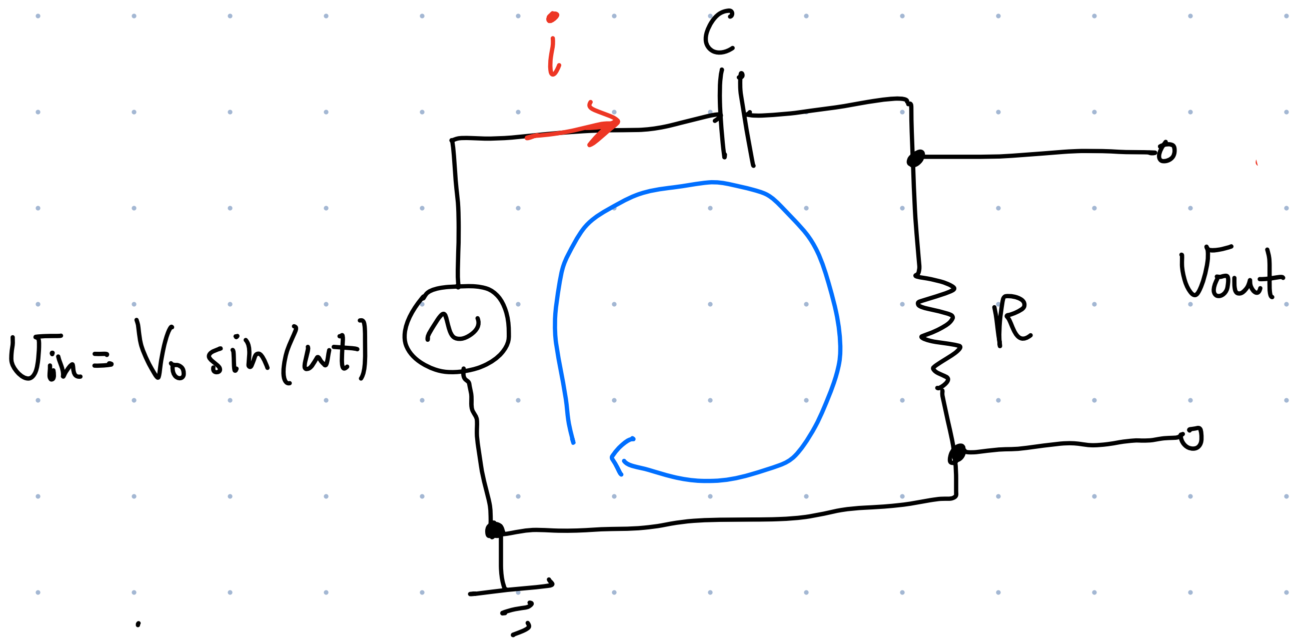
PHYS 231 — Sept. 26, 2025

- Assignment #2 on Course website
- No lab next week
- Lab #3 notebook due Thurs. Oct. 9 @ start of lab 4a

Last Time: Frequency Response of an RC circuit



Found that:

$$i = I_0(\omega)\sin\left[\omega t - \phi(\omega)\right]$$

where

$$\boxed{I_0(\omega) = \frac{\omega V_0 C}{\sqrt{1+(\omega RC)^2}} \qquad \tan\phi(\omega) = -\frac{1}{\omega RC}}$$

Practical of this RC circuit w/ $V_{out} = iR$

$$\therefore V_{out} = \underbrace{I_0 R}_{\text{amplitude of output}} \sin[\omega t - \phi]$$

$$I_0 R = \frac{\omega V_0 RC}{\sqrt{1+(\omega RC)^2}}$$

The ratio of input amplitude (V_0) to output amplitude ($I_0 R$)

$$\boxed{\frac{I_0 R}{V_0} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}}$$

Transfer fcn of RC circuit.

consider 3 case: (i) $\omega RC \ll 1$ (low freq)

(ii) $\omega RC \gg 1$ (high freq)

(iii) $\omega RC = 1$

(i) low freq. $\sqrt{1+(\omega RC)^2} \approx 1$

$$\frac{I_0 R}{V_0} = \omega RC \sim \omega$$

(ii) high freq. $\omega RC >> 1$

$$\sqrt{1+(\omega RC)^2} \approx \omega RC$$

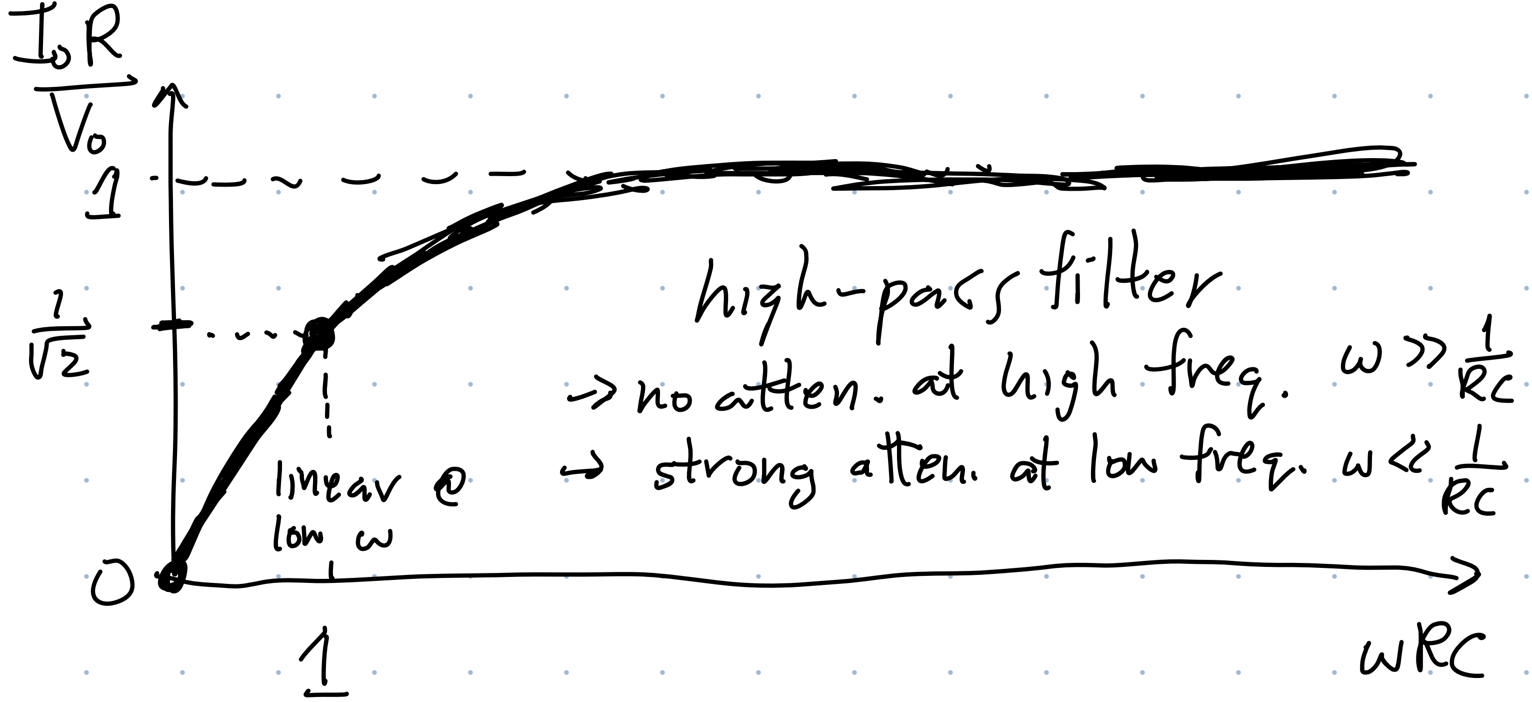
$$\therefore \frac{I_0 R}{V_0} \approx \frac{\omega RC}{\omega RC} = 1 \quad (\text{const})$$

(iii) $\omega RC = 1$ (intermediate freq)

denom: $\sqrt{1+(\underbrace{\omega RC}_{1})^2} = \sqrt{2}$

numer: $\omega RC = 1$

$$\frac{I_0 R}{V_0} = \frac{1}{\sqrt{2}} = 0.707$$



$$\left(\omega = \frac{1}{RC},\ f = \frac{1}{2\pi RC}\right)$$

Solving the RC circuit driven by a sine wave wasn't too difficult, but involve lots of algebra and trig fcns.... .

We need new mathematical tools so that we can solve these problems more efficiently.

Develop the idea of complex nos.

Definition: $\sqrt{-1} = \pm j$

This definition is useful because it allows to solve some problem that previously had no obvious sol'n.

Eg. Consider $x^2 - 2x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{aligned} a &= 1 \\ b &= -2 \\ c &= 4 \end{aligned}$$

$$= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2}$$

$$= 1 \pm \frac{\sqrt{-12}}{2}$$

$$= 1 \pm \frac{\overset{j}{\sqrt{-1}}\,\cancel{\sqrt{4}}\,\sqrt{3}}{\cancel{2}}$$

$$x = \begin{cases} 1 + j\sqrt{3} \\ 1 - j\sqrt{3} \end{cases}$$

In general, a complex no. z has a "real" part and an "imaginary".

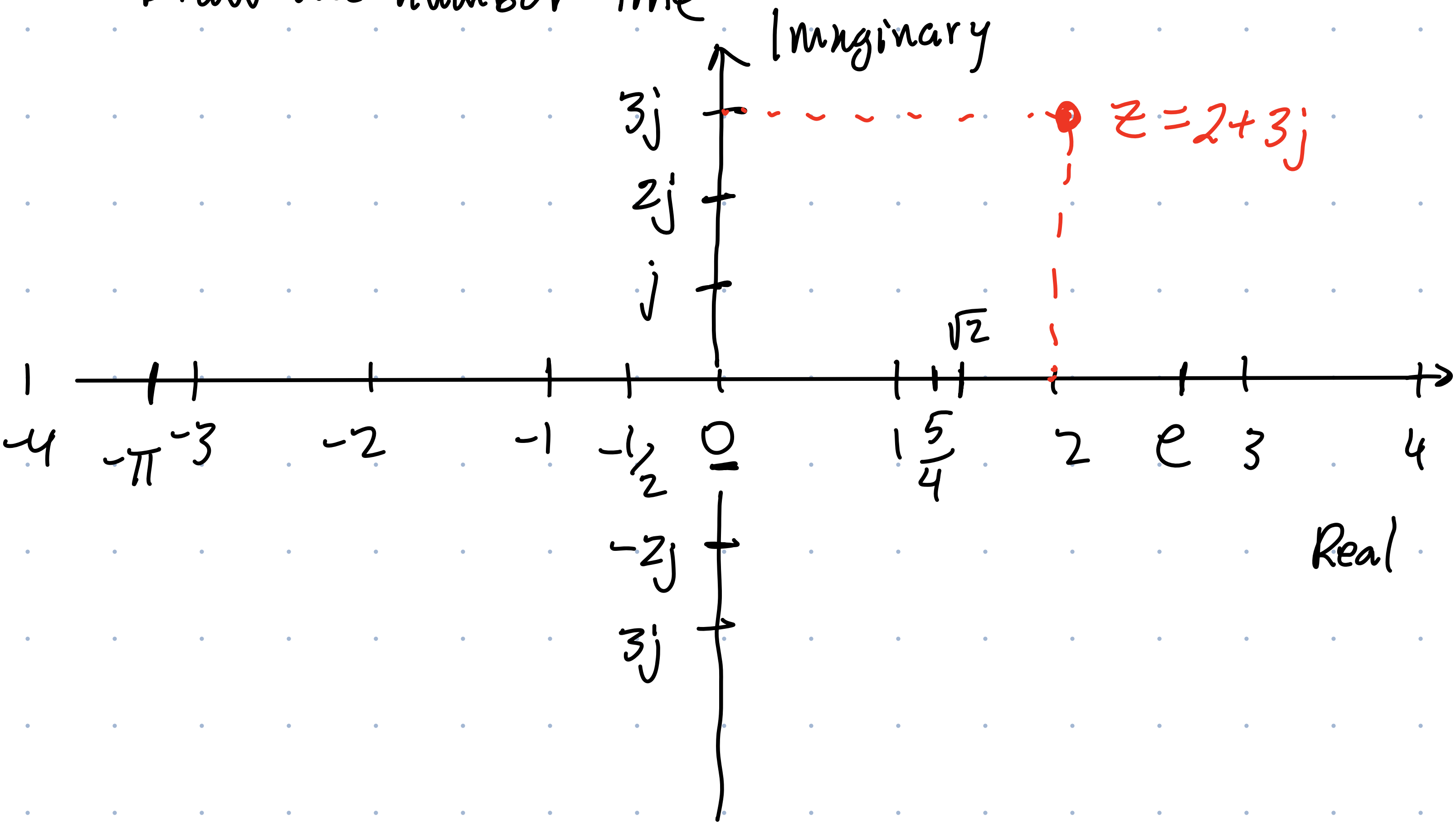
$$z = x + jy$$

Real part of complex no z : $\text{Re}[z] = x$ anything in z that doesn't involve j

Imaginary part of z : $\text{Im}[z] = y$ anything that multiplies j, but not including j

both x & y are real numbers b/c they don't involve j directly.

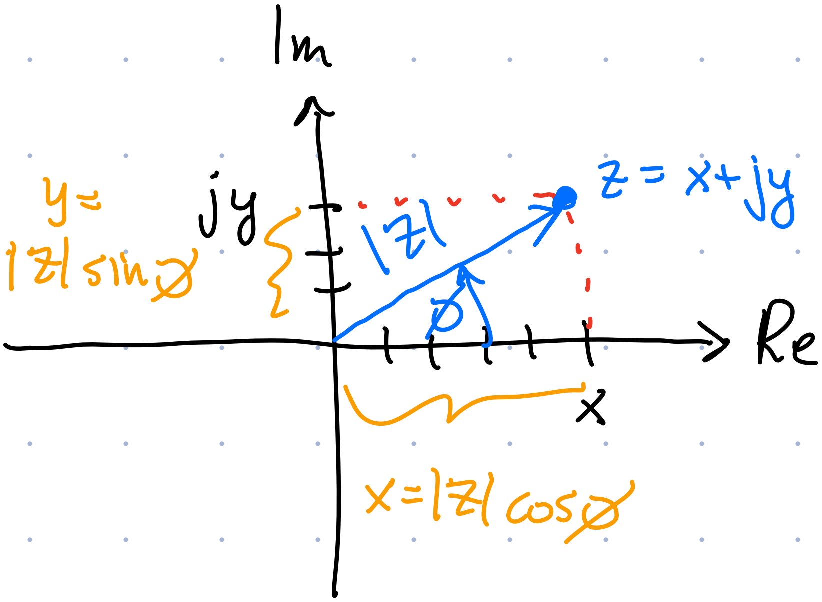
Draw the number line

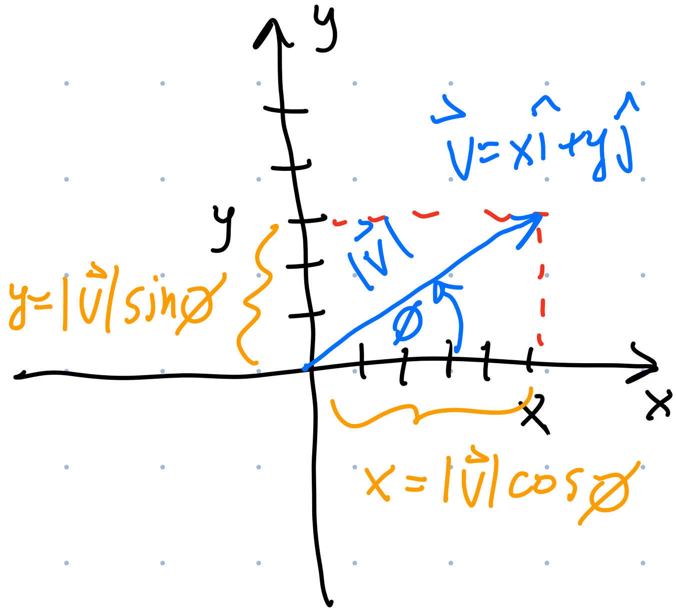


To extend the number line to include complex nos, we add a $\perp$ axis for the imaginary part of z.

Place $z = 2+3j$ on our coord. system.

Notice that, in many ways, $z = x+jy$ is similar to a 2-D vector $\vec{v} = x\hat{i} + y\hat{j}$.





complex plane

$$z = x + jy$$

Likewise, we can specify z as a dist. from origin to the point $z \Rightarrow |z|$

magnitude or length of z

and an angle ϕ meas. from real axis.

$$\mathrm{Re}[z] = x = |z| \cos\phi \quad (1)$$

$$\mathrm{Im}[z] = y = |z| \sin\phi \quad (2)$$

$$\vec{V} = x\hat{i} + y\hat{j}$$

$\vec{V} \Rightarrow |\vec{V}|$ and dir'n ϕ

length of vector

$$x = |\vec{V}| \cos\phi$$

$$y = |\vec{V}| \sin\phi$$

$$|\vec{V}| = \sqrt{x^2 + y^2}$$

$$\tan\phi = \frac{y}{x}$$

$$|z| = \sqrt{x^2+y^2}$$

$$\tan\phi = \frac{y}{x}$$

If $z = x + jy$

$= |z|\cos\phi + j|z|\sin\phi$

$= |z|(\cos\phi + j\sin\phi)$

Can represent any complex number in two ways:

$$\boxed{\begin{array}{ll} \text{"component form"} & z = x + jy \\ \text{"magnitude \& phase"} & z = |z|(\cos\phi + j\sin\phi) \end{array}}$$