PHYS 301: Electricity & Magnetism.

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https://cmps-people.ok.ubc.ca/jbobowsk/phys301.html

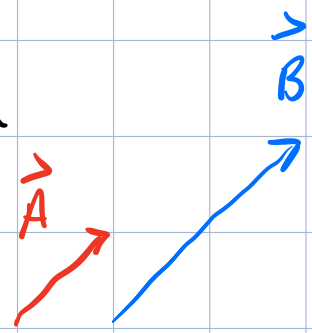
Griffiths "Introduction to Electrodynamics"
5th Ed.

Chapter 1. Vector Analysis / Calculus

Three types of vector multiplication.

1. Scalar multiplication

$$\vec{B} = a\vec{A}$$



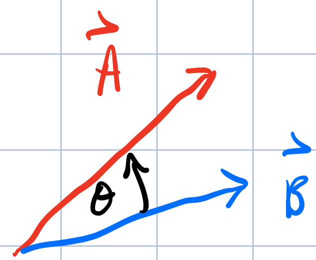
for $a \approx 2$.

Changes in the length or mag. of the vector.

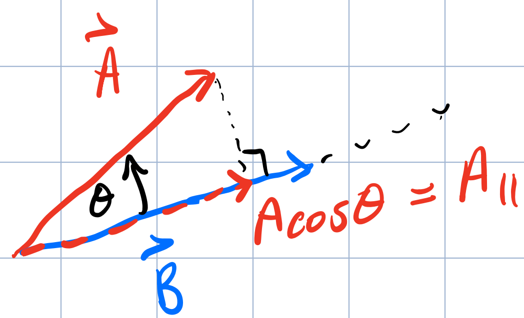
2. Dot Product.

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = AB\cos\theta.$$

if $A = |\vec{A}|$
$B = |\vec{B}|$

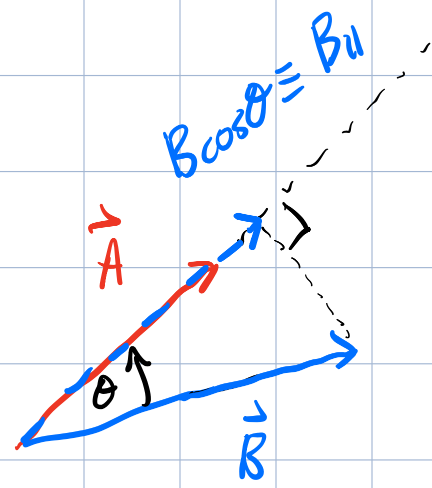


$$\vec{A}\cdot\vec{B} = (A\cos\theta)B$$



$A_{\parallel}$ is the component of $\vec{A}$ that is parallel to $\vec{B}$.

Alternatively:



$$\boxed{\therefore \quad \vec{A}\cdot\vec{B} = AB\cos\theta = A_{\parallel} B = A B_{\parallel}}$$

Result of a dot product is a scalar quantity.

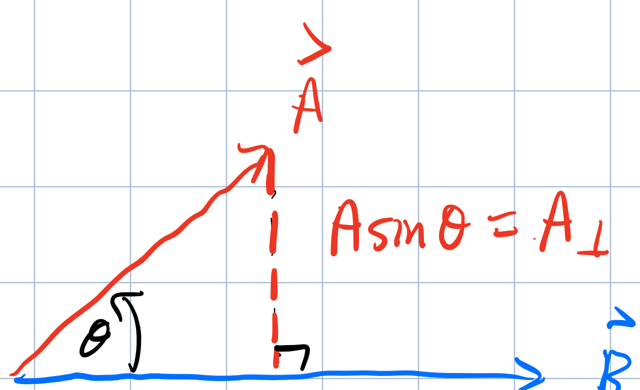
3. Cross Product.

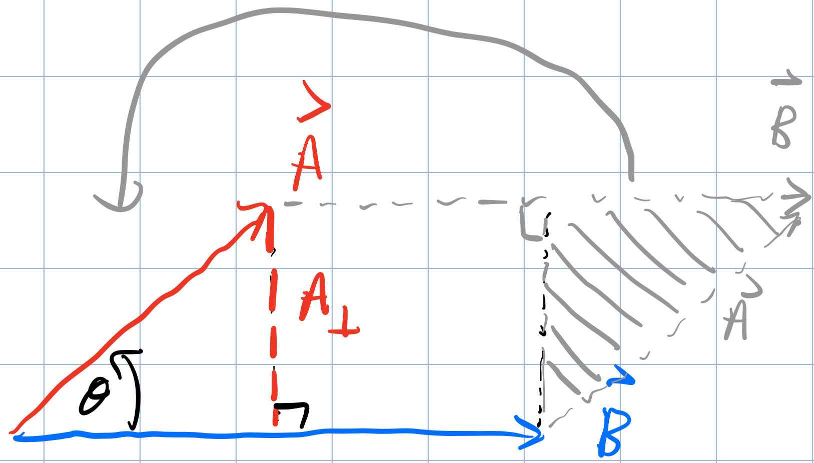
$$\vec{A}\times\vec{B} = \vec{C}$$

Result of a cross product is another vector.

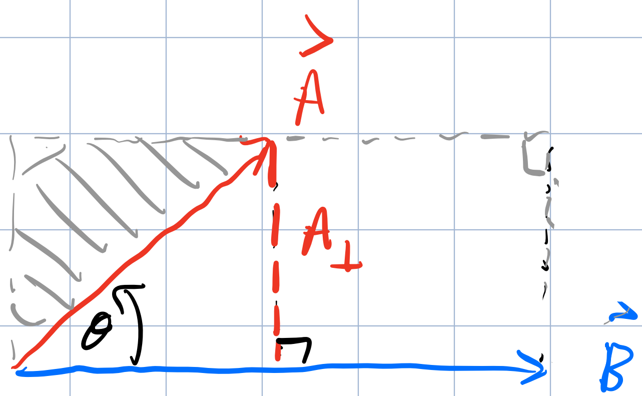
Magnitude of $\vec{C}$: $|\vec{C}| = |\vec{A}\times\vec{B}| = AB\sin\theta$

$A\sin\theta \equiv A_{\perp}$





Vectors $\vec{A}$ & $\vec{B}$ can be used to define a parallelogram.



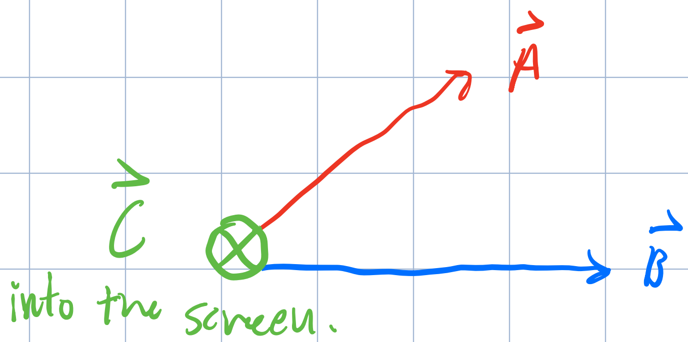
The above rectangle & parallelogram have the same area:

$$\begin{aligned} \text{area} = |\vec{C}| &= A_\perp B \\ &= (A\sin\theta)\, B \\ &= AB\sin\theta \\ &= |\vec{A}\times\vec{B}| \end{aligned}$$

Magnitude of $\vec{A} \times \vec{B}$ finds area of parallelogram.

If $\vec{C} = \vec{A} \times \vec{B}$, need the dir'n of $\vec{C}$.

By definition, $\vec{C}$ is $\perp$ to both $\vec{A}$ & $\vec{B}$.



Since $\vec{A}$ & $\vec{B}$ form a plane in the screen, $\vec{C}$ must be either into or out of the screen. Use right-hand rule (RHR) to find the dir'n.

Note that $\vec{D} = \vec{B} \times \vec{A}$ has dir'n <u>out</u> of the screen.

$$\boxed{\therefore \quad \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

__Reminder:__ Can use determinant of a matrix to evaluate cross products.

In Cartesian coordinates:

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$$\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y \text{ ②} & \text{①} B_z \end{vmatrix}$$

← unit vectors

← components of first vector

← components of 2nd vector

$$= \hat{x}\left(A_y B_z - A_z B_y\right) - \hat{y}\left(A_x B_z - A_z B_x\right)$$

$$+ \hat{z}\left(A_x B_y - A_y B_x\right)$$

Triple Product Identities

(i) Scalar Triple Product:

$$\vec{A} \cdot (\vec{B} \times \vec{C})$$

Work the identity by expressing the 3 vector in Cartesian coord.

$$(A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) \cdot \Big[(B_y C_z - B_z C_y) \hat{x} - (B_x C_z - B_z C_x) \hat{y} + (B_x C_y - B_y C_x) \hat{z} \Big]$$

$$= A_x (\underline{B_y} C_z - \underline{B_z} C_y) - A_y (\underline{B_x} C_z - \underline{B_z} C_x) + A_z (\underline{B_x} C_y - \underline{B_y} C_x)$$

$$= B_x (A_z C_y - A_y C_z) + B_y (A_x C_z - A_z C_x)$$
$$B_z (A_y C_x - A_x C_y)$$

$$= \vec{B}\cdot\left(\vec{C}\times\vec{A}\right)$$

$$\therefore\ \vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \vec{B}\cdot\left(\vec{C}\times\vec{A}\right) = \vec{C}\cdot\left(\vec{A}\times\vec{B}\right)$$

preserves the order of
$$\begin{tikzcd} & \vec{A} \arrow[dr] & \\ \vec{C} \arrow[ur] & & \vec{B} \arrow[ll] \end{tikzcd}$$

(ii) Vector Triple Product:

Exercise for the student. Show that:

$$\boxed{\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}\left(\vec{A}\cdot\vec{C}\right) - \vec{C}\left(\vec{A}\cdot\vec{B}\right)}$$

ABC → BAC − CAB

$\vec{A}\cdot\vec{C}$ & $\vec{A}\cdot\vec{B}$ are scalars.

$\therefore$ Vector Triple Product can be re-expressed as simple vector subtraction.