PHYS 232 Jan. 17, 2023

Fourier Series: Any periodic fcn can be expressed as:

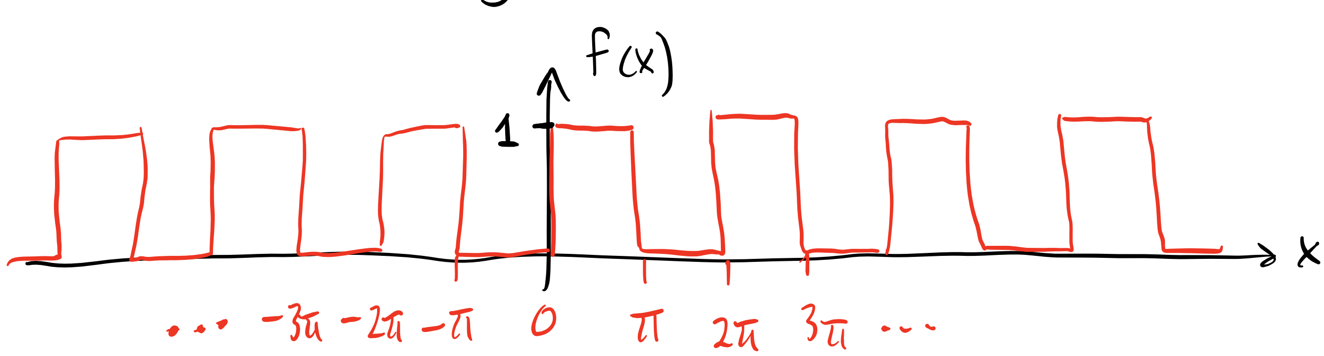
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx \qquad n = 1, 2, 3, \ldots$$

Today: Find the Fourier series of the following square wave.



$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

Start by finding $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$$a_0 = \frac{1}{\pi}\left[\underbrace{\int_{-\pi}^{0} 0\cdot dx}_{0} + \underbrace{\int_{0}^{\pi} 1\cdot dx}_{\pi}\right]$$

$$\therefore \boxed{a_0 = 1}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\underbrace{\int_{-\pi}^{0} 0\cdot\cos nx\,dx}_{0} + \int_{0}^{\pi} 1\cdot\cos nx\,dx\right]$$

$$= \frac{1}{\pi n}\sin nx\Big|_{0}^{\pi} = \frac{1}{\pi n}\left[\cancelto{0}{\sin n\pi} - \cancelto{0}{\sin 0}\right]$$

$$\boxed{\therefore\ a_n = 0 \quad \forall\ n = 1, 2, 3, \dots}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$$

$$= \frac{1}{\pi}\left[\underbrace{\int_{-\pi}^{0} 0\cdot \sin nx\, dx}_{0} + \int_{0}^{\pi} 1\cdot \sin nx\, dx\right]$$

$$b_n = -\frac{1}{\pi n}\cos nx \Big|_{0}^{\pi}$$

$$= -\frac{1}{\pi n}\left[\underbrace{\cos n\pi}_{(-1)^n} - \underbrace{\cos 0}_{1}\right]$$

n	$\cos n\pi$	$(-1)^n$
1	-1	-1
2	1	1
3	-1	-1
4	1	1
$\vdots$	$\vdots$	$\vdots$

$$\therefore\ b_n = \frac{1}{\pi n}\left[1 - (-1)^n\right]$$

n	b_n
1	$\frac{2}{\pi}$
2	0
3	$\frac{2}{3\pi}$
4	0
5	$\frac{2}{5\pi}$
$\vdots$	

$$\boxed{\begin{array}{l} n \text{ even}, \quad b_n = 0 \\ n \text{ odd}, \quad b_n = \dfrac{2}{n\pi} \end{array}}$$

Fourier series for our square wave is:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

Square wave:

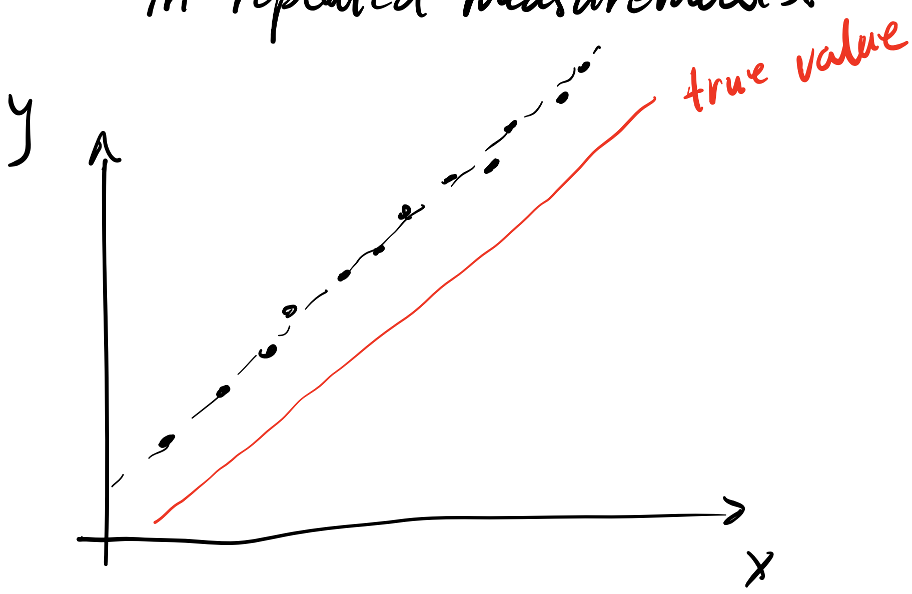
$$f(x) = \frac{1}{2} + \frac{2}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right]$$

In any meas. of a quantity x, must determine from experimental conditions an estimate of our confindence in the measured value

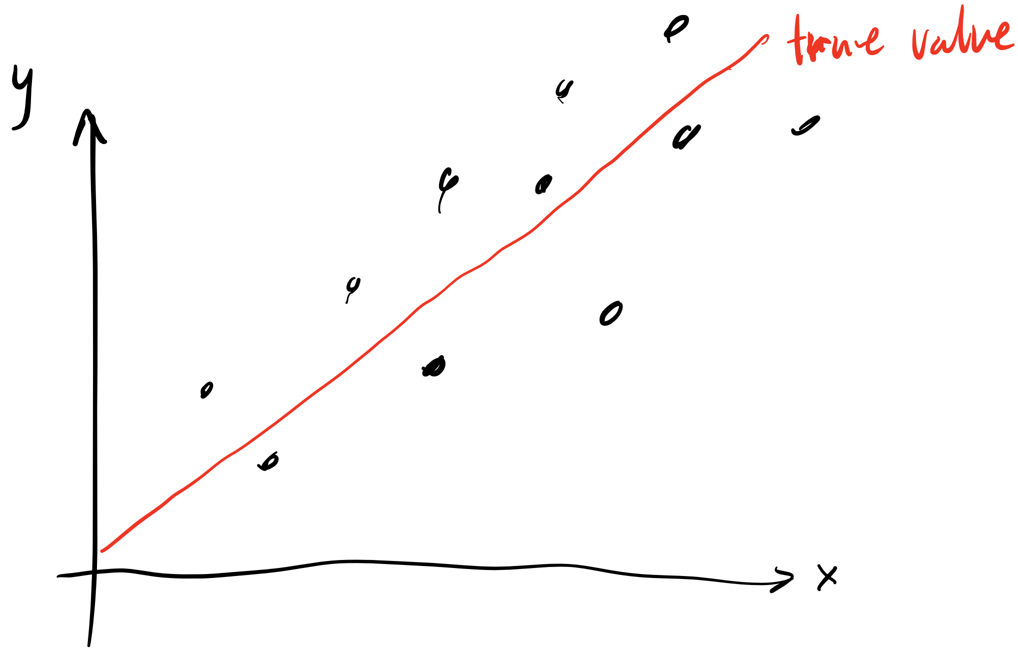
$\Rightarrow \Delta x$ (uncertainty in meas. result).

Accuracy: how close a meas. is to true value.

Precision: how much fluctuation is there in repeated measurements.



A precise, but inaccurate measurement.



An inpercise but accurate measurement.

Accuracy of an experiment is largely determined by systematic errors.

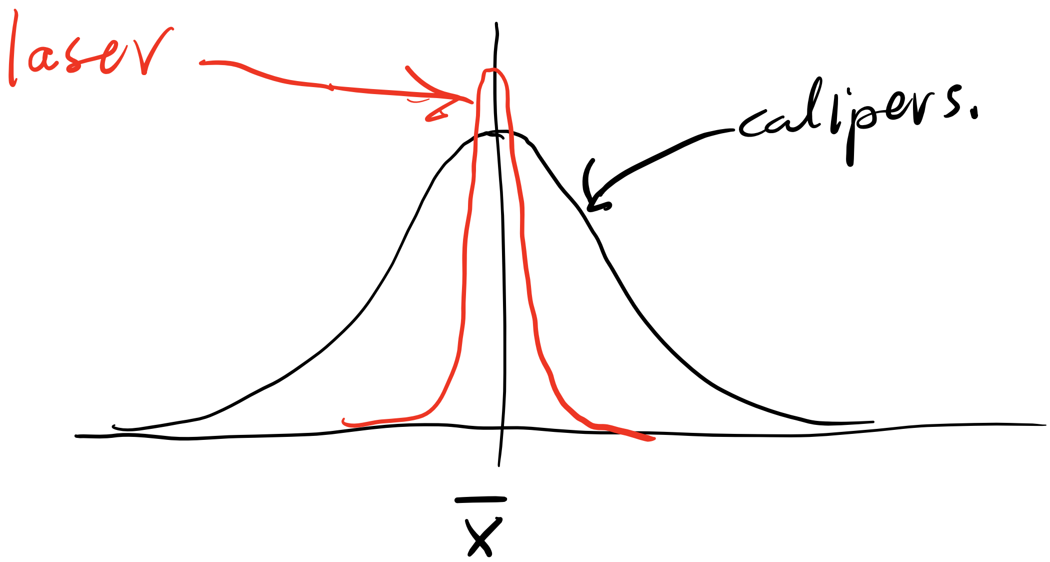
Errors that make a result reproducibly different than the true value.

→ faulty equipment (uncalibrate calipers)
→ bias (slow reaction time)

If you are aware of a systematic error, you would correct for it.

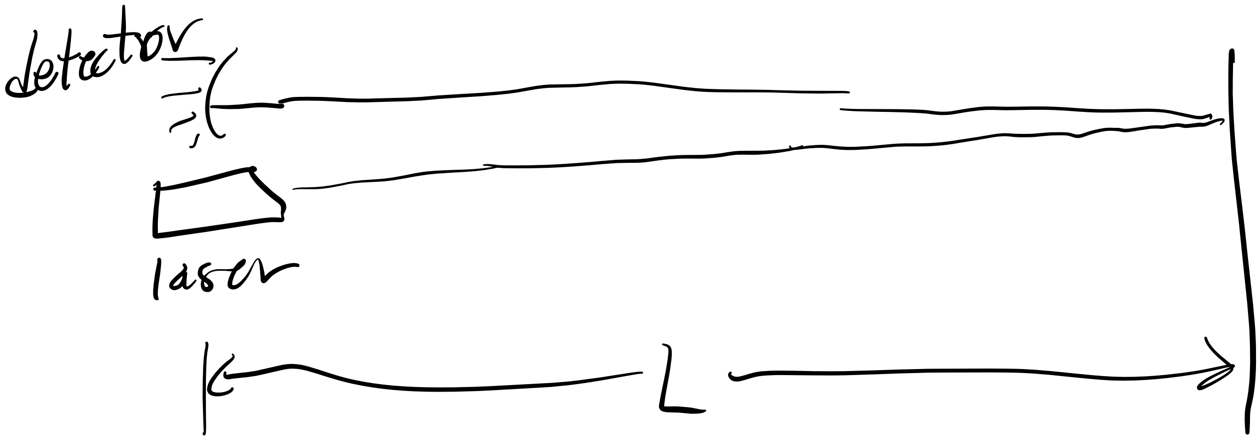
Random Errors determine the precision of an experiment.

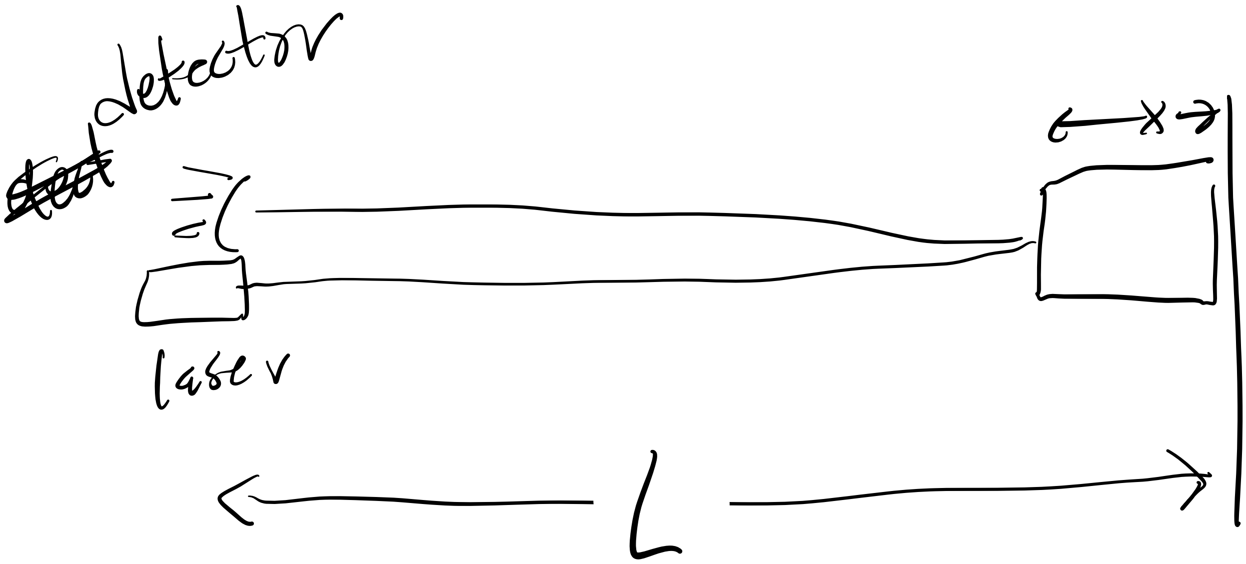
Eg. meas the length of a block of wood w/ calipers. Won't get the same reading each time. Instead, get a dist'n of valves.



Reducing random errors is acheived through better experimental design or improved equipment.

Eg. Use a laser & photodetector to meas. length of block.





Distributions

You have a radioactive sample that emits electrons (Beta decay).

$$\overset{p+n}{\searrow}\ {}^{A}_{Z}N \longrightarrow {}^{A}_{Z+1}N' + e^- + \bar{\nu}_e$$

p

$$(n \rightarrow p + e^- + \bar{\nu}_e)$$

You meas. no. of e^- in a fixed time interval to determine β decay rate.

trial	
1	12
2	18
3	13
4	17
5	22
⋮	⋮
N	8

Construct a histogram by binning the data & counting the no. of trials fall within each bin.

Eg. bin (12,13)
(14,15)
(16,17) ---

distn
of meas.