PHYS 232 Jan. 29, 2025

- Assignment #2 is on the course website.
  - due Feb. 7 @ 09:00

Last Time:

$\boxed{\text{Discrete distributions:}}$

$$\mu = \sum_{j=1}^{n} x_j P(x_j)$$

$$\sigma^2 = \sum_{j=1}^{n} (x_j - \mu)^2 P(x_j)$$

$$= \sum_{j=1}^{n} x_j^2 P(x_j) - \mu^2 = \overline{x^2} - \bar{x}^2$$

$\boxed{\text{Continuous Distributions}}$

$$\mu = \int_{-\infty}^{\infty} x P(x) dx$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 P(x) dx$$

$$= \int_{-\infty}^{\infty} x^2 P(x) dx - \mu^2 = \overline{x^2} - \bar{x}^2$$

<u>Today</u>: The Binomial Dist'n - an important example of a discrete dist'n.

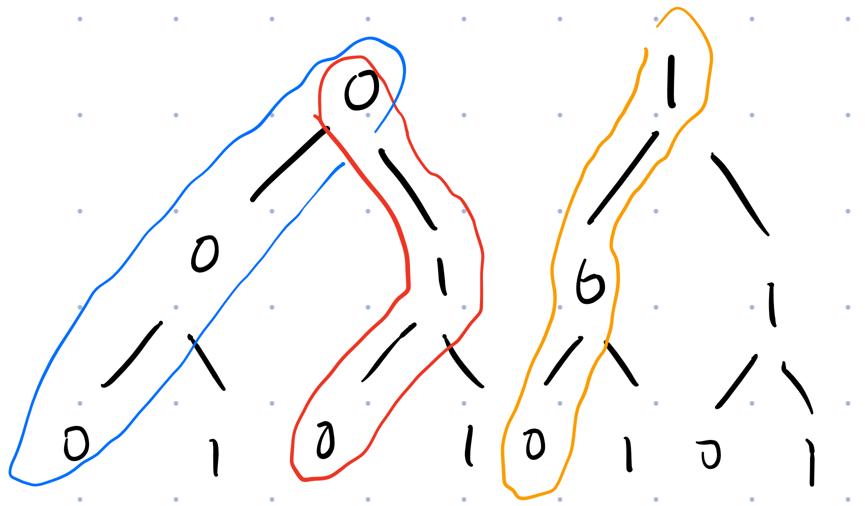
Use when the result of an experiment can be only one of a small no. of outcomes.

- rolling die
- flipping a coin
- drawing a card
- random walk

⋮

If toss n coins in air, how many possible outcomes are there?

First coin has 2 possible outcomes
2nd " " " " "



2^1 outcomes

$2^2 = 4$ outcomes

$2^3 = 8$ outcomes

⋮

2^n outcomes for n coins.

The prob. of each individual outcome are all equal to $\frac{1}{2^n}$

For $n=5$ prob. of getting 00000 sames as prob. of getting exactly 01100 in this order.

coin 1 2 3 4 5

However, there are many ways of getting 2 heads & 3 tails.

11000   01100   00110   00011
10100   01010   00101
10010   01001
10001

10

In general, if we toss n coins, how many ways or Permutations $P_m(n, x)$ are there of x heads & n-x tails when we do n trials?

① Select from n coins in front of you & put one into heads piles

n choices

② Select from n-1 remaining coins & add 1 to heads pile

n-1 choices

⋮

ⓧ Select from $n-(x-1)$ coins remaining & add one to heads pile.

$n-x+1$ choices.

The total no. of permutations is:

$$P_m(n,x) = n(n-1)(n-2)\cdots(n-x+2)(n-x+1)$$

$$= \frac{n!}{(n-x)!}$$

Eg. $n=5$, $x=2$

$$P_m(5,2) = \frac{5!}{3!} = 5 \cdot 4 = \boxed{20}$$

Why is $P_m(5,2) = 20$ different than 10 possibilities that we drew?

Consider the 11000 case.

$P_m(n,x)$ considers

$$1_1 \, 1_2 \, 0 \, 0 \, 0$$

$$1_2 \, 1_1 \, 0 \, 0 \, 0 \quad \text{as distinct outcomes.}$$

Choosing coin 1 first or second is distinct.

If have x heads, there are $x!$ ways to order them in the heads pile.

Eg. $x=2$

$1_1 \quad 1_2$

$1_2 \quad 1_1$

$x! = 2! = 2$

$x=3$

$1_1 \quad 1_2 \quad 1_3$

$1_3 \quad 1_1 \quad 1_2$

$1_2 \quad 1_3 \quad 1_1$

$1_3 \quad 1_2 \quad 1_1$

$1_1 \quad 1_3 \quad 1_2$

$1_2 \quad 1_1 \quad 1_3$

$x! = 3!$
$= 2 \cdot 3 = 6$

The no. of combinations $C(n, x)$ of x heads from n coins when the order of x doesn't matter is:

$$C(n,x) = \frac{P_m(n,x)}{x!} = \frac{n!}{x!\,(n-x)!}$$

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$ read as "n choose x"

Eg. $n=5,\ x=2$

$$\binom{5}{2} = \frac{5!}{2!\,(5-2)!} = \frac{5!}{2!\,3!} = \frac{4 \cdot 5}{2} = 10$$

✓

Probability: Since each outcome has the same prob. $\left(\frac{1}{2^n}\right)$ of occuring the prob. of getting x heads from n coins is:

$$P(x;n) = \binom{n}{x}\frac{1}{2^n} = \underbrace{\frac{n!}{x!(n-x)!}}_{\text{no. combinations}}\ \underbrace{\frac{1}{2^n}}_{\text{Prob. of any particular outcome}}$$

Valid only when prob. of getting result x in any single trial is $\frac{1}{2}$.

In general, the prob. of the possible outcomes could be different.

Eg. Weighted coin

Prob. of getting heads $\quad P_H = p$ (prob. of "success")

prob. of getting tails $\quad P_T = q = 1-p$

(prob. of "failure")

Thus, the prob. of x heads & $n-x$ tails is:

$$p^x q^{n-x} = p^x (1-p)^{n-x}$$

The binomial distribution is:

$$P_B(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

no. of "successes"

no. of trials

prob of success in any individual trial

$$\boxed{P_B(x;n,p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}}$$

Binomial Distribution.

"Guess" at what we expect the mean of P_B to be:

n trials.

prob. for success is p.

$$\boxed{\therefore \mu = np}$$

We will verify this result.

We will also show that $\sigma^2 = npq$

$= np(1-p)$