PHYS 232 Mar. 13, 2024

- ✓ Assignment #5 due Fri, Mar. 22
- ✓ Sign up for Experiment #5 by the end of the day.

Last Time: Found best-fit parameters for weighted linear fits.

Intercept: $a \pm \sigma_a$

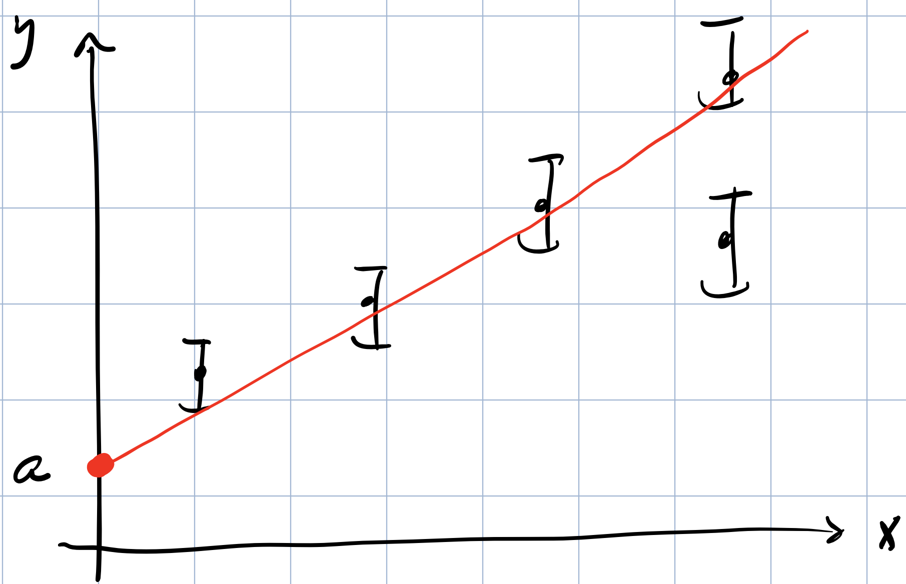
Slope: $b \pm \sigma_b$

$$\boxed{\begin{aligned} a &= \frac{1}{\Delta}\left(\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2}\right) \\ b &= \frac{1}{\Delta}\left(\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2}\right) \end{aligned}}$$

where: $\boxed{\Delta = \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2}$

$$\therefore \boxed{\sigma_a^2 = \frac{1}{\Delta}\sum \frac{x_i^2}{\sigma_i^2}} \qquad \therefore \boxed{\sigma_b^2 = \frac{1}{\Delta}\sum \frac{1}{\sigma_i^2}}$$

Today: Linearizing Experimental Data.



$y = a + bx$

Can determine best-fit values for $a \pm \sigma_a$ & $b \pm \sigma_b$ based on meas. of

$$(x_i, y_i \pm \sigma_i) \text{ for } i = 1..N$$

Easy to use when have obvious linear dependence.

Eg. Object in free fall

m

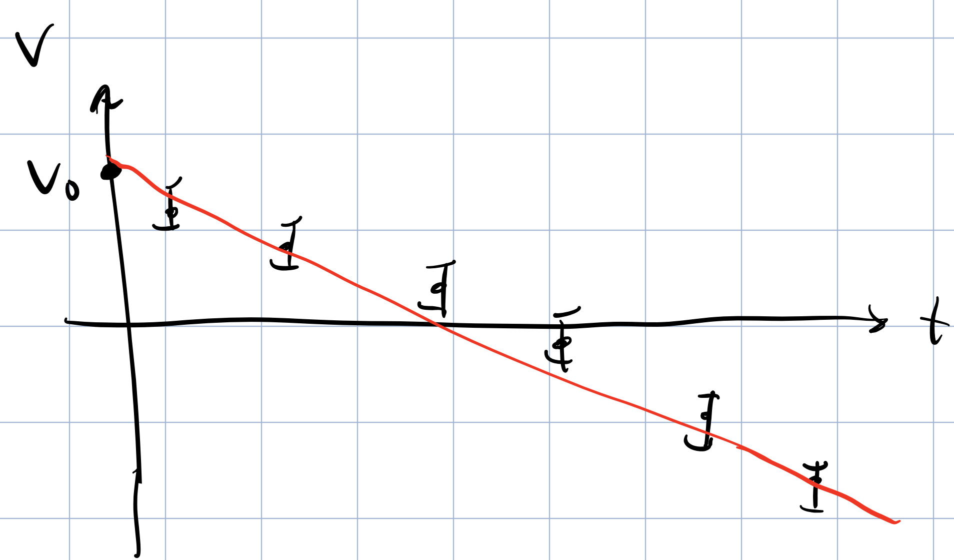
$m\vec{g}$

$$v = v_0 - g t$$

$$y = a + b x$$

meas. $v \pm \sigma_v$ as a fcn of t.

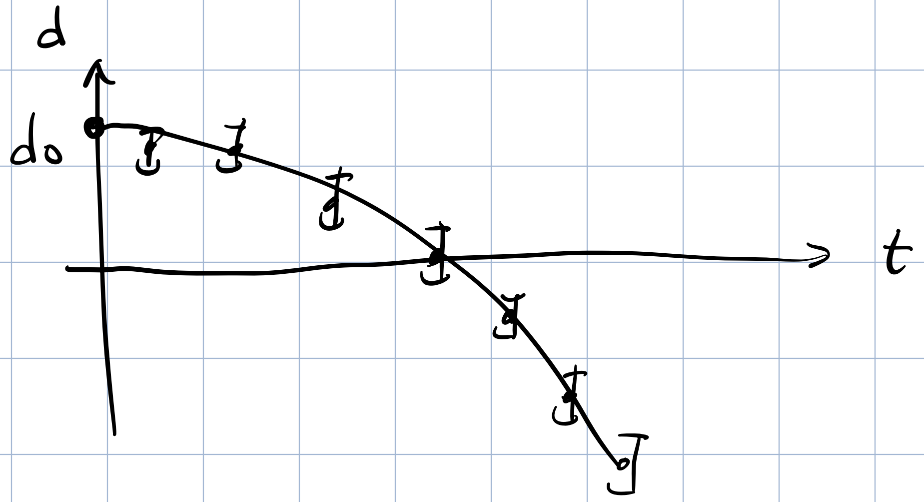
set $y = v$, $x = t$ $\Longrightarrow$ $a = v_0$, $b = -g$



Sometimes a pair of variables may not be strictly linearly related, but often can still use a linear fit to interpret the data.

Eg. Object in free fall (start rest)

$$d = d_0 - \frac{1}{2} g t^2$$

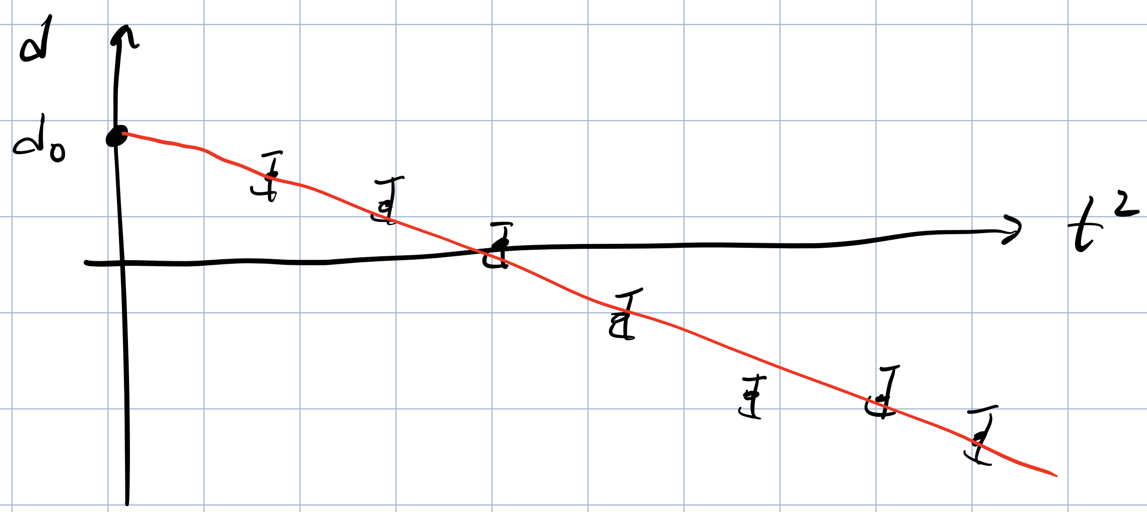


Instead, can <u>linearize</u> data by plotting

$y = d$ vs $x = t^2$

$$d = d_0 - \frac{1}{2} g t^2$$
$$y = a + b x$$

In this case a plot of d vs t^2 has intercept $a = d_0$ & slope $b = -g/2$.



Another Example. In thermal waves, the amplitude of fund. Fourier component is

$$A_1 = \left(\frac{4T_0}{\pi}\right)\exp\left(-\sqrt{\frac{\omega}{2\alpha}}\,x\right)$$

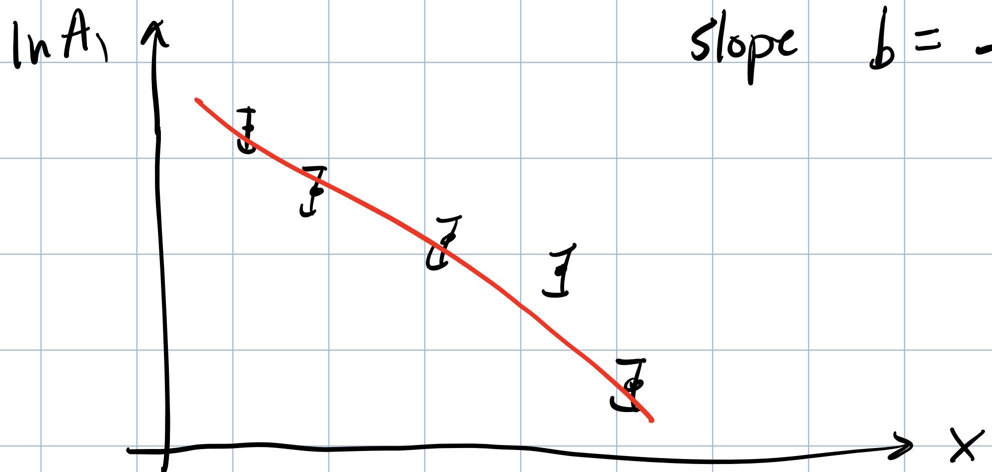
To linearize, take ln of both sides:

$$\ln A_1 = \ln\left(\frac{4T_0}{\pi}\right) - \sqrt{\frac{\omega}{2\alpha}}\,x$$

$$y = a + b\,x$$

Plot $\ln A_1$ vs x, get intercept $a = \ln\left(\frac{4T_0}{\pi}\right)$

slope $b = -\sqrt{\frac{\omega}{2\alpha}}$



In thermal waves exp't use the slope b to find the thermal diffusivity α of copper

$$b = -\sqrt{\frac{\omega}{2\alpha}} \Rightarrow b^2 = \frac{\omega}{2\alpha}$$

$$\Rightarrow \boxed{\alpha = \frac{\omega}{2b^2}}$$

$$\sigma_\alpha^2 = \left(\frac{\partial \alpha}{\partial b}\sigma_b\right)^2 + \left(\frac{\partial \alpha}{\partial \omega}\sigma_\omega\right)^2$$

$$= \left(-\frac{2\omega}{2b^3}\sigma_b\right)^2 + \left(\frac{1}{2b^2}\sigma_\omega\right)^2$$

$$\boxed{\therefore \sigma_\alpha = \sqrt{\left(\frac{\omega\sigma_b}{b^3}\right)^2 + \left(\frac{\sigma_\omega}{2b^2}\right)^2}}$$

Another example (used in current research)

Low-Temperature thermal conductivity of a conductor is typically of the form:

$$K = \underbrace{c_1 T}_{\substack{\text{electron} \\ \text{contribution}}} + \underbrace{c_3 T^3}_{\substack{\text{lattice/phonon} \\ \text{contribution}}}$$

Meas. K as a fcn of T.

Does not appear to be easily linearized at first glance.

$$y = a + b x$$

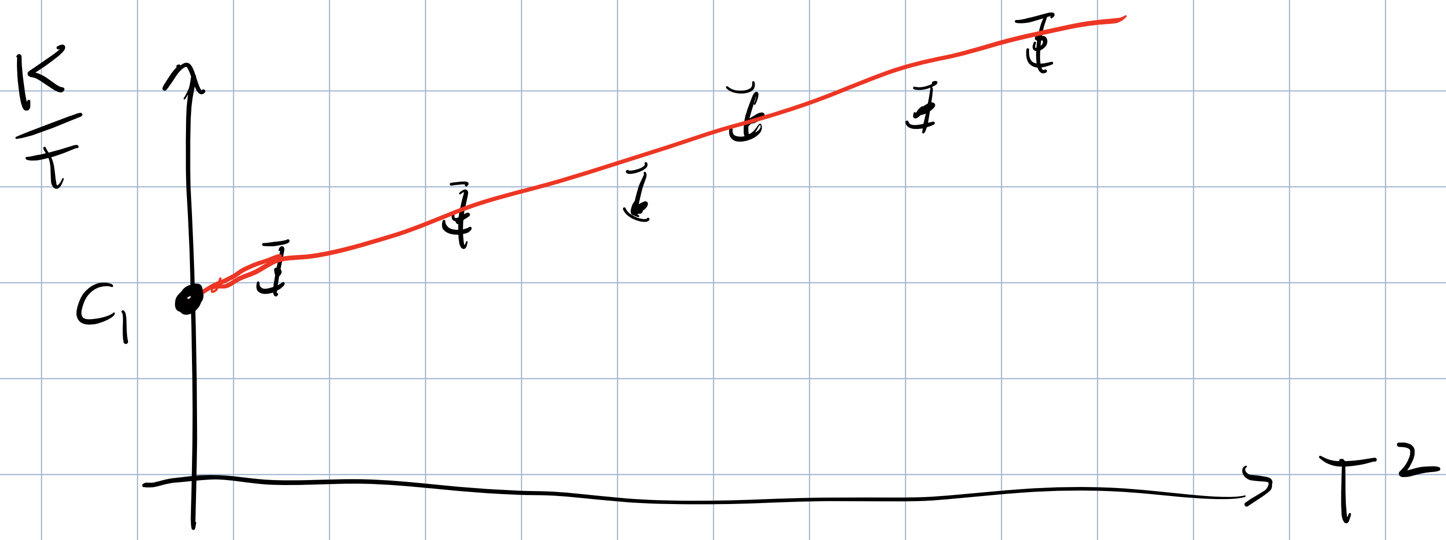
Try $\frac{K}{T}$.

$$\underbrace{\frac{K}{T}}_{y} = c_1 + c_3 \underbrace{T^2}_{x}$$

Plot $\frac{K}{T}$ vs T^2

$\Rightarrow$ intercept of $a = c_1$

slope of $b = c_3$



Another Example. Sometimes a pair of variables cannot be strictly linearized.

In Blackbody radiation

$$I_\lambda = \frac{2c^2h}{\lambda^5}\left[\frac{1}{e^{hc/k_BT\lambda}-1}\right]$$

meas. I_λ vs T which is <u>not</u> linear.

However, under certain conditions, can make valid approximations that all for the data to be linearized.

In this case, if $\dfrac{hc}{k_B T \lambda} \gg 1$

then $e^{hc/k_B T\lambda} - 1 \approx e^{hc/\lambda k_B T}$

$$I_\lambda \approx \frac{2c^2 h}{\lambda^5} e^{-hc/\lambda k_B T}$$

Now it makes sense to take ln of both sides

$$\ln I_\lambda \approx \ln\left(\frac{2c^2 h}{\lambda^5}\right) - \left(\frac{hc}{\lambda k_B}\right)\frac{1}{T}$$

$$y = a + b\,x$$

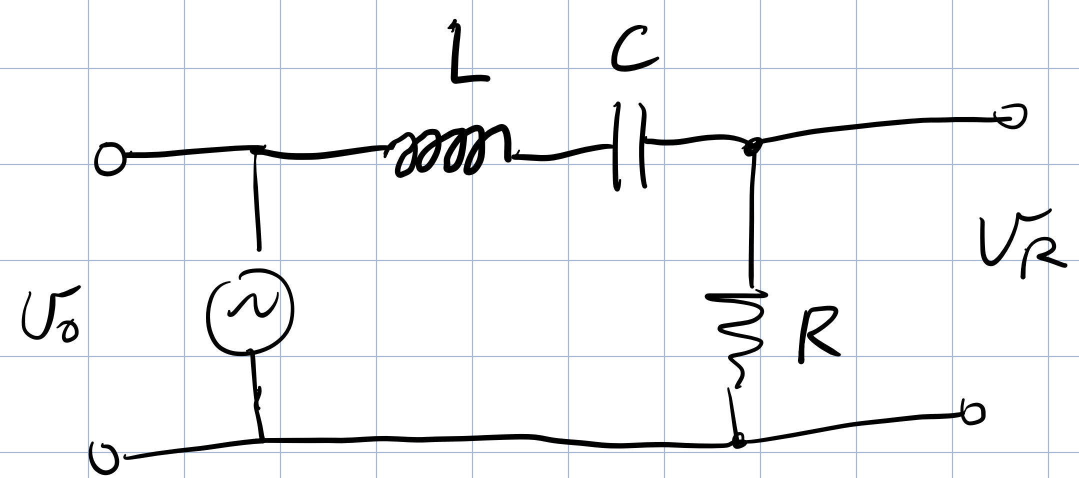
Plot of $\ln I_\lambda$ vs $\frac{1}{T}$ has intercept $a = \ln\left(\dfrac{2c^2 h}{\lambda^5}\right)$

$$b = -\frac{hc}{\lambda k_B}$$

Find $h = \dfrac{-\lambda k_B b}{c}$

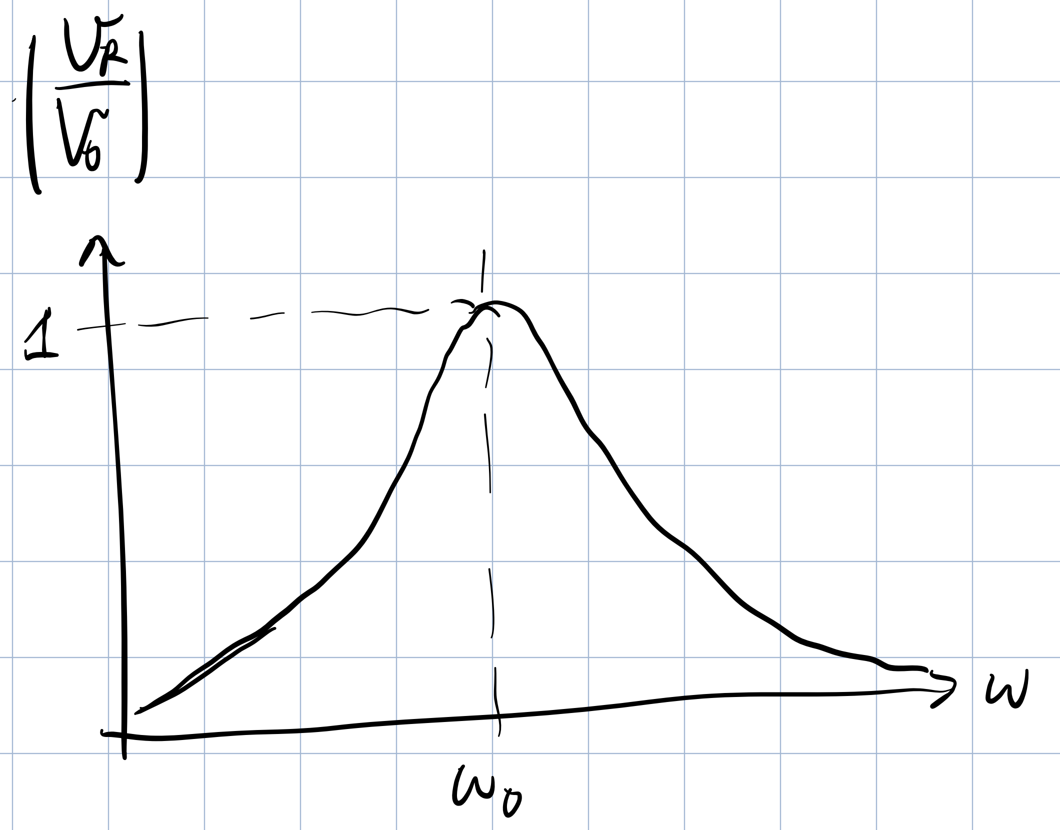
Sometimes it's just not possible to linearize a data set.

Eg. LRC resonance



meas. $\left|\frac{V_R}{V_0}\right|$ vs ω ← freq.

$$\left|\frac{V_R}{V_0}\right| = \frac{1}{\sqrt{1+\left(\omega\frac{L}{R} - \frac{1}{\omega RC}\right)^2}}$$



Need different fitting techniques to extract desired parameters from non-linear datasets.