PHYS 232 April, 4 2025

Due dates:

- Experiment #5 lab notebooks : Now
- Formal Report : Mon. Apr. 7 @ 14:00
  - Bring printed and stapled report to SCI 241.
- Bring formula sheet (8.5" × 11", both sides) and calculator to final exam.

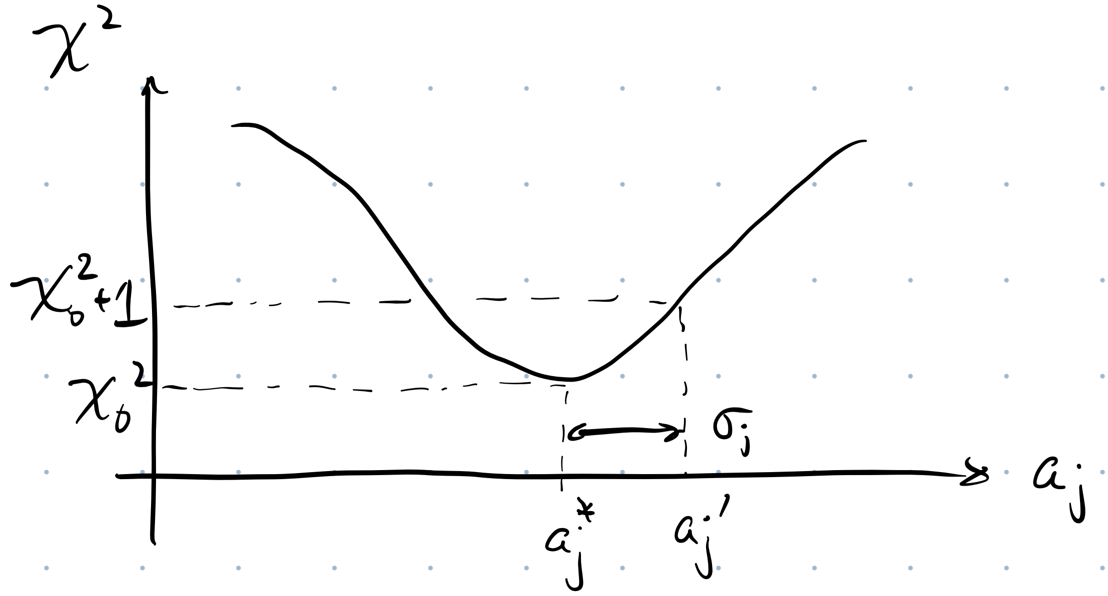
Last Time:

If meas. $(x_i, y_i \pm \sigma_i)$ for $i = 1..N$ & fit the data to a model with m parameters $a_1, a_2, \ldots, a_m$ then we have $\nu = N - m$ degrees of freedom and we expect

$$\chi^2 \approx \nu = N - m$$

The reduced chi-squared χ_ν^2 is defined to be $\chi_\nu^2 = \frac{\chi^2}{\nu}$. For a good fit, expect

$$\chi_\nu^2 \approx 1.$$



If a_j^* corresponds to minimum value of χ^2 which is denoted χ_0^2 and...

a_j' corresponds to a value of χ^2 equal to $\chi_0^2 + 1$ (an increase of 1 above the minimum value), then...

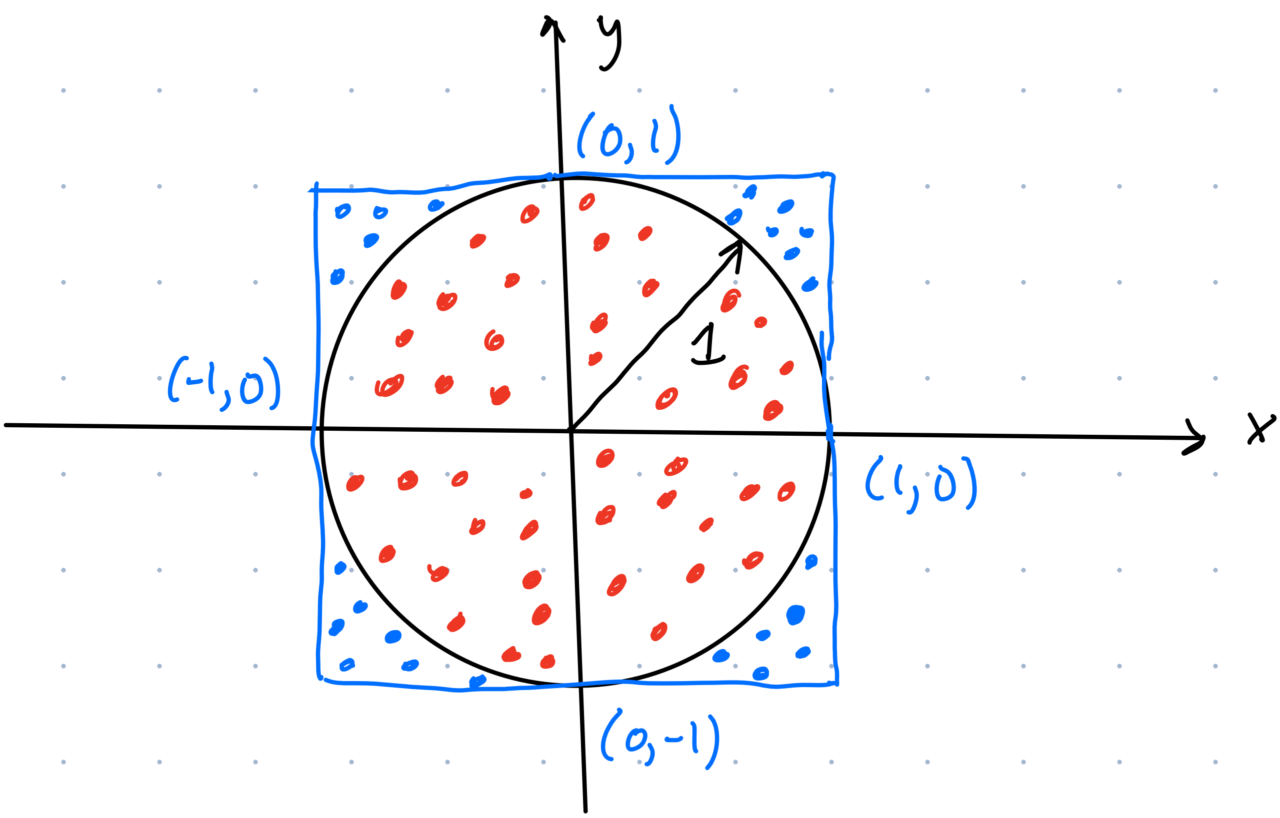
$|a_j' - a_j^*|$ is an estimate of σ_j which is the uncertainty in the best-fit value of parameter a_j.

Monte Carlo Simulations (Brief Introduction)

- useful for numerical integration (esp. high-dimension integrals)

The Monte Carlo method uses repeated random sampling to obtain numerical results.

Example 1: Computing π



Assume that we can produce random nos uniformly drawn from the interval $[-1, 1]$

Repeatedly draw sample pts (x_i, y_i) from $[-1,1] \times [-1,1]$

Prob. that radom pt lands within the unit circle is:

$$p = \frac{\text{area of circle}}{\text{area of square}} = \frac{\pi(1)^2}{2 \cdot 2} = \frac{\pi}{4}$$

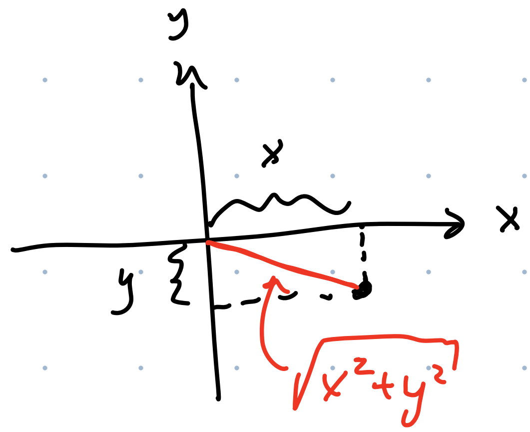
$$\therefore \boxed{\pi = 4p} \quad ①$$

Have expressed desired quantity π in terms of a prob. p. Est. value of p using a Monte Carlo simulation.

To determine the value of p, we general n random coordinates (x, y). Prob. that we get a pt inside the circle (hit) is

$$p = \frac{\#\text{hits}}{\#\text{trials}} = \frac{z}{n} \quad \begin{array}{l} \leftarrow \text{hits} \\ \leftarrow \text{trials.} \end{array} \quad ②$$

We get a hit when the distance from the origin is less than the radius of the circle $(r=1)$.



require

$$x^2+y^2<1$$

for a hit.

$$\boxed{\pi = 4p = 4\frac{z}{n}}$$

If we count the no. of hits z, then from the Poisson dist'n, the uncertainty in z is $\sigma_z = \sqrt{z}$ (counting expts)

$$\therefore \sigma_\pi = \left|\frac{\partial \pi}{\partial z}\sigma_z\right| = \frac{4}{n}\sqrt{z}$$

$\therefore \sigma_\pi$ decrease as n increases.

$$\propto \frac{1}{\sqrt{n}}$$

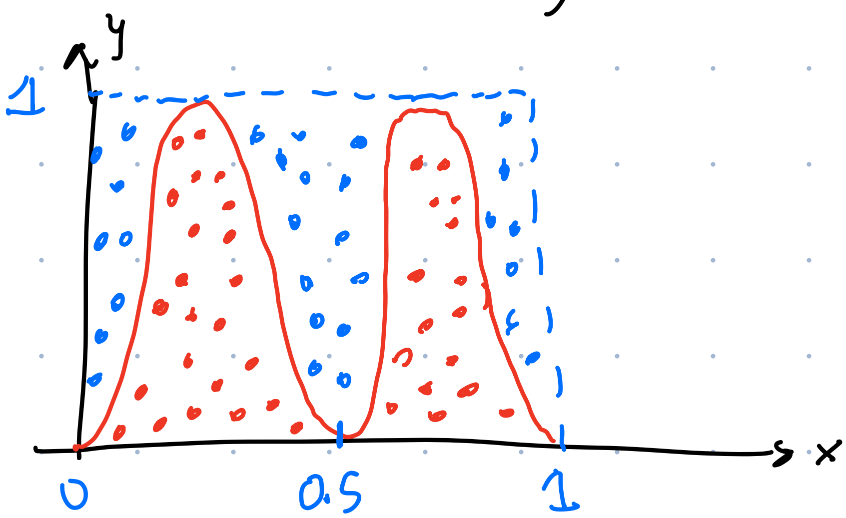
Hit & Miss Monte Carlo Integration.

Assume that we want to evaluate

$$\int_a^b f(x)\,dx = \text{area under a curve.}$$

Eg. $a=0, \quad b=1$

$$f(x) = \frac{1}{27}\Big(-65536x^8 + 262144x^7 - 409600x^6 + 311296x^5 - 114688x^4 + 16384x^3\Big)$$

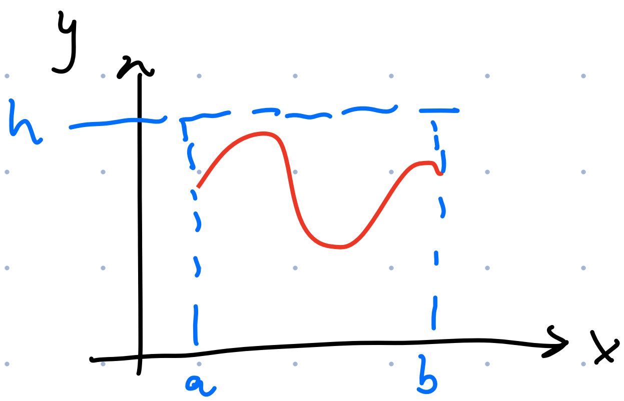


On $0 \leq x \leq 1$, this fcn $f(x)$ fits inside a unit square.

Use the same hit & miss simulation to find prob. of a pt landing beneath the curve.

1. Draw a pt (x, y)
2. If $f(x) > y$, then we get a hit $z \to z+1$

$$p = \frac{z}{n} = \frac{\text{area under curve}}{\text{area of square}} = \frac{\int_a^b f(x)\,dx}{(b-a)h}$$



In our example, $a=0$, $b=h=1$

$$\boxed{I = \int_0^1 f(x)\,dx = \frac{z}{n}}$$

$$\sigma_I \simeq \left| \frac{\partial I}{\partial z} \sigma_z \right| = \frac{\sqrt{z}}{n} \underset{\sim}{\propto} \frac{1}{\sqrt{n}}$$