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Measurement of Boltzmann's constant

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Inman and Miller (1973) have shown how to use a power transistor to measure e/k, where e is the magnitude of the electron charge, and k is Boltzmann's constant. Knowing e, the results may be used to deduce k. The purpose of this short article is to draw attention to this method and to show how the experiment may be performed using cheap, readily available apparatus. As the basic equation used depends on the Boltzmann distribution law, this important relation is also verified. (Ogborn mentions this aspect of the Inman and Miller experiment in the revised Nuffield A-level course.)

A p–n junction in a semiconductor is a region where the potential energy of an electron changes due to what is essentially a contact potential difference between the p-type and the n-type regions. In particular, the potential step is such that an electron moving from the n- to the p-type region gains potential energy and loses kinetic energy (figure 1a). It therefore follows that, for an electron to move from the n- to the p-type region, it must have an energy greater than eV_0, where V_0 is the contact potential difference. The number of electrons in the n-type region with an energy greater than eV_0 is determined by $\exp(-eV_0/kT)$, the Boltzmann distribution law. Hence there will be diffusion of electrons from the n- to the p-type region, resulting in an electron current, i_0, given by

$$i_0=a\exp(-eV_0/kT), \qquad (1)$$

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where a is a constant of proportionality which depends on V_0, T and the nature of the semiconductor.

There is also an electron current from the p- to the n-type region. The source of these electrons is the freeing of electrons in the p-type region by the breaking of bonds (i.e. the thermal creation of an electron–hole pair). Some of these electrons will diffuse to the junction region, where the electric field caused by the contact potential difference will drive them across the junction. In equilibrium, the magnitude of this reverse current must be equal to the diffusion current, and so it is also given by equation (1).

If now a forward bias, V, is applied to the junction, the height of the potential step now alters to (V_0-V) and the forward electron current is now proportional to $\exp[-e(V_0-V)/kT]$ (figure 1b). The reverse current is unaffected by the change in bias, so the observed electron current is now

$$I=a\exp[-e(V_0-V)kT]-a\exp(-eV_0/kT)$$
$$=i_0[\exp(eV/kT)-1]. \qquad (2)$$

Thus measuring the forward current of a diode as a function of the bias voltage V would seem to enable one to find experimentally the value of e/k. Unfortunately, this is not the case, as it turns out there are other contributions to the diode current, which have the effect of modifying equation (2) to

$$I=i_0[\exp(eV/mkT)-1], \qquad (3)$$

where the value of the number m can be anywhere between 1 and 2.5, depending on diode type and also the bias voltage (Clayton 1979 p171, Neudeck 1983 pp67–8).

Fortunately, equation (2) can be applied directly to a transistor circuit. The base–emitter junction is a p–n junction, and it is found that if the collector current I_c is measured as a function of the base–emitter voltage V_{be}, with the collector–base voltage V_{cb} kept at zero then equation (2) holds exactly†. The majority carrier and surface current effects which give the factor of m in equation (3) are still present and do contribute to the emitter current, but these currents flow through the *base* contact of the transistor, and so will not be measured. In fact, the base current which must be supplied to a transistor arises partially from the need to supply these extraneous diode currents. Equation (2) is, in this context, called the Ebers–Moll equation.

For the values of V_{be} used in this experiment, it turns out that $\exp(eV_{be}/kT)>>1$, hence the collec-

†The collector current will include a leakage current given by equation (3) with $V=V_{cb}$. By keeping $V_{cb}=0$ at all times, this contribution to I_c is always zero (Clayton 1979 p173).

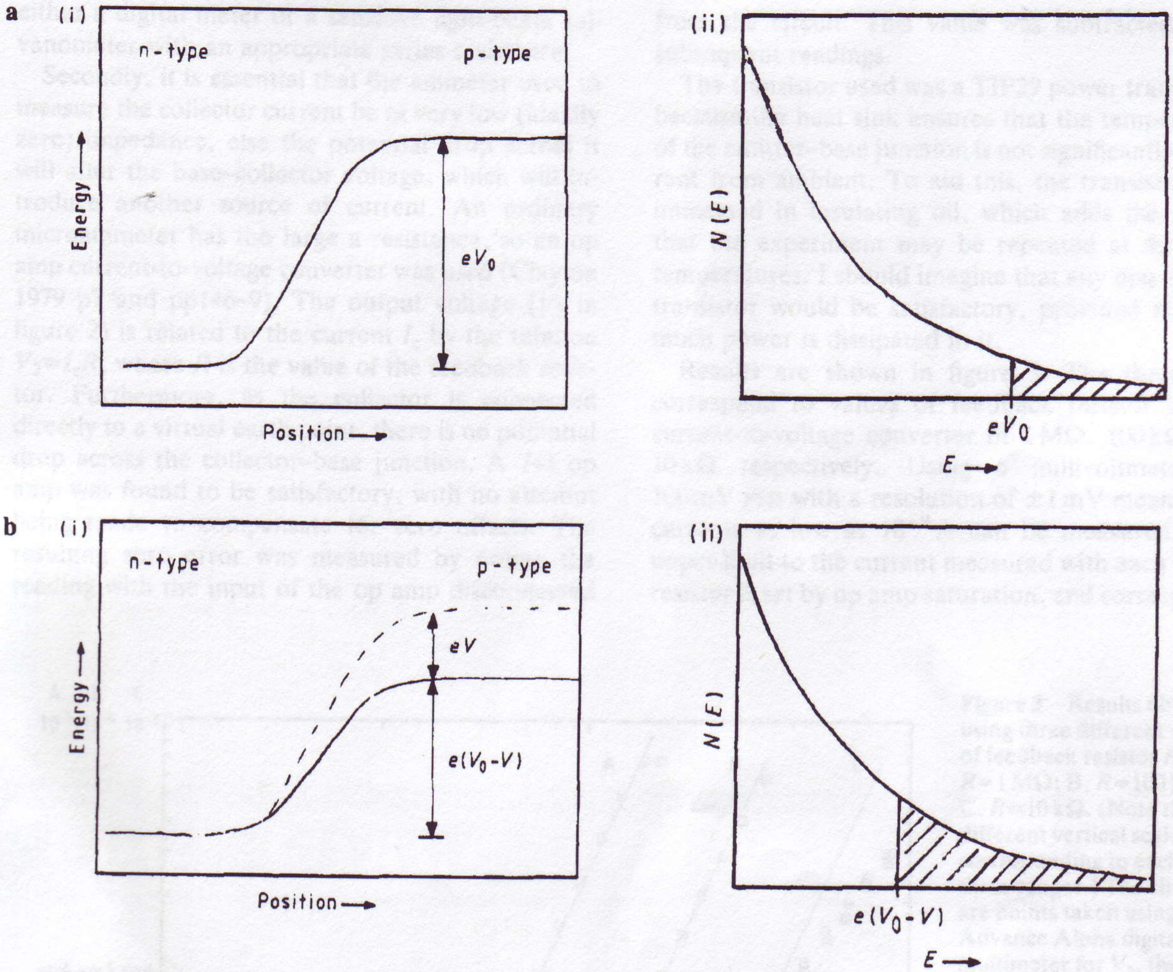


Figure 1a Graphs of (i) the potential energy of an electron and (ii) the Boltzmann distribution for a p–n junction with zero bias. The number of electrons with sufficient energy to cross the junction is proportional to the shaded area. **b** Potential energy and Boltzmann distribution graphs for a p–n junction with a forward bias voltage V. The shaded area now shows that many more electrons can cross the junction

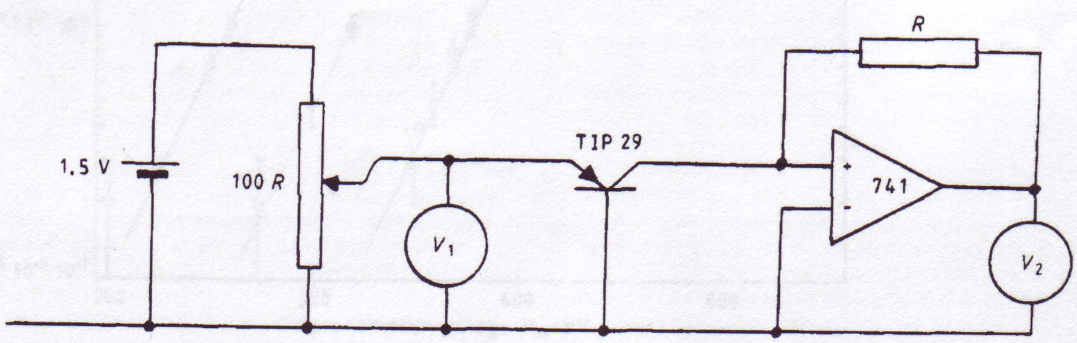
tor current is given to a good approximation by

$$I_c = \exp(eV_{be}/kT). \qquad (4)$$

A graph of $\ln I_c$ against V_{be} should therefore be linear, with a slope of e/kT.

The circuit used to measure V_{be} and I_c is shown in figure 2 (Inman and Miller 1973). Some care needs to be exercised in the choice of instruments to measure these quantities. First, a small change in V_{be} causes a large change in I_c. Hence V_{be} has to be measured to a high precision, typically ± 1 mV in about 200–500 mV (see figure 3). Such a resolution is beyond most pointer instruments and requires

Figure 2 The circuit used. The feedback resistor R was varied between 1 MΩ and 10 kΩ, depending on the current to be measured



either a digital meter or a sensitive light-beam galvanometer with an appropriate series resistance.

Secondly, it is essential that the ammeter used to measure the collector current be of very low (ideally zero) impedance, else the potential drop across it will alter the base–collector voltage, which will introduce another source of current. An ordinary microammeter has too large a resistance, so an op amp current-to-voltage converter was used (Clayton 1979 p7 and pp146–9). The output voltage (V_2 in figure 2) is related to the current I_c by the relation $V_2 = I_c R$, where R is the value of the feedback resistor. Furthermore, as the collector is connected directly to a virtual earth point, there is no potential drop across the collector–base junction. A 741 op amp was found to be satisfactory, with no attempt being made to compensate for zero offsets. The resulting zero error was measured by noting the reading with the input of the op amp disconnected from the circuit. This value was subtracted from subsequent readings.

The transistor used was a TIP29 power transistor, because the heat sink ensures that the temperature of the emitter–base junction is not significantly different from ambient. To aid this, the transistor was immersed in insulating oil, which adds the bonus that the experiment may be repeated at different temperatures. I should imagine that any npn or pnp transistor would be satisfactory, provided not too much power is dissipated in it.

Results are shown in figure 3. The three sets correspond to values of feedback resistor in the current-to-voltage converter of 1 MΩ, 100 kΩ and 10 kΩ respectively. Using a millivoltmeter of 100 mV FSD with a resolution of ±1 mV means that currents as low as 10^{-9} A can be measured. The upper limit to the current measured with each range resistor is set by op amp saturation, and corresponds

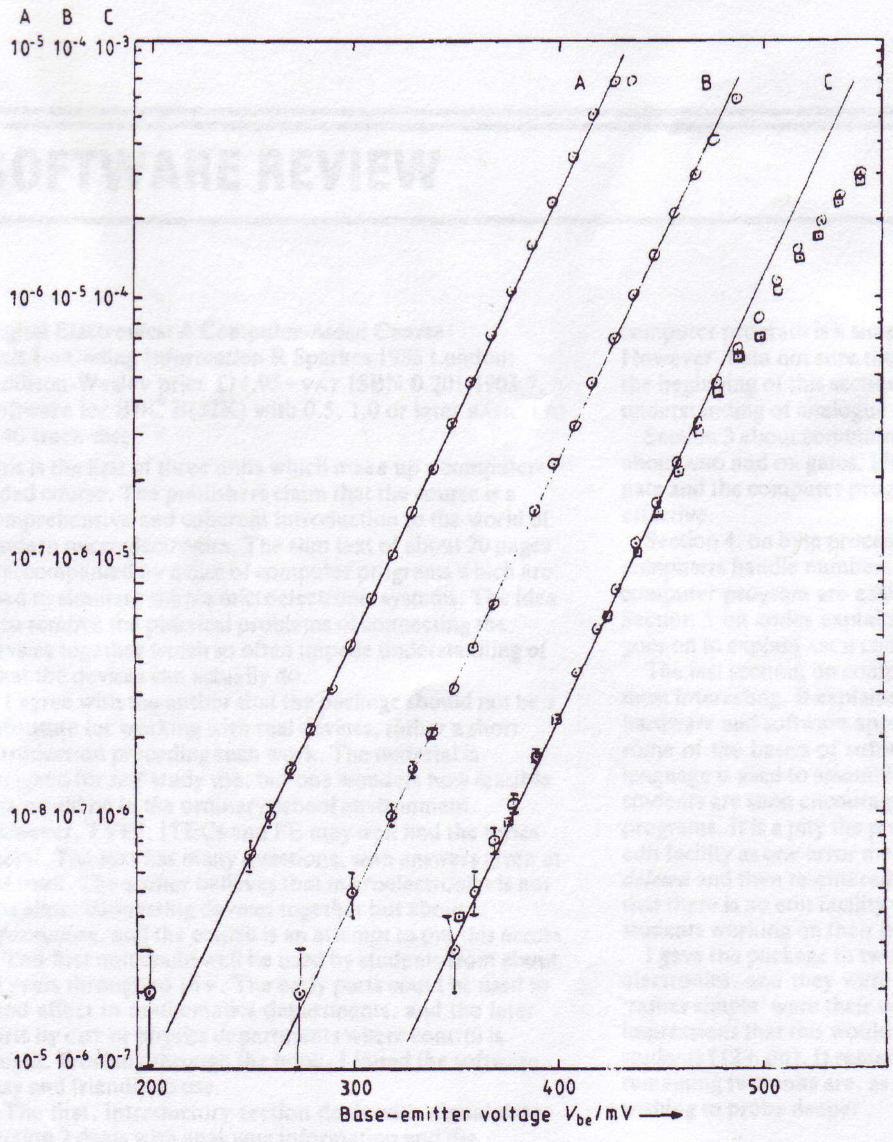


Figure 3 Results obtained using three different values of feedback resistor R: A, $R = 1$ MΩ; B, $R = 100$ kΩ; C, $R = 10$ kΩ. (Note the different vertical scales corresponding to each of these ranges.) The circles are points taken using an Advance Alpha digital multimeter for V_2, the squares are points taken using an AVO Mark II analogue multimeter. Ambient temperature was 290.1 K. The error bars correspond to the ±1 digit resolution of the multimeter. A digital multimeter was also used for V_1

to V_2 (about 10 V). Note that each range overlaps its neighbour by two decades of current; corresponding readings on each range were found to agree to within the error of reading of the voltmeter. For most readings an Advance Alpha digital multimeter was used, but readings on the highest range ($R=10\,k\Omega$, $V_2=1$ mV to 10 V, $I_c=10^{-7}$ A to 10^{-3} A) were repeated using an AVO Mark II multimeter. The slope of the line A was calculated using a least squares fitting routine. In this, the lowest value of I_c was omitted because of its low precision, the highest value because of evidence that the op amp was saturating. The lines B and C are drawn parallel to line A. Note that the linearity is preserved over nearly five orders of magnitude of current. The departure from linearity just below 10^{-4} A is probably due to ohmic voltage drop across the bulk material of the transistor, though it is possible that self heating at the junction could be present.

The final result of $e/k=(1.132\pm0.002)\times10^4\,C\,K\,J^{-1}$ differs from the accepted value of $1.160\times10^4\,C\,K\,J^{-1}$ by 3%, which is within the tolerance of the meters used and the feedback resistors in the current-to-voltage converter. The latter were ordinary radio resistors and were not specially selected. This demonstration shows that using simple equipment a good measure of an important atomic constant may be achieved. The good linearity attained also verifies the Boltzmann distribution law.

References

Clayton G B 1979 *Operational Amplifiers* (2nd edn) (London: Newnes-Butterworths)

Inman F W and Miller C E 1973 *Am. J. Phys.* **41** 349–51

Neudeck G W 1983 *The PN Junction Diode* (Reading, Mass: Addison-Wesley)

Nuffield Advanced Level Physics (Revised Version) Teachers' Guide 2 (Harlow: Longman) p 295

SOFTWARE REVIEW

Digital Electronics: A Computer-Aided Course Unit 1—Coding Information R Sparkes 1986 London: Addison-Wesley price £14.95+VAT ISBN 0 201 1903 7. Software for BBC B(32K) with 0.5, 1.0 or later BASIC 1 or 2 40 track disc

This is the first of three units which make up a computer-aided course. The publishers claim that the course is a comprehensive and coherent introduction to the world of modern microelectronics. The slim text of about 20 pages is accompanied by a disc of computer programs which are used to simulate simple microelectronic systems. The idea is to remove the practical problems of connecting the devices together which so often impede understanding of what the devices can actually do.

I agree with the author that the package should not be a substitute for working with real devices, rather a short introduction preceding such work. The material is designed for *self*-study use, but one wonders how feasible this would be in the ordinary school environment. However, TVEI, ITECs and FE may well find the series useful. The text has many questions, with answers given at the back. The author believes that microelectronics is not just about connecting devices together but about *information*, and the course is an attempt to put this across.

This first unit could well be used by students from about 12 years through to 16+. The early parts could be used to good effect in mathematics departments, and the later parts by CDT or physics departments where control is taught. Working through the book, I found the software easy and friendly to use.

The first, introductory section deals with digital data. Section 2 deals with analogue information and the computer program is a simple aid to understanding. However, I am not sure that the inclusion of Ohm's law at the beginning of this section is necessary for an understanding of analogue information.

Section 3 about combining digital data is really just about AND and OR gates. I liked the example for the AND gate and the computer program is *very* simple but effective.

Section 4, on byte processes, is concerned with how computers handle numbers. The first part and the computer program are easier to follow than the later part. Section 5 on codes explains the need for them and then goes on to explain ASCII coding.

The last section, on computers in control, is by far the most interesting. It explains the difference between the hardware and software approach and goes on to introduce some of the basics of software engineering. A LOGO-like language is used to assemble control sequences and students are soon encouraged to set up their own programs. It is a pity the programmer did not include an edit facility as one error means the whole program must be *deleted* and then re-entered. The note on page 16 warning that there is no edit facility will not be of any help to students working on their own!

I gave the package to two students about to take O-level electronics, and they were not impressed by the it—'rather simple' were their words. This confirmed my earlier impressions that this would be more useful with junior students (12+ on). It remains to be seen whether the remaining two units are, as stated in the preface, 'for those wishing to probe deeper'.

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