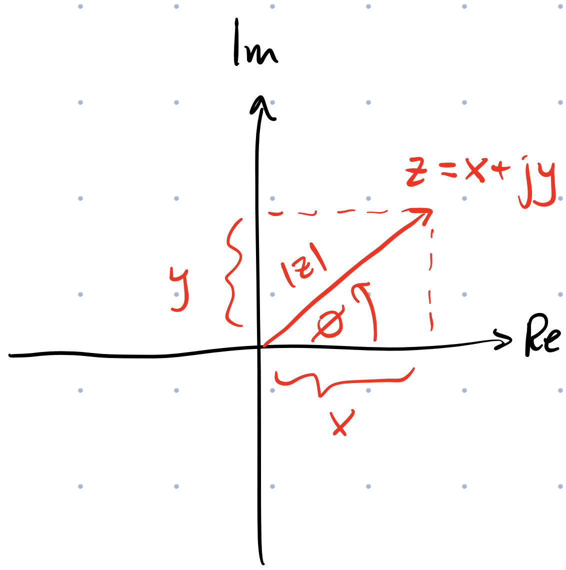
PHYS 231

Oct. 21, 2024

$\boxed{\text{Assignment \#3 on course website}}$

Complex Numbers.

$$z = x + jy = |z| e^{j\phi}$$



$$x = |z| \cos\phi$$

$$y = |z| \sin\phi$$

$$|z| = \sqrt{x^2 + y^2}$$

$$\tan\phi = \frac{y}{x}$$

One more definition: The complex conjugate z^*

If $\quad z = x + jy = |z| e^{j\phi}$

then $\quad z^* = x - jy = |z| e^{-j\phi}$ (replace all j's w/ $-j$)

$\Rightarrow$ reverse sign in front of all j's when taking complex conjugate.

Consider $zz^* = (x+jy)(x-jy)$

$= x^2 - \cancel{jxy} + \cancel{jxy} - \underbrace{j^2}_{+1} y^2$

$= x^2 + y^2 = |z|^2$

$\therefore zz^* = |z|^2$ square of the mag. or length of z.

Analogous to finding length of

$\vec{V} = V_x \hat{i} + V_y \hat{j}$

$|\vec{V}|^2 = \vec{V} \cdot \vec{V} = V_x^2 + V_y^2$

What $zz^* = (\underbrace{|z|e^{j\phi}}_{z})(\underbrace{|z|e^{-j\phi}}_{z^*})$

$= |z|^2$

Another use for the complex conjugate.

→ remove j's from denominator of a fraction.

want to find $z = x + jy$

Take $z = \frac{z_1}{z_2}$

given that

$z_1 = s + jt$

$z_2 = u + jv$

$\therefore z = \frac{s + jt}{u + jv}$

want to find x & y for $z = x + jy$.

Want to remove j from denominator.

$z_2 = u + jv, \quad z_2^* = u - jv$

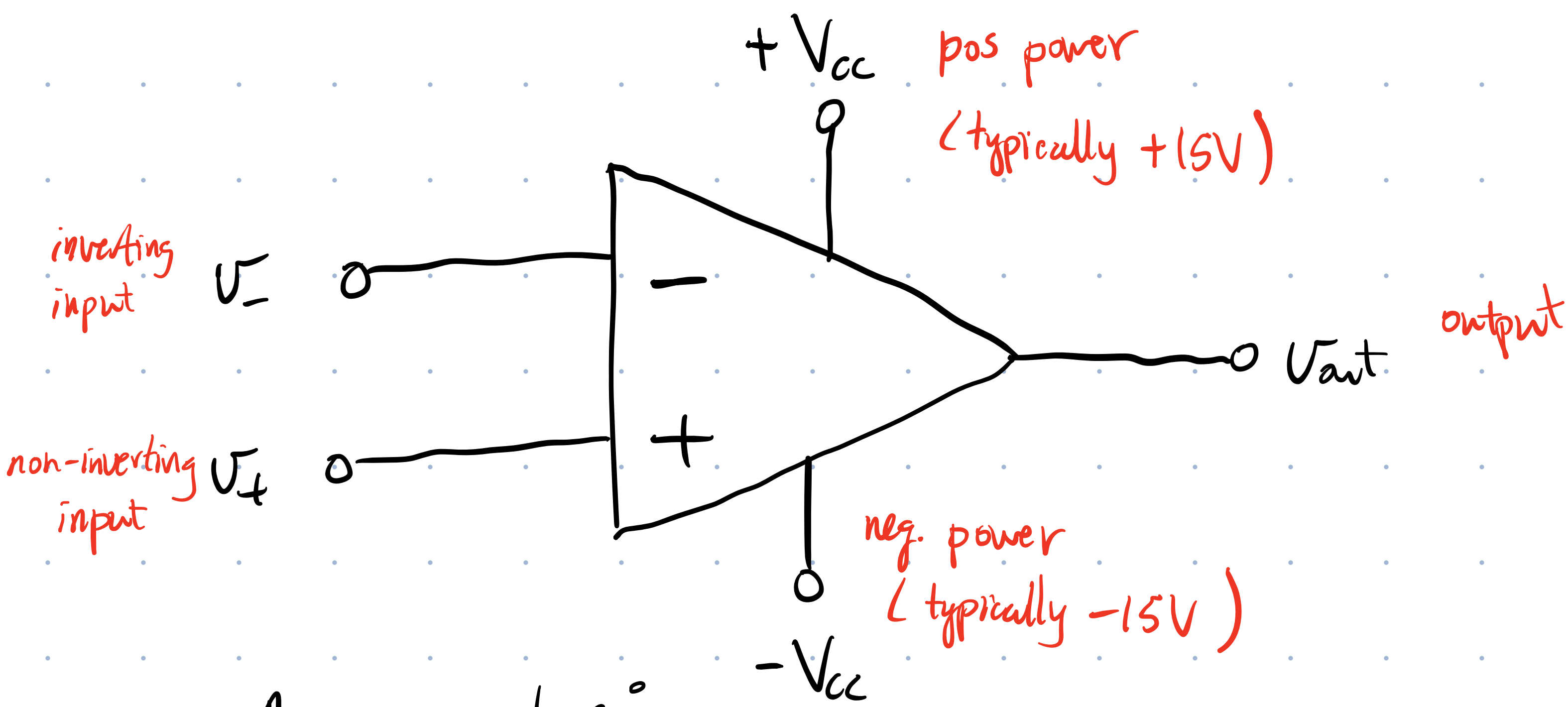
Mult. top & btm of our fraction by z_2^*

$$z = \frac{(s+jt)}{(u+jv)} \frac{(u-jv)}{(u-jv)} = \frac{(su \cancelto{+1}{-j^2} tv) + jtu - jsv}{u^2+v^2}$$

$$z = x + jy = \frac{(su+tv) + j(tu-sv)}{u^2+v^2}$$

$$\boxed{x = \frac{su+tv}{u^2+v^2} \qquad y = \frac{tu-sv}{u^2+v^2}}$$

Operational Amplifier (Op Amps)



An op amp has:

- two inputs (v_-, v_+)
- one output (v_{out})
- two power terminals

The two inputs v_- & v_+ can each be pos. or neg. voltages.

Treat the op amp as a black-box device that performs the following fcn:

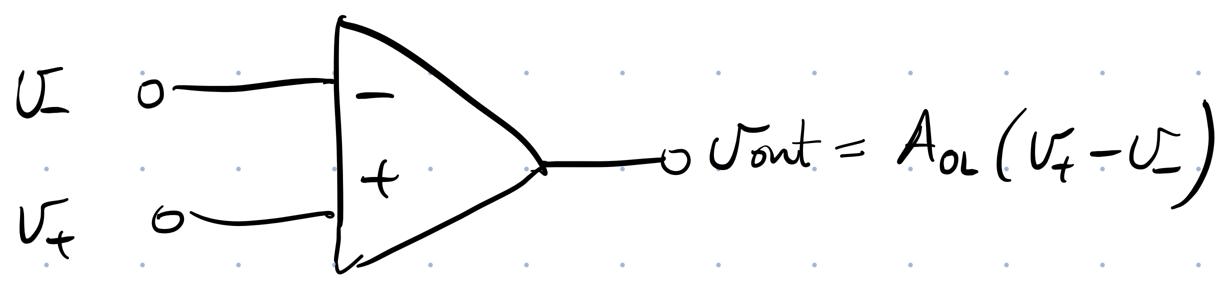
$$V_{out} = A_{OL}(V_+ - V_-)$$

"open-loop" gain.

We will use the LM741 op amp in the lab & it is designed to have an open-loop gain of about:

$$A_{OL} \approx 2\times10^5.$$

In circuit diagrams, the power terminals $\pm V_{cc}$ are often omitted from diagrams. However, the op amp <u>always</u> needs to be powered to fcn properly.



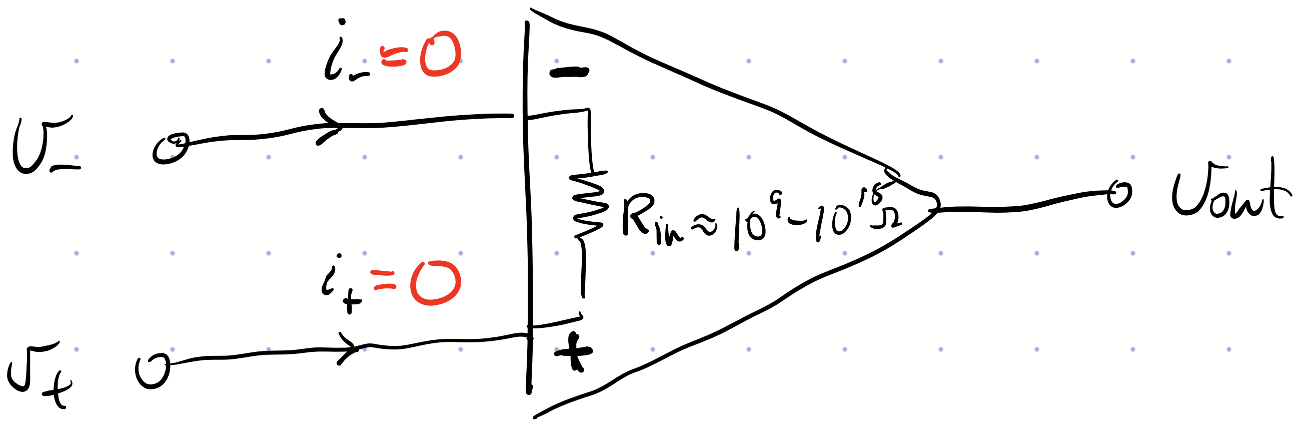
Limitations:

saturation voltage

① $|v_{out}| \leq V_{sat} \approx V_{cc} - 1$

If $V_{cc} = 15V$, $V_{sat} \approx 14V$.

② The input impedance/resistance of the op amp is very large ($10^9 - 10^{15}\,\Omega$)



Assume $R_{in} \to \infty$ s.t. $i_- = i_+ = 0$

$\boxed{\text{No current flows into or out of the op amp inputs.}}$

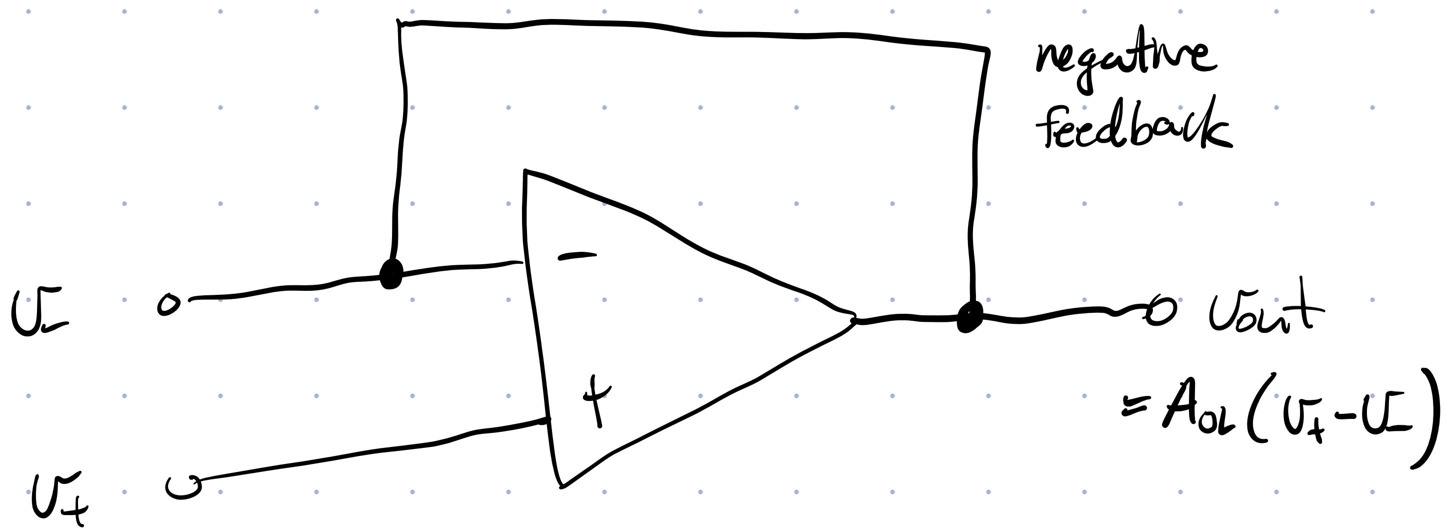
Note: Output can have non-zero current.

Op Amp Golden Rules

1. Op Amp input impedance $\to \infty$

$$\boxed{i_- = i_+ = 0}$$

2. The second op amp Golden rule applies to op amp circuits using <u>negative feedback</u>



Know $V_{out} = A_{OL}(V_+ - V_-)$

w/ negative feedback, we require:

$$V_{out} = V_-$$

$$V_- = A_{OL}(V_+ - V_-)$$ solve for V_-,

$$\therefore V_- + V_- A_{OL} = V_+ A_{OL}$$

$$\therefore V_-(A_{OL}+1) = A_{OL} V_+$$

$$\therefore V_- = \left(\frac{A_{OL}}{A_{OL}+1}\right) V_+$$

If $A_{OL} \gg 1$ (always the case)

then $\boxed{V_- = V_+}$ 2nd Golden Rule

only true when using negative feedback!

First Op Amp Application: (Lab #5)

Inverting Amplifier.