PHYS 231 Sept. 18, 2024

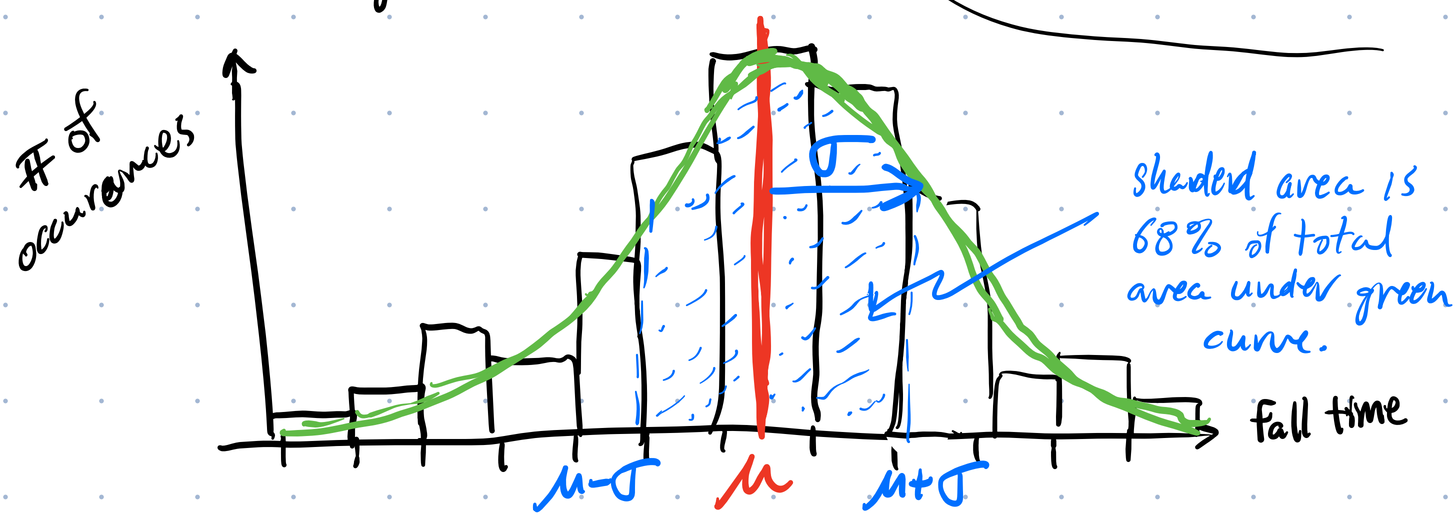
Last Time:

trial #	time (s)
1	148.25
2	151.36
3	146.20
4	153.67
⋮	⋮
N	149.71

Repeated measurements plotted as a histogram.

If fluctuations in measurement are radom, histogram approaches the shape of a Gaussian dist'n.

Plot a histogram of the N trials.



$$\mu = \frac{1}{N}\sum_{i=1}^{N} t_i \qquad \sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(t_i-\mu)^2$$

Expect 68% of measuments to fall within one σ of the mean μ.

If we meas. a quantity N times using the same experimental set up, the our best estimate of the quantitie's true value is the mean μ.

Expect our estimate of μ to improve when we increase the number of trials, N.
However, the standard deviation is <u>independent</u> of the number of trials N.

$\therefore$ The Std. dev. σ, by itself, is <u>not</u> a good estimate of the uncertainty in the mean.

The std. dev. is determined by exp'tal setup/meas. method. To decrease σ, need to improve exptal setup/proceedures.

The proper est. of the uncertainty in μ is given by the stanard error $\sigma_\mu = \frac{\sigma}{\sqrt{N}}$

$$\boxed{\sigma_\mu = \frac{\sigma}{\sqrt{N}}}$$

std. dev.

no. of trials.

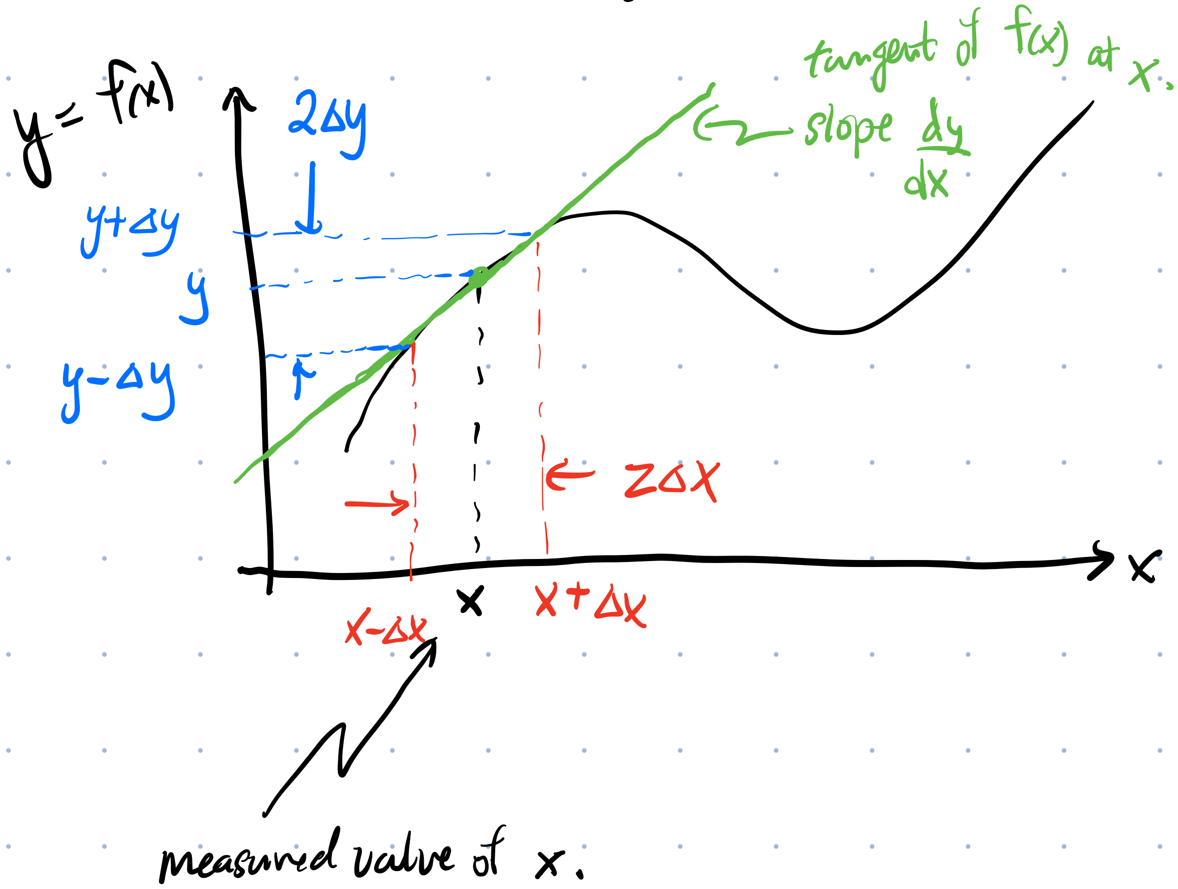
standard error
(uncertainty in the mean)

σ_μ has the desired property that it decreases as the no. of trials N is increased.

<u>Summary:</u> When we repeat a meas. N times, final result that should be reported is:

$$\boxed{\mu \pm \frac{\sigma}{\sqrt{N}}}$$

If we meas $x \pm \Delta x$, then what is the uncertainty in $y = f(x)$. i.e. calc. y based on value of x. What is Δy?



$$\text{Slope} = \frac{dy}{dx} = \frac{\text{rise}}{\text{run}} = \frac{\cancel{2}\Delta y}{\cancel{2}\Delta x} \quad \text{solve for } \Delta y$$

$$\boxed{\therefore \Delta y = \left|\frac{dy}{dx}\right| \Delta x}$$

If we know $x \pm \Delta x$, & $y = f(x)$. Then Δy is uncertainty in y.

If we meas. $x_1 \pm \Delta x_1$ & $x_2 \pm \Delta x_2$, then
what is the uncertainty in: $y = x_1 + x_2$?

i.e. what is Δy?

Turns that Δy is given by the "quadrature" sum
of Δx_1 & Δx_2:

$$\Delta y = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2} < \Delta x_1 + \Delta x_2$$

Now suppose we meas $u \pm \Delta u$ and $V \pm \Delta V$ and
we want to calc. $y = f(u,v)$. What is Δy?

First, we find the uncertainty in y due to the
u variable:

$$\Delta y_u = \left| \frac{\partial y}{\partial u} \right| \Delta u$$

Then, find the contribution to Δy due to V variable:

$$\Delta y_v = \left| \frac{\partial y}{\partial v} \right| \Delta V$$

Finally, we combine the two contributions using the quadrature sum:

$$\Delta y = \sqrt{(\Delta y_u)^2 + (\Delta y_v)^2}$$

$$\boxed{\therefore \Delta y = \sqrt{\left(\frac{\partial y}{\partial u}\Delta u\right)^2 + \left(\frac{\partial y}{\partial v}\Delta v\right)^2}}$$

In general, for a fcn of any no. of variables

$$y = f(x_1, x_2, \ldots, x_N)$$

$$\boxed{\Delta y = \sqrt{\left(\frac{\partial y}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial y}{\partial x_2}\Delta x_2\right)^2 + \cdots + \left(\frac{\partial y}{\partial x_N}\Delta x_N\right)^2}}$$

Propagation of errors