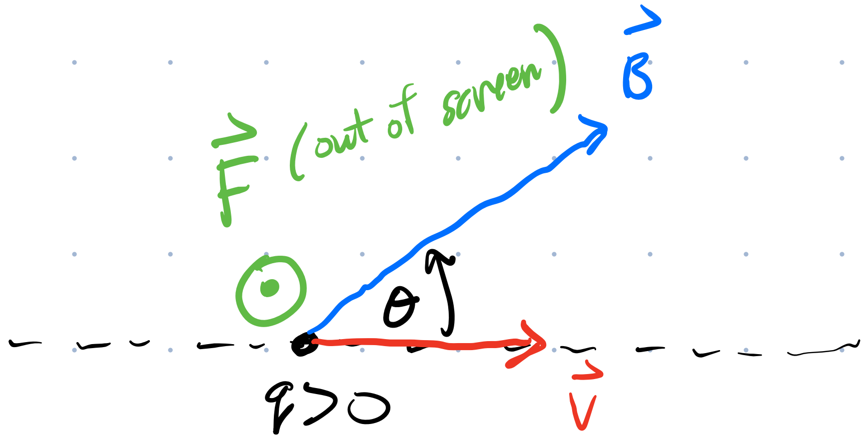


If one was to experimentally investigate the force on moving charge due to magnetic field $\vec{B}$, they would find some unusual properties.

- $F \propto q$
- $F \propto v$
- $F \propto B$

} not unusual

- $F \propto \sin\theta$ — if $\theta = \pm\frac{\pi}{2}$, F is max. if $\theta = 0, \pi$ F is zero
- $\vec{F}$ is $\perp$ to both $\vec{v}$ & $\vec{B}$.



These observations can be summarized as follows:

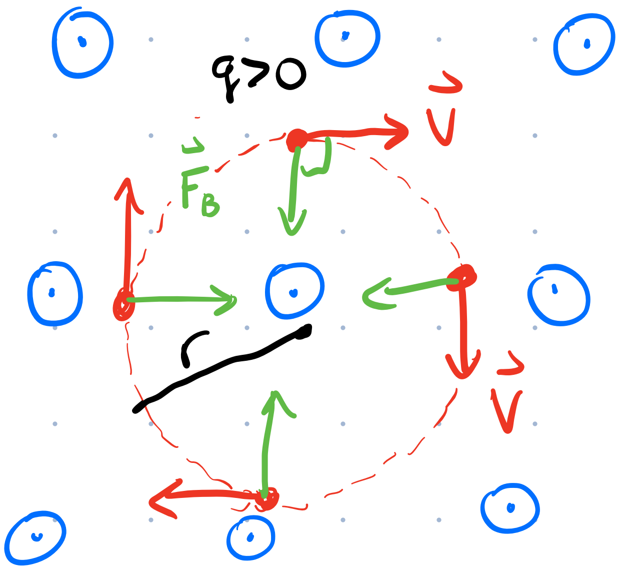
$$\vec{F}_B = q\vec{v} \times \vec{B}$$ Lorentz Force Law

If there are both electric & magnetic fields in region of space through which a charge moves, then

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Consequences of Lorentz force law.

① q moving through a <u>uniform</u> magnetic field w/ $\vec{v} \perp \vec{B}$



Since $\vec{F}_B \perp \vec{v}$, <u>speed</u> of the charge doesn't change. <u>Only</u> its dir'n of motion changes

$\therefore$ K.E. $= \frac{1}{2}mv^2$ is const.

$$W = \int_{\vec{a}}^{\vec{b}} \vec{F}_B \cdot d\vec{l} = \Delta K = 0$$

the magnetic force on q does <u>zero</u> work.

A charge moving $\perp$ to a uniform magnetic field undergoes circular motion.

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$|\vec{F}_B| = q|\vec{v} \times \vec{B}| = qvB\underbrace{\sin\theta}_{1 \text{ since } \theta = \frac{\pi}{2}}$$

$$F_B = qvB$$

Net Force on an object in circular motion is

$$F = ma_c = m\frac{v^2}{r}$$

$$\therefore \quad m\frac{v^2}{r} = qvB$$

$$\boxed{r = \frac{mv}{qB}}$$ radius of circular motion.

The time it takes to complete one loop (period, T) is:

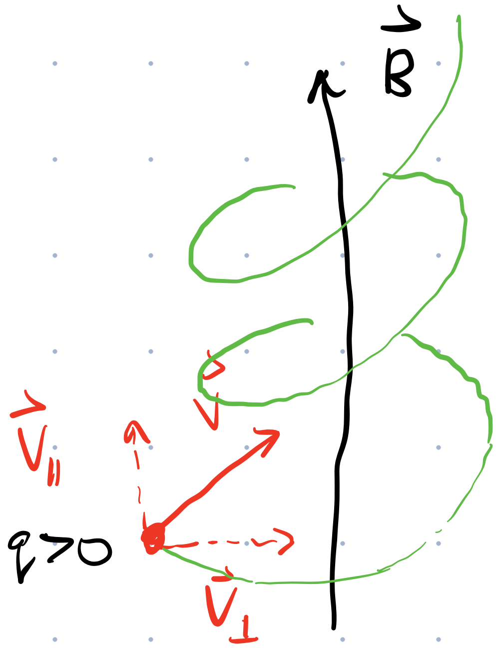
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB} \Rightarrow \boxed{\omega_c = \frac{2\pi}{T} = \frac{qB}{m}}$$

cyclotron freq.

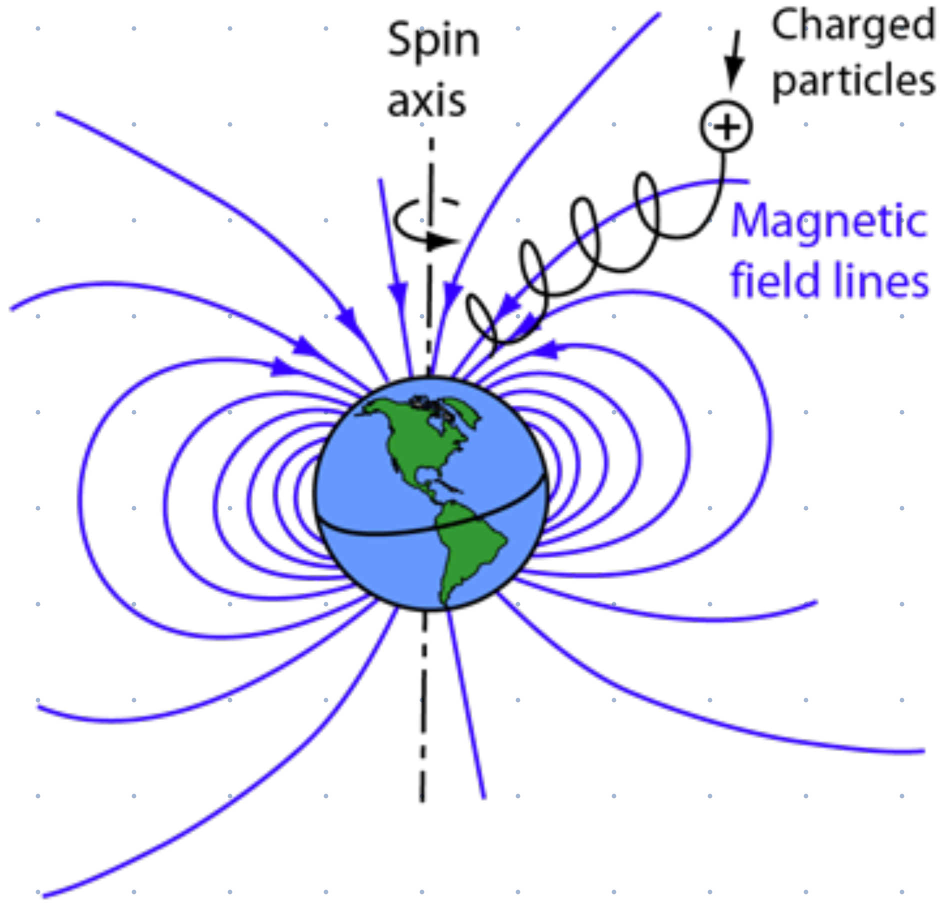
TRIUMF

If the particle's motion has components $\perp$ & $\parallel$ to $\vec{B}$, then parallel component of velocity is unaffected by $\vec{B}$.



Particle spirals around $\vec{B}$.

The Northern lights or Aurora Borealis is due to charged particles from the sun (solar wind) that spiral around the Earth's magnetic field near the poles


Spin
axis
Charged
particles
Magnetic
field lines