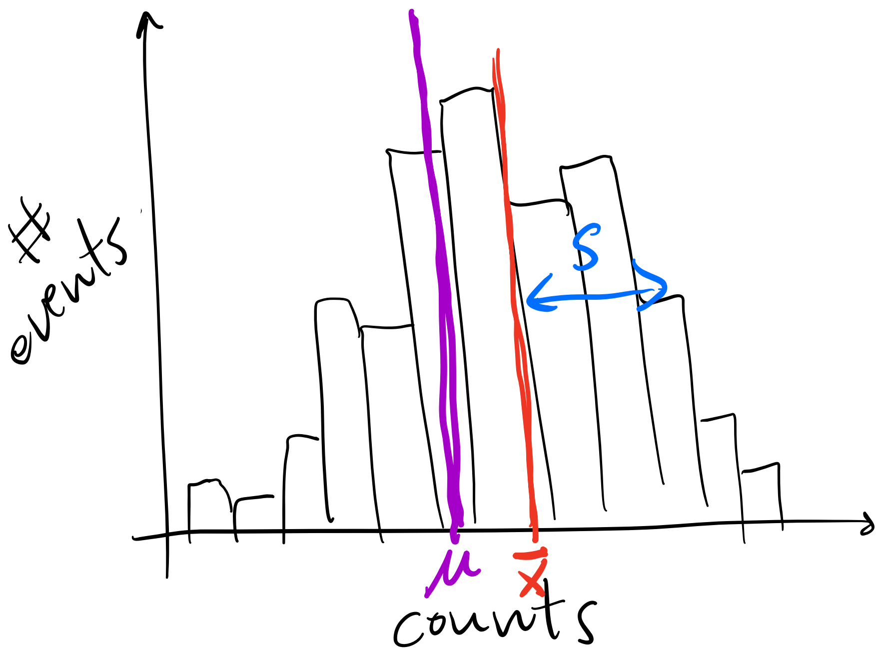
PHYS 232 Jan. 19, 2023

✓ Assignment #1 due next class (Wednesday)

Last Time: Count the number of radioactive decays in time interval T.

trial	
1	12
2	18
3	13
4	17
5	22
⋮	⋮
N	8

bin the data & plot as a histogram.



The mean or average of no. of counts is:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{N}\sum x_i$$

The standard deviation s is given by

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2$$

If we were to take a large no. of meas. ($N \to \infty$), then our sample distribution (finite set of N meas) would approach the so-called parent dist'n (true dist'n for $N \to \infty$)

mean of partent dist'n

$$\mu = \lim_{N \to \infty} \left(\frac{1}{N} \sum x_i \right)$$

std. dev. of parent dist'n

$$\sigma^2 = \lim_{N \to \infty} \left(\frac{1}{N} \sum (x_i - \mu)^2 \right)$$

Why does s^2 have a factor of $\frac{1}{N-1}$ instead of $\frac{1}{N}$ like σ?

(see online notes for detailed discussion).

Intutitive argument:

$$\sum (x_i - \bar{x})^2$$ calculates deviation from $\bar{x}$, but $\bar{x}$ was calculated from x_i data. As a result, $\bar{x}$ is artificially close to the x_i meas.

$\rightarrow$ $\frac{1}{N}\sum(x_i-\bar{x})^2$ would underestimate σ^2. The correction factor turns out to be

$$\frac{N}{N-1}.$$

Mean & Std. Dev. of Distributions.

Discrete distn.

Suppose the result of a meas. can only be one of n possible results.

Eg. Rolling a die, must get one of

$x_1 = 1, \quad x_2 = 2, \quad \ldots, \quad x_6 = 6$

If we make a total of N measurements, we will find:

x_1 a total of m_1 times
x_2 " " " m_2 times

$\vdots$

x_6 " " " m_6 times

require $\displaystyle\sum_{i=1}^{n} m_i = N$ (In this example $n=6$)

For a large no. of meas (N large),
prob. of meas. x_n is $P_n = \lim\limits_{N \to \infty} \dfrac{m_n}{N}$

$$\mu = \lim_{N\to\infty}\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

$$= \lim_{N\to\infty} \frac{1}{N}\left(m_1 x_1 + m_2 x_2 + \dots + m_6 x_6\right)$$

$$= \lim_{N\to\infty}\left(\frac{m_1}{N} x_1 + \frac{m_2}{N} x_2 + \dots + \frac{m_6}{N} x_6\right)$$

$$= P_1 x_1 + P_2 x_2 + \dots + P_6 x_6$$

prob. of getting x_1.

no. of possible outcomes of expt.

$$\boxed{\mu = \sum_{j=1}^{n} x_j P_j = \sum_{j=1}^{n} x_j \underbrace{P(x_j)}}$$

prob. that
a meas. results
in outcome x_j

Likewise can show (homework)

$$\boxed{\sigma^2 = \sum_{j=1}^{n} \left[(x_j - \mu)^2 P(x_j) \right]}$$

manipulation of σ^2 expression.

$$\sigma^2 = \sum_{j=1}^{n} \left[\left(x_j^2 - 2\mu x_j + \mu^2 \right) P(x_j) \right]$$

distribute the sum

$$\sigma^2 = \sum_{j=1}^{n} x_j^2 P(x_j) + \sum_{j=1}^{n} \left(-2\mu x_j P(x_j) \right)$$

$$+ \sum_{j=1}^{n} \mu^2 P(x_j)$$

Pull the constants -2μ & μ^2 outside sums.

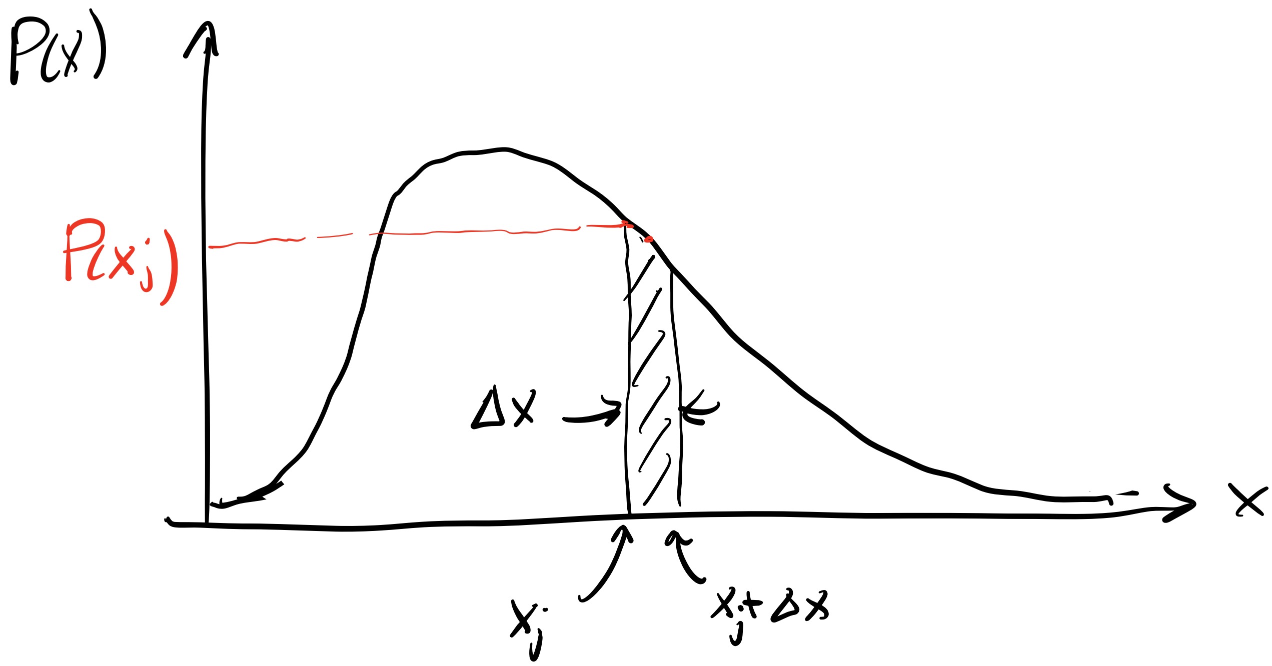
$$\sigma^2 = \sum_{j=1}^{n} x_j^2 P(x_j) - 2\mu \underbrace{\sum_{j=1}^{n} x_j P(x_j)}_{\mu} + \mu^2 \underbrace{\sum_{j=1}^{n} P(x_j)}_{1}$$

prob. of getting x_1 or x_2 or $x_3 \ldots$ or x_{n-1} or x_n

$$\boxed{\sigma^2 = \sum_{j=1}^{n} x_j^2 P(x_j) - \mu^2}$$

Continuous Distributions

For a continuous dist'n $P(x)$ is a "probability density".



For a normalized $P(x)$, the area under $P(x)$ vs x plot must be 1.

Prob. of meas. a value between x_j & $x_j + \Delta x$ is given by the shaded area which is approx $P(x_j)\,\Delta x$

If we did N meas., we will get a result between x_j & $x_j + \Delta x$:

$$m_j = N \underbrace{P(x_j)\,\Delta x}_{\substack{\text{prob. of getting} \\ x \text{ between } x_j \text{ \& } x_j + \Delta x}}$$

$$\mu = \sum_{j=1}^{n} \frac{m_j}{N} x_j$$

$$= \sum_{j=1}^{n} x_j \, P(x_j)\,\Delta x$$

In the limit $\Delta x \to 0$.

$$\boxed{\mu = \int_{\text{all } x} x \, P(x)\, dx}$$

for std. dev.

$$\boxed{\sigma^2 = \int_{\text{all } x} (x-\mu)^2 P(x)\, dx}$$