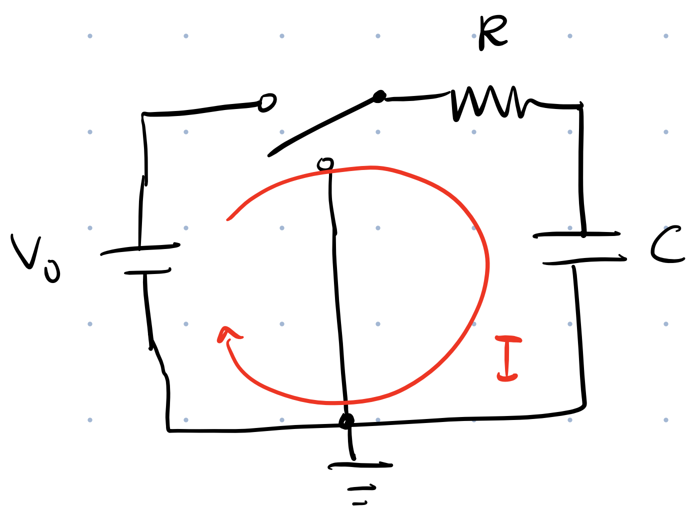
PHYS 231 Sept. 16, 2024

Last Time: Series RC Circuit



Start w/ switch down $\Rightarrow Q=0$,

$I = \frac{dQ}{dt} = 0$

@ $t=0$, flip switch up, $I \neq 0$.

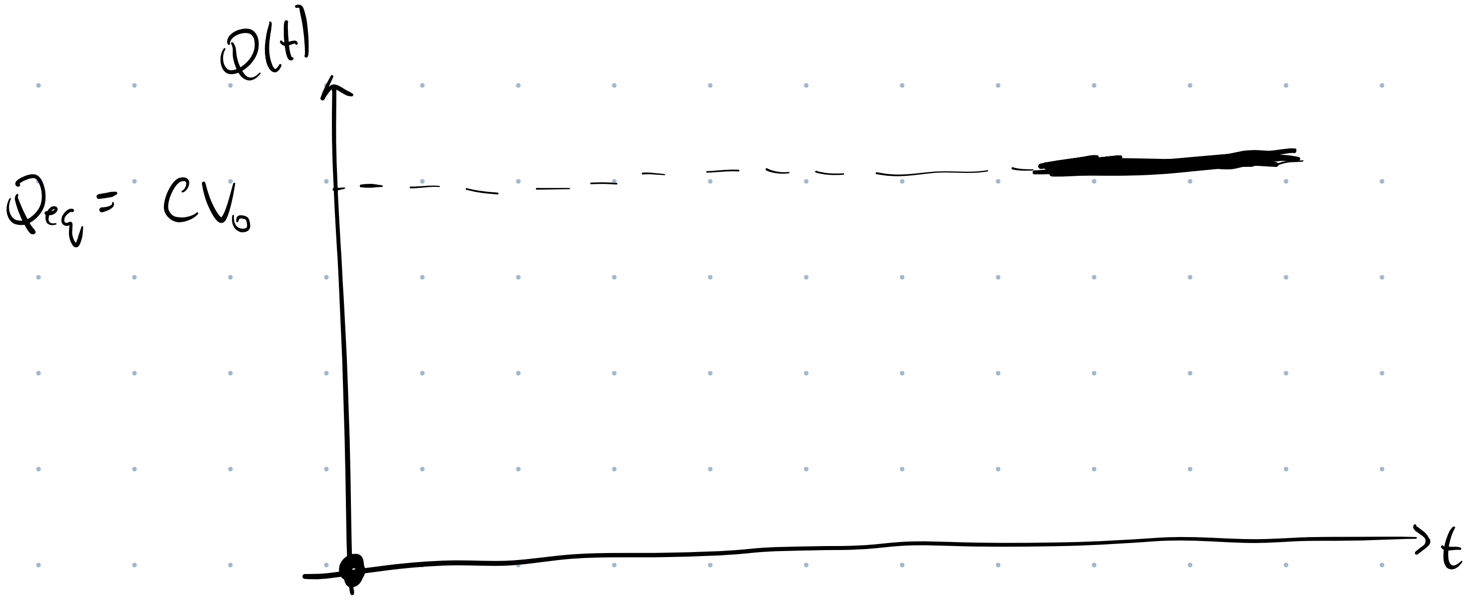
KLR: $$\boxed{\frac{dQ}{dt} + \frac{1}{RC} Q = \frac{V_0}{R}} \quad (*)$$

Solve for $Q(t)$ subject to initial condition $Q(0) = 0$.

After switch has been up for a long time, system reaches a new equil. s.t.

$Q = Q_{eq}$ (const) $\}$ $I = \frac{dQ}{dt} = 0$

$$\therefore \quad \frac{1}{RC} Q_{eq} = \frac{V_0}{R} \Rightarrow \boxed{Q_{eq} = CV_0}$$



Express $Q(t) = Q_h(t) + \underbrace{Q_{eq}}_{\text{const.}}$ $\quad Q_{eq} = CV_0$

$$Q(t) = Q_h(t) + CV_0$$

sub into ⊛ s.t.

$$\frac{V_0}{R} = \underbrace{\frac{d}{dt}\left(Q_h(t) + CV_0\right)}_{\frac{dQ_h(t)}{dt}} + \frac{1}{RC}\left(Q_h(t) + CV_0\right)$$

since CV_0 is const.

$$\therefore \cancel{\frac{V_0}{R}} = \frac{dQ_h(t)}{dt} + \frac{1}{RC} Q_h(t) + \cancel{\frac{V_0}{R}}$$

0

$$(\#) \quad \boxed{0 = \frac{dQ_h(t)}{dt} + \frac{1}{RC} Q_h(t)}$$

Homogeneous 1st order diff. eq'n.

→ LHS is zero.

→ $Q_h(t)$ called the homogeneous sol'n.

Goal is to solve (#) for $Q_h(t)$. Then, we will know the capacitor charge at all times since

$$Q(t) = Q_h(t) + CV_0.$$

Start by defining $\tau = RC$ (time const.)

$$0 = \frac{dQ_h(t)}{dt} + \frac{1}{\tau} Q_h(t).$$

$$\therefore -\frac{1}{\tau} Q_h(t) = \frac{dQ_h(t)}{dt} \qquad \left(\text{mult. by } dt\right)$$

$$\therefore -\frac{dt}{\tau} Q_h(t) = dQ_h(t) \qquad \left(\text{divide by } Q_h(t)\right)$$

$$\therefore -\frac{dt}{\tau} = \frac{dQ_h(t)}{Q_h(t)} \quad \text{(integrate both sides)}$$

$$\therefore \int\left(-\frac{dt}{\tau}\right) = \int \frac{dQ_h(t)}{Q_h(t)}$$

$$\therefore -\frac{1}{\tau}\int dt = \int \frac{dQ_h(t)}{Q_h(t)}$$

integration constant.

$$\therefore -\frac{t}{\tau} = \ln Q_h(t) + B$$

$$\therefore \ln Q_h(t) = -\left(\frac{t}{\tau} + B\right) \quad \text{(exponentiate)}$$

$$Q_h(t) = e^{-\left(\frac{t}{\tau}+B\right)} = e^{-t/\tau}\underbrace{e^{-B}}_{\equiv B' \text{ (const).}}$$

$$\therefore Q_h(t) = B' e^{-t/\tau}$$

use our initial condition $Q(t) = Q_h(t) + CV_0$

Require $Q(0) = 0$

$$\therefore 0 = Q_h(0) + CV_0$$

But $Q_h(0) = B' e^{-0/\tau} = B'$

$$\therefore 0 = B' + CV_0 \Rightarrow B' = -CV_0$$

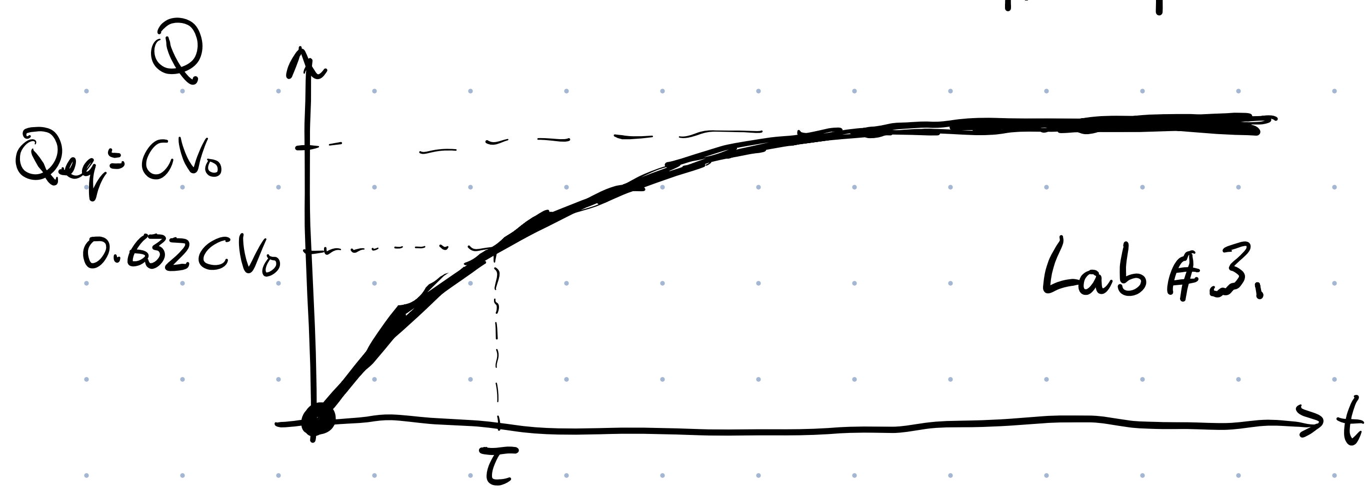
$$\therefore Q_h(t) = -CV_0 e^{-t/\tau}$$

$$\therefore Q(t) = Q_h(t) + CV_0$$

$$= -CV_0 e^{-t/\tau} + CV_0$$

$$\boxed{\therefore Q(t) = CV_0\left(1 - e^{-t/\tau}\right)}$$

Charge on cap. at all times after switch flipped up.



Lab #3.

Consider the time $t = \tau$ $\quad (\tau = RC)$

$$Q(t=\tau) = CV_0\left(1 - e^{-\tau/\tau}\right)$$

$$= CV_0\underbrace{\left(1 - \frac{1}{e}\right)}_{0.632}$$

The capacitor reaches 63% of its final charge after one time const.

---

aside

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-t/\tau} = 1 - \frac{t}{\tau} + \underbrace{\frac{1}{2!}\left(\frac{t}{\tau}\right)^2 - \frac{1}{3!}\left(\frac{t}{\tau}\right)^3 + \dots}_{\text{small c.t. } t/\tau\text{, linear}}$$

To find the time dependence of the current, evaluate

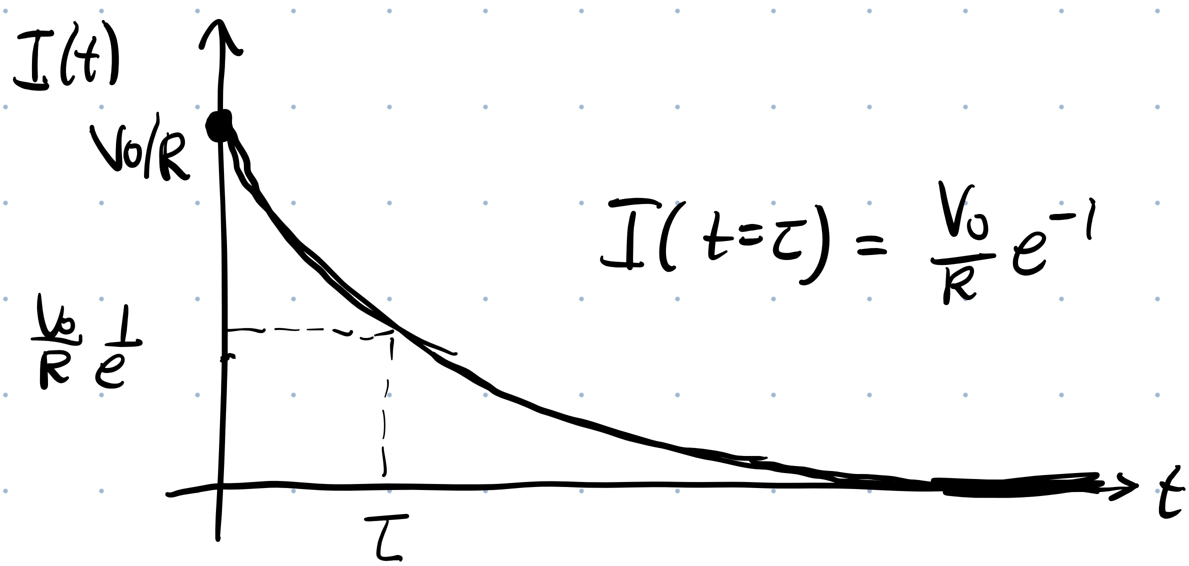
$$I(t) = \frac{dQ}{dt} = \frac{d}{dt}\left[CV_0\left(1 - e^{-t/\tau}\right)\right]$$

$$= 0 - CV_0 \frac{d}{dt}\left(e^{-t/\tau}\right)$$

$$= \cancel{-}\, CV_0 \left(\cancel{-}\,\frac{1}{\tau} e^{-t/\tau}\right)$$

$$\therefore I(t) = \frac{\cancel{C}V_0}{R\cancel{C}} e^{-t/\tau}$$

$$\boxed{I(t) = \frac{V_0}{R} e^{-t/\tau}}$$



$$I(t=\tau) = \frac{V_0}{R} e^{-1}$$

Uncertainties & Propagation of Errors

Every quantity that is measured has an associated uncertainty.

Experiment: Drop a penny from a tall building & meas. time for it to reach the gnd. Make a table of repeated trials.

trial #	time (s)
1	148.25
2	151.36
3	146.20
4	153.67
⋮	⋮
N	149.71

$$\mu = \frac{1}{N}\sum_{i=1}^{N} t_i$$

Plot a histogram of the N trials.