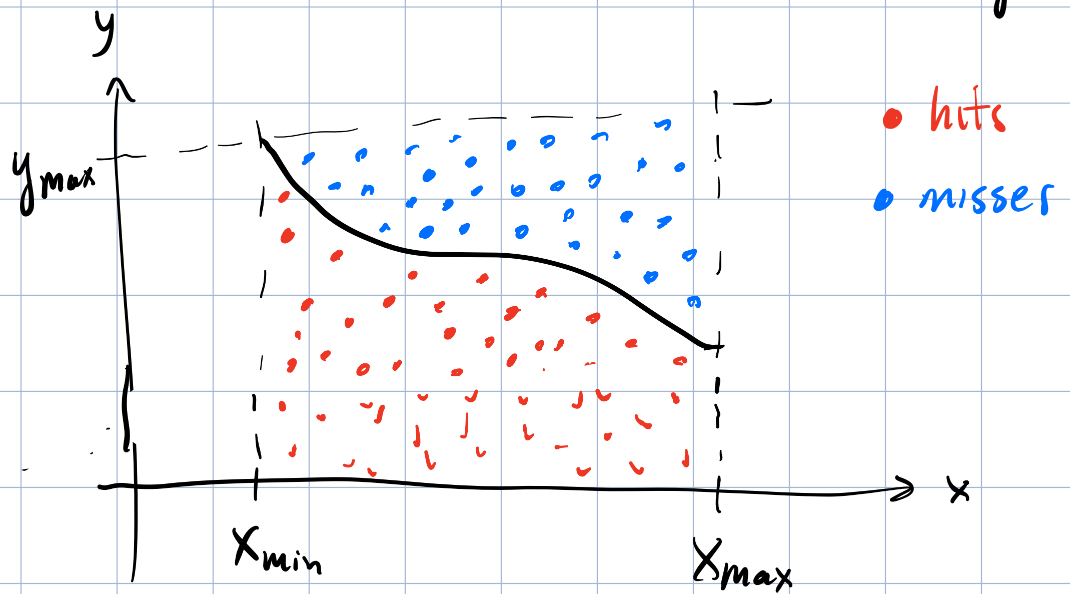
PHYS 232 April 10, 2024

✓ Final Exam: Can bring on 8.5"x11" piece of paper with anything written on it (both sides). Bring a calculator. Graphing calculators are fine.

Last Time: Hit & Miss Integration

$$\text{Prob. of hit} \quad \mathbb{P} = \frac{Z_n}{n} = \frac{\int_{x_{min}}^{x_{max}} f(x)\,dx}{(x_{max}-x_{min})\,y_{max}}$$

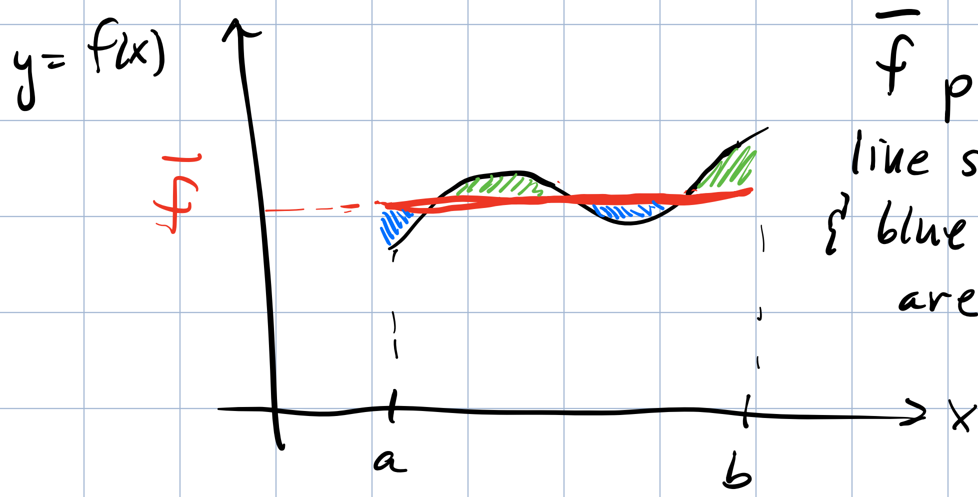


$$\boxed{\therefore I = \int_{x_{max}}^{x_{min}} f(x)\,dx = (x_{max}-x_{min})\,y_{max}\,\frac{Z_n}{n}}$$

uncertainty in I : $\boxed{\sigma_I \propto \frac{1}{\sqrt{n}}}$

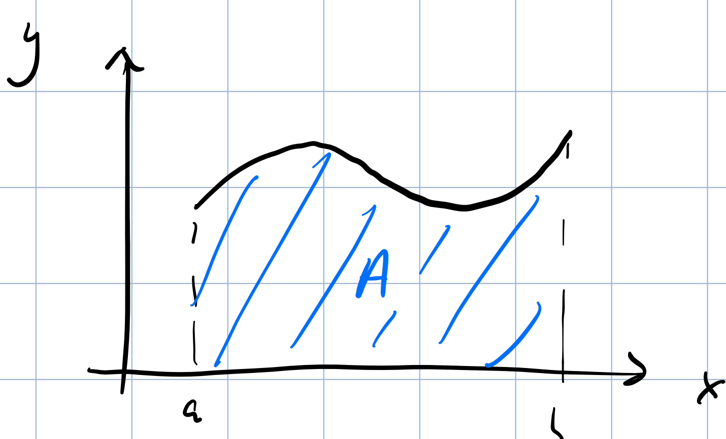
Monte Carlo Integration - Method #2.

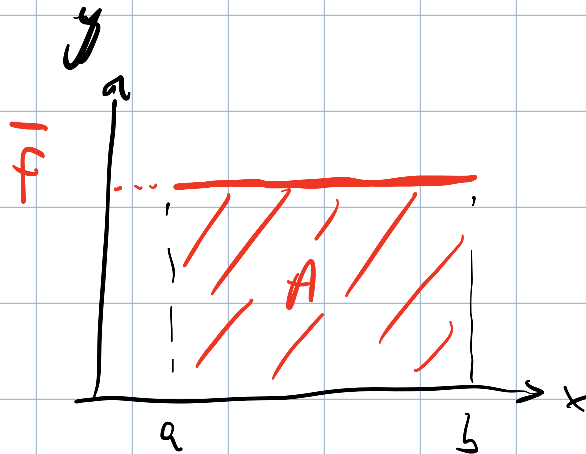
Average value of $f(x)$ over $a < x < b$



$\bar{f}$ place red line s.t. green & blue areas are equal.

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$





Notice that area under $f(x)$: $\int_a^b f(x)\,dx$

Is the same as area under red line representing $\bar{f}$: $\bar{f}(b-a)$

$$\boxed{\therefore \int_a^b f(x)\,dx = \bar{f}(b-a)}$$

We want a monte carlo method for calculating $\bar{f}$ so that it can be used to estimate

$$I = \int_a^b f(x)\,dx$$

Implemention :

① Generate random no x_i that lies between integration limits

$$a \le x_i \le b$$

② Calculate $y_i = f(x_i)$ using the value of x_i from step ①.

③ Keep a running total of all y_i value

$$y_{tot} = y_{tot} + y_i$$

④ Return to ① & repeat n times.

$$\bar{f} = \frac{y_{tot}}{n} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n} f(x_i)$$

$$I = \int_a^b f(x)\,dx \simeq (b-a)\bar{f}$$

Expect uncertainty in our estimate of I to decrease as n increases.

By Prop. of errors:

$$I = (b-a)\bar{f}$$

$$\sigma_I = \left| \frac{\partial I}{\partial \bar{f}} \sigma_{\bar{f}} \right| = (b-a)\,\sigma_{\bar{f}}$$

std dev. squared $\quad \sigma_{\bar{f}}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{f})^2$

for large $n \quad \frac{1}{n-1} \approx \frac{1}{n}$

$$\boxed{\therefore\ \sigma_{\bar{f}} \propto \frac{1}{\sqrt{n}}}$$

same as we had for hit & miss method.