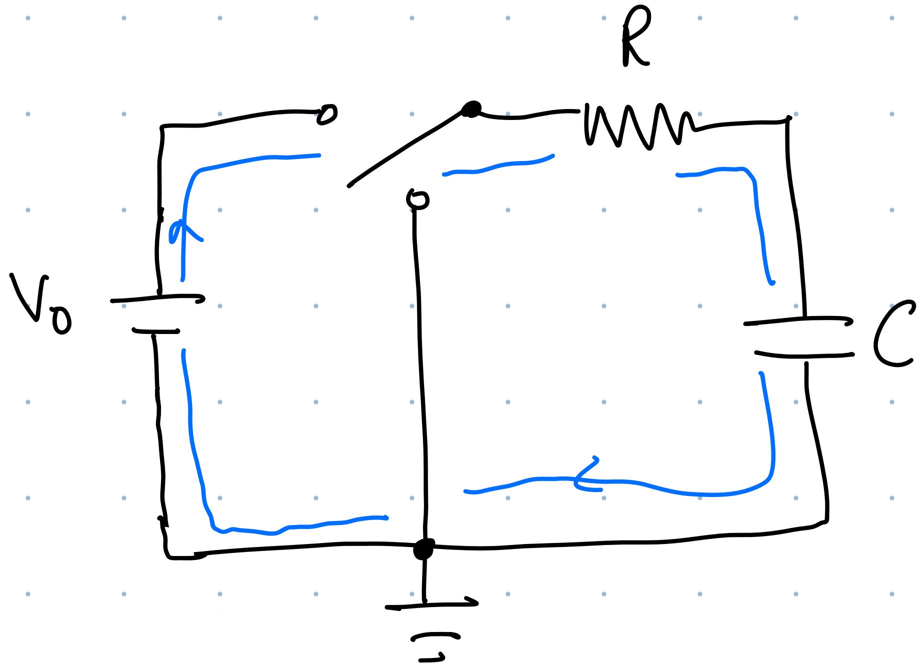
PHYS 231 Sept. 12, 2025

Previously: $|\Delta V_R| = IR = R\frac{dQ}{dt}$

$$|\Delta V_C| = \frac{1}{C}Q$$

Today: RC Circuit Transient Response.



Start w/ switch down s.t. $Q=0$
$I=0$

At $t=0$, flip switch up. Find $Q(t)$ as system approaches new equilibrium.

Kirchhoff Voltage loop rule.

$$V_0 - IR - \frac{Q}{C} = 0.$$

$$\therefore \quad R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \qquad \text{divide by } R$$

$$\therefore \quad \frac{dQ}{dt} + \frac{1}{RC}Q = \frac{V_0}{R} \qquad \text{define } \tau = RC$$

(time constant)

$$\boxed{\therefore \quad \frac{dQ}{dt} + \frac{1}{\tau}Q = \frac{V_0}{R}} \quad (*)$$

inhomogenous first order differential eq'n.

Goal: Solve for $Q(t)$ subject to initial condition

$\rightarrow Q(0) = 0.$

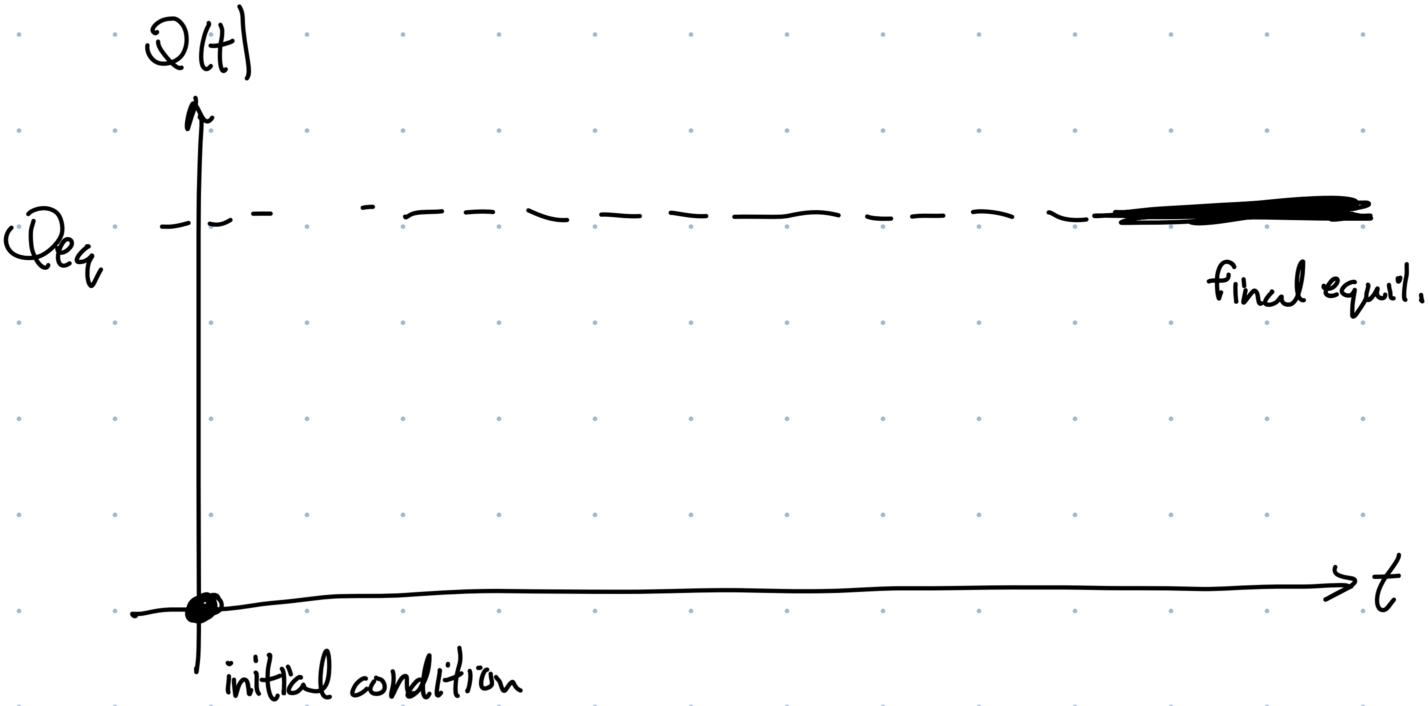
After switch has been up for a long time, capacitor is fully charged s.t. Q is const. and $\frac{dQ}{dt} = 0$.

$t \to \infty$, expect $\frac{dQ}{dt} \to 0 \quad Q \to Q_{eq}$

sub into (*)

$$0 + \frac{1}{\tau} Q_{eq} = \frac{V_0}{R}$$

$$\therefore \; Q_{eq} = \tau \frac{V_0}{R} = \cancel{R}C \frac{V_0}{\cancel{R}} = CV_0 .$$



Express $Q(t) = Q_h(t) + \underbrace{Q_{eq}}_{CV_0 \text{ (const)}}$.

Sub into (*)

(*) Requires $\dfrac{dQ}{dt} = \dfrac{d}{dt}\left(Q_h(t) + CV_0 \right)$

$$= \frac{dQ_h(t)}{dt}$$

$\therefore$ (A) becomes:

$$\frac{dQ_h(t)}{dt} + \frac{1}{\tau}\left(Q_h(t) + CV_0\right) = \frac{V_0}{R}$$

$$\therefore \frac{dQ_h(t)}{dt} + \frac{1}{\tau}Q_h(t) + \cancel{\underbrace{\frac{1}{\tau}CV_0}_{\frac{1}{RC}CV_0 = \frac{V_0}{R}}} = \cancel{\frac{V_0}{R}} \quad 0$$

$$\boxed{\therefore \frac{dQ_h(t)}{dt} + \frac{1}{\tau}Q_h(t) = 0} \quad (\#)$$

homogenous first order differential Eq'n
(easier to solve).

multiply (#) by dt

$$dQ_h(t) + Q_h(t)\frac{dt}{\tau} = 0$$

move 2nd term to right.

$$dQ_h(t) = -Q_h(t)\frac{dt}{\tau}$$

divide by $Q_h(t)$

$$\frac{dQ_h(t)}{Q_h(t)} = -\frac{dt}{\tau}$$

integrate.

$$\int \frac{dQ_h(t)}{Q_h(t)} = -\frac{1}{\tau}\int dt$$

integration const.

$$\therefore \ln Q_h(t) = -\frac{1}{\tau}t + B$$

$$\therefore Q_h(t) = e^{-t/\tau + B} = e^{-t/\tau}\underbrace{e^{B}}_{=B'}$$

just another const.

$$\therefore Q_h(t) = B' e^{-t/\tau}$$

To get the final $Q(t) = \underbrace{Q_h(t)}_{B'e^{-t/\tau}} + \underbrace{Q_{eq}}_{CV_0}$

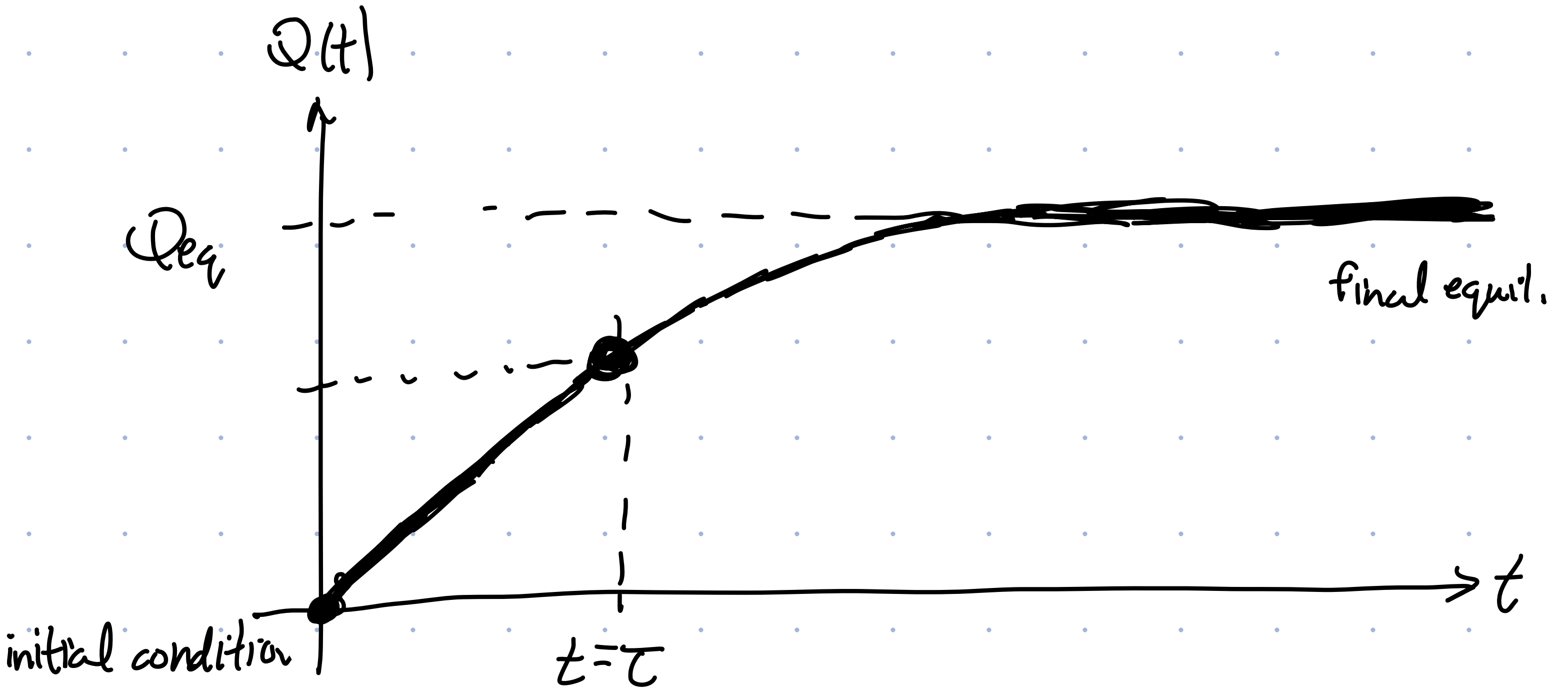
$$\therefore Q(t) = B'e^{-t/\tau} + CV_0 .$$

Last step is to use the initial condition $Q(0) = 0$ to determine B'.

$$\underbrace{Q(0)}_{0} = B'\cancel{e^{0}}^{\,1} + CV_0 \qquad \therefore B' = -CV_0$$

Finally $$\boxed{Q(t) = CV_0\left[1-e^{-t/\tau}\right] \quad \text{where } \tau = RC}$$

Transient response of cap. charging from $Q(0)=0$ to Q_{eq}.



Consider where $t=\tau$

$$Q(\tau) = CV_0\left[1-e^{-\tau/\tau}\right]$$

$$= CV_0\left(1-\frac{1}{e}\right)$$

0.632

When charging a cap through a resistor, it take ~ 4-5 time constants $\tau = RC$ to get close to fully charged.

We will investigate RC circuit transients in Lab # 3.

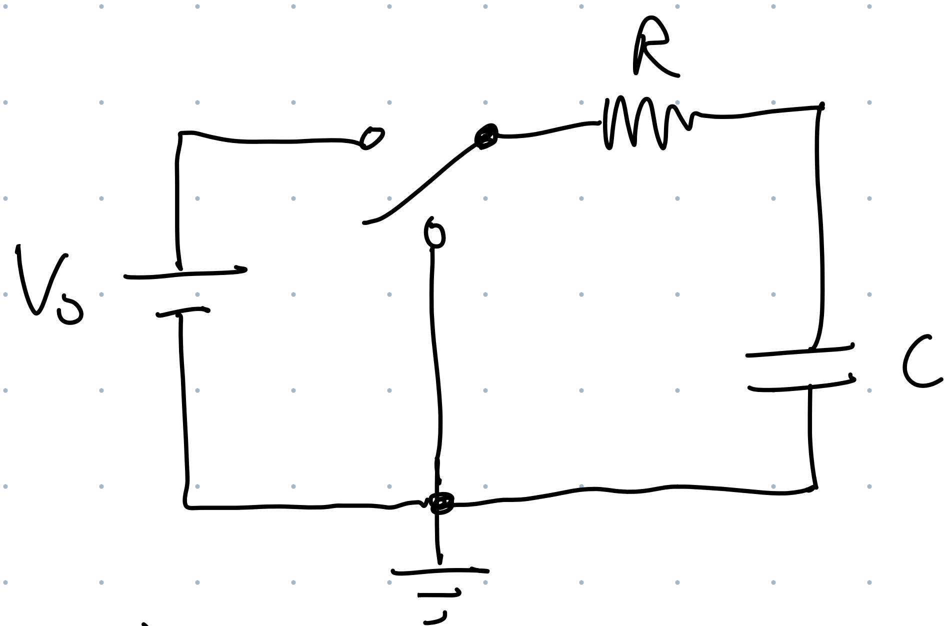
Having found $Q(t)$, we can find the current $I(t) = \frac{dQ}{dt}$

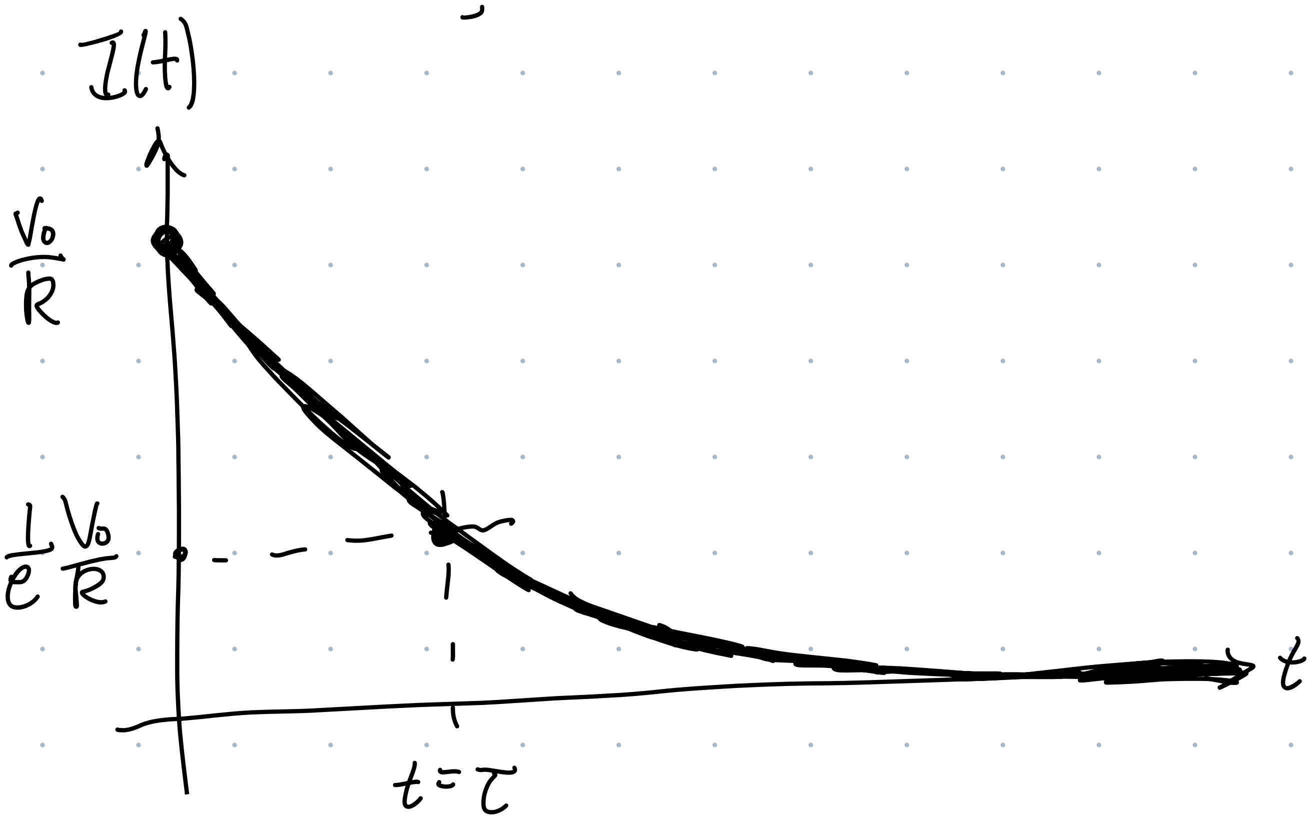
$$I(t) = \frac{d}{dt}\left\{ CV_0\left[1 - e^{-t/\tau}\right]\right\}$$

$$= CV_0 \frac{d}{dt}\left[1 - e^{-t/\tau}\right]$$

$$= \frac{CV_0}{\tau} e^{-t/\tau} = \frac{\cancel{C}V_0}{R\cancel{C}} e^{-t/\tau}$$

$$\boxed{\therefore \; I(t) = \frac{V_0}{R} e^{-t/\tau}}$$





Consider $t = \tau$: $\quad I(\tau) = \frac{V_0}{R} e^{-\tau/\tau} = \frac{1}{e}\frac{V_0}{R}$

Uncertainties & Propagation of Errors.

Every quantity that is measured has an associated uncertainty.

Recall first-year prop. of error rules:

Meas. $x \pm \Delta x$, $y \pm \Delta y$

① $z = x \pm y \implies \Delta z = \Delta x + \Delta y$

② $z = xy$ or $z = \frac{x}{y} \implies \frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$

③ $z = x^n \implies \frac{\Delta z}{z} = n\frac{\Delta x}{x}$

One of our goals will be to discover the origin of these rules and then generalize to cover all cases, including things like

$z = e^x$, $z = \ln x$, $z = \sin x$ ...