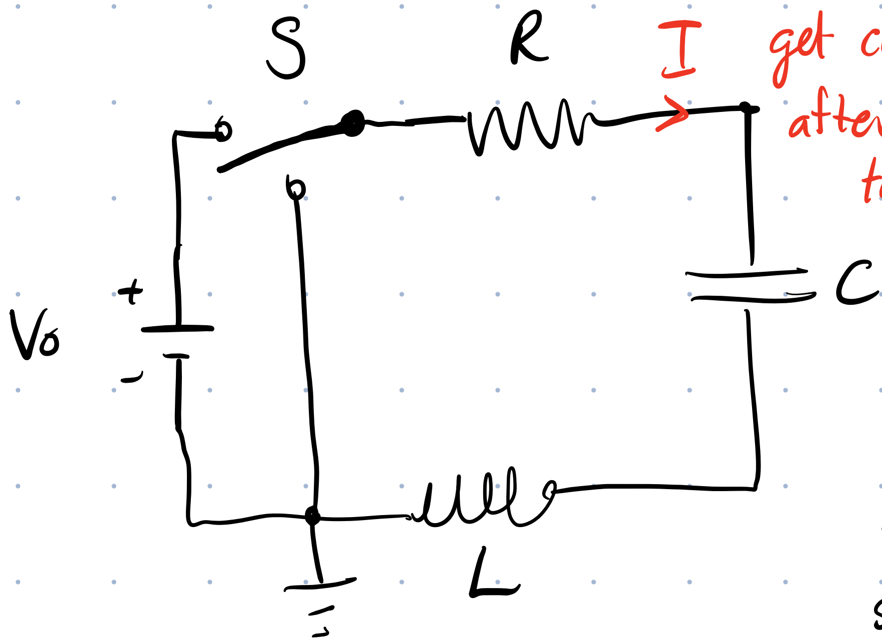
PHYS 231 Sept. 11, 2024

Last time

$$|\Delta V_C| = \frac{Q}{C}, \quad |\Delta V_R| = IR = R\frac{dQ}{dt}, \quad |\Delta V_L| = L\frac{dI}{dt} = L\frac{d^2Q}{dt^2}$$

LRC circuit



get current I after switch is moved to "up" position.

Start w/ switch in "down" position for a long time s.t. $Q=0$, $I=0$

Kirchhoff Loop Rule

$$V_0 - \frac{1}{C}Q - R\frac{dQ}{dt} - L\frac{d^2Q}{dt^2} = 0$$

2nd order differential eq'n. → Math 225, PHYS 216

Solve for $Q(t)$. → Called time-domain or a transient analysis.

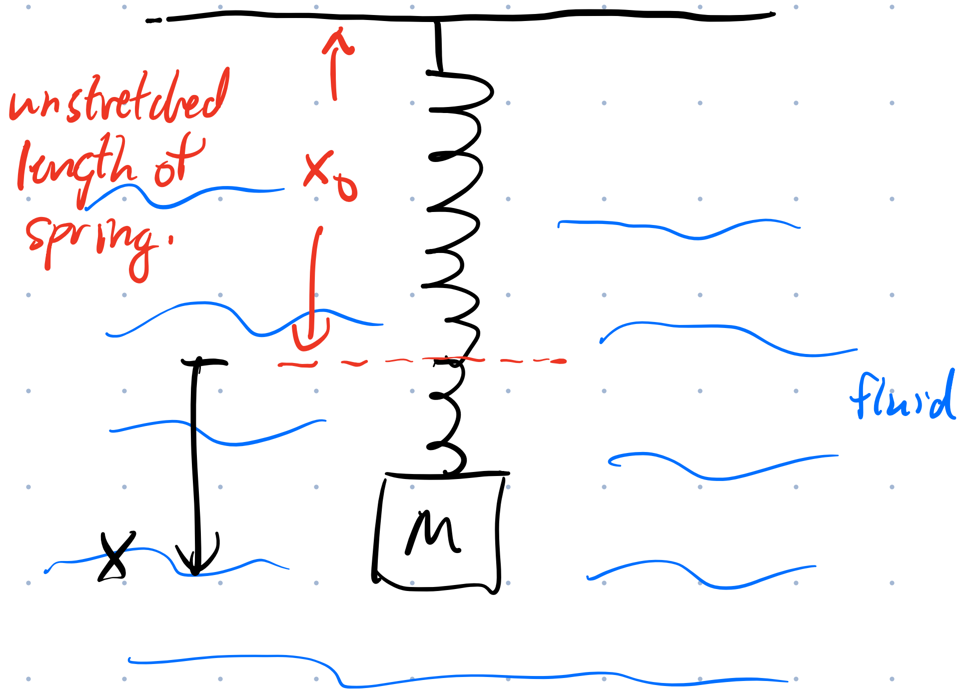
Rewrite diff. eq'n as:

$$\boxed{\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = \frac{V_0}{L}} \quad (a)$$

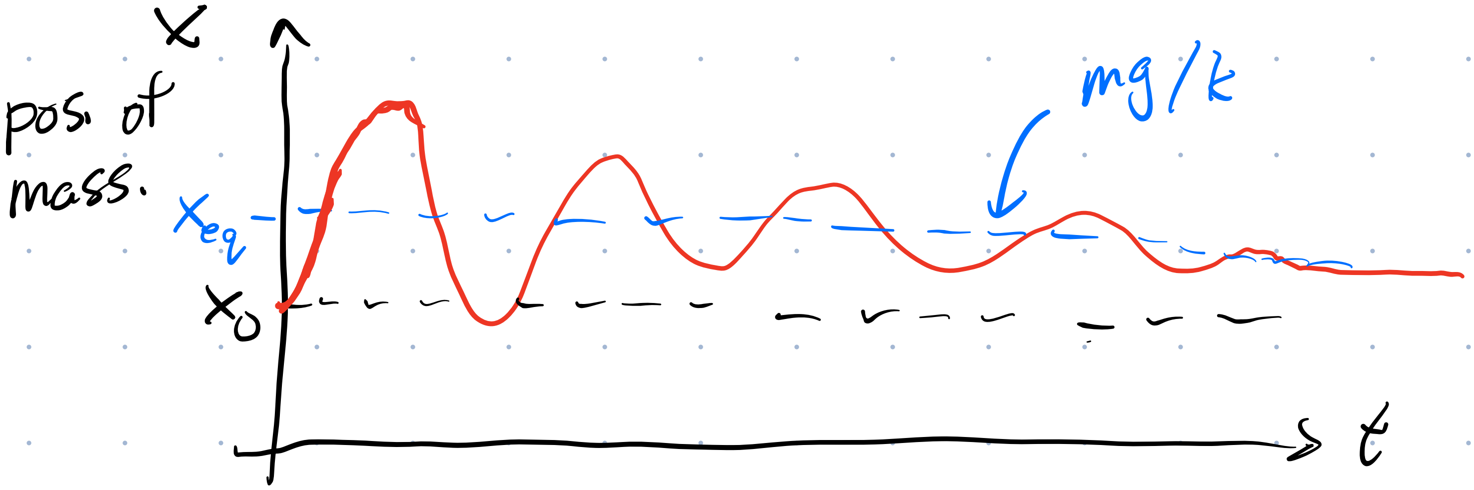
If you like, see full sol'n to (a) on PHYS 231 website (optional)

We will demonstrate that the LRC circuit has a mechanical analogy that we already understand intuitively.

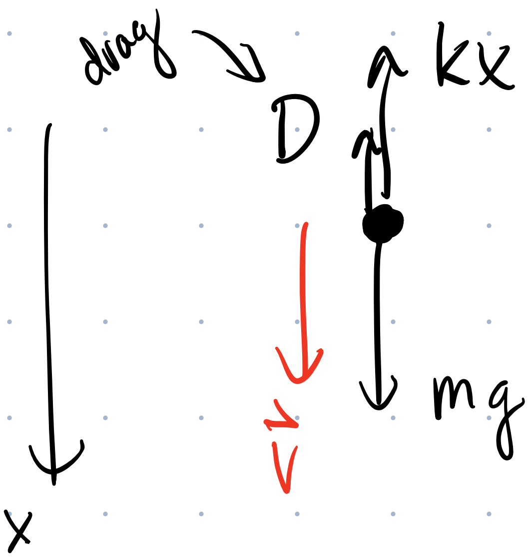
Consider a mass hanging from ceiling by a spring. Mass is also submerged in a viscous fluid.



If we stretch & release the mass, it will begin to oscillate about an equil. position. The drag force on the mass due to fluid converts some of the system's mechanical energy into heat. This dissipation, causes the amplitude of the osc. to decrease w/ time (damping).



FBD of mass on spring:



Assume that the drag force D is given by $D = bv$

drag coefficient.

Newton's 2nd Law

$$\underbrace{ma}_{\text{net force.}} = mg - kx - bv$$

rewrite $v = \frac{dx}{dt}$, $a = \frac{d^2x}{dt^2}$, eqn:

$$m\frac{d^2x}{dt^2} = mg - kx - b\frac{dx}{dt}$$

$$\boxed{\therefore \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = g} \quad (b)$$

Mathematically, Eq'ns (a) & (b) are identical.

$\therefore$ Sol'n for $Q(t)$ in (a) is of the same form as the sol'n for $x(t)$ in (b).

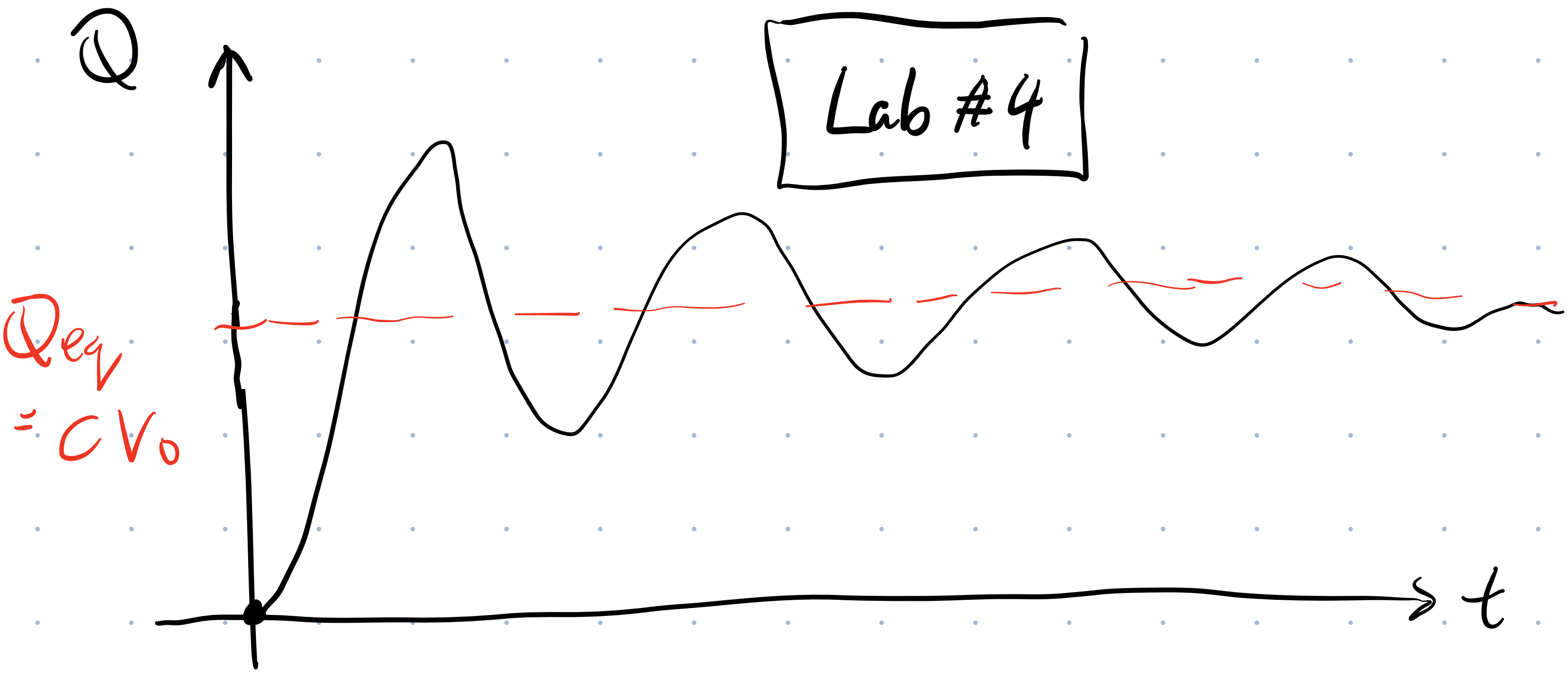
For the mass on a spring, we can find the final equilib. position by setting $\frac{dx}{dt} = v$ & $\frac{d^2x}{dt^2} = a$ to be equal to zero.

$$\frac{k}{m}x_{eq} = g \Rightarrow \boxed{x_{eq} = \frac{mg}{k}}$$

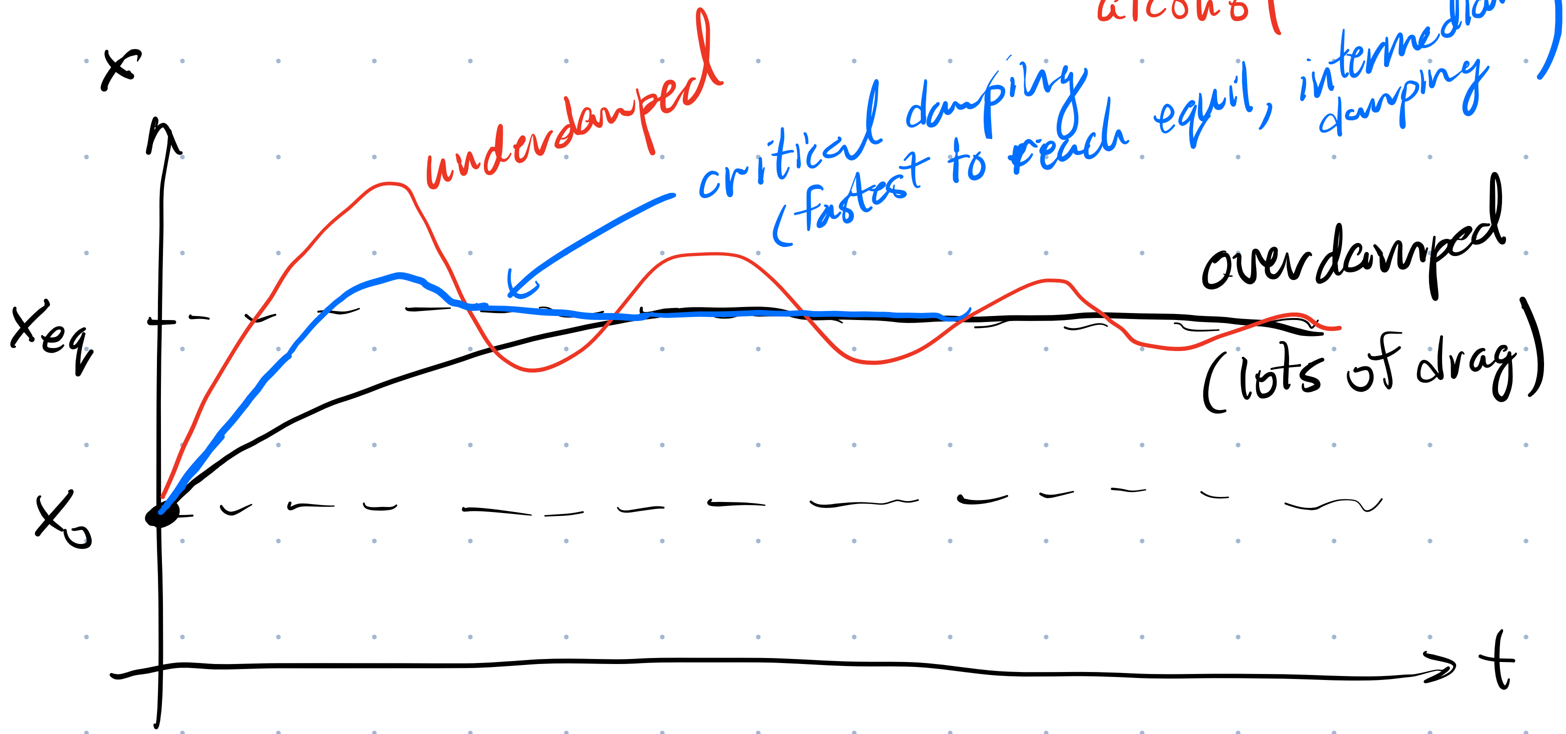
For LRC circuit, can find final equil. charge on cap. by setting $I = \frac{dQ}{dt}$ & $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$ to zero in (a)

$$\frac{1}{\cancel{L}C} Q_{eq} = \frac{V_0}{\cancel{L}}$$

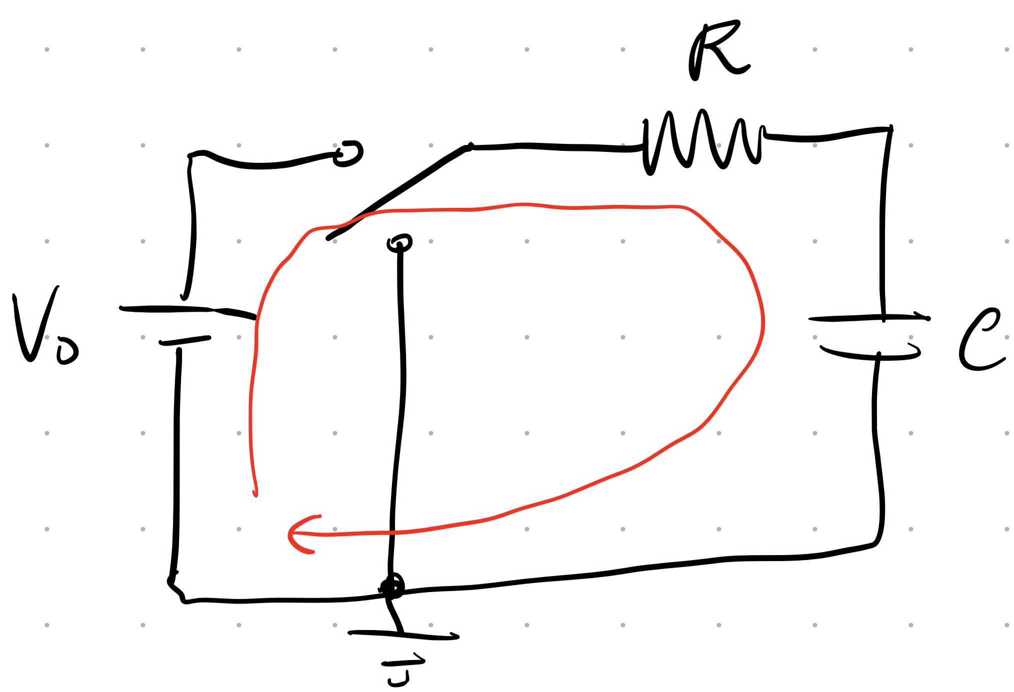
$$\boxed{\therefore Q_{eq} = C V_0}$$



Mass on a spring submerged in honey.



Next time, we'll analyze the RC series circuit.



Start w/ switch down s.t.

$Q = 0$

$I = 0$

At $t=0$, flip switch up. Find $Q(t)$ as system approaches new equil.

KLR w/ switch up.

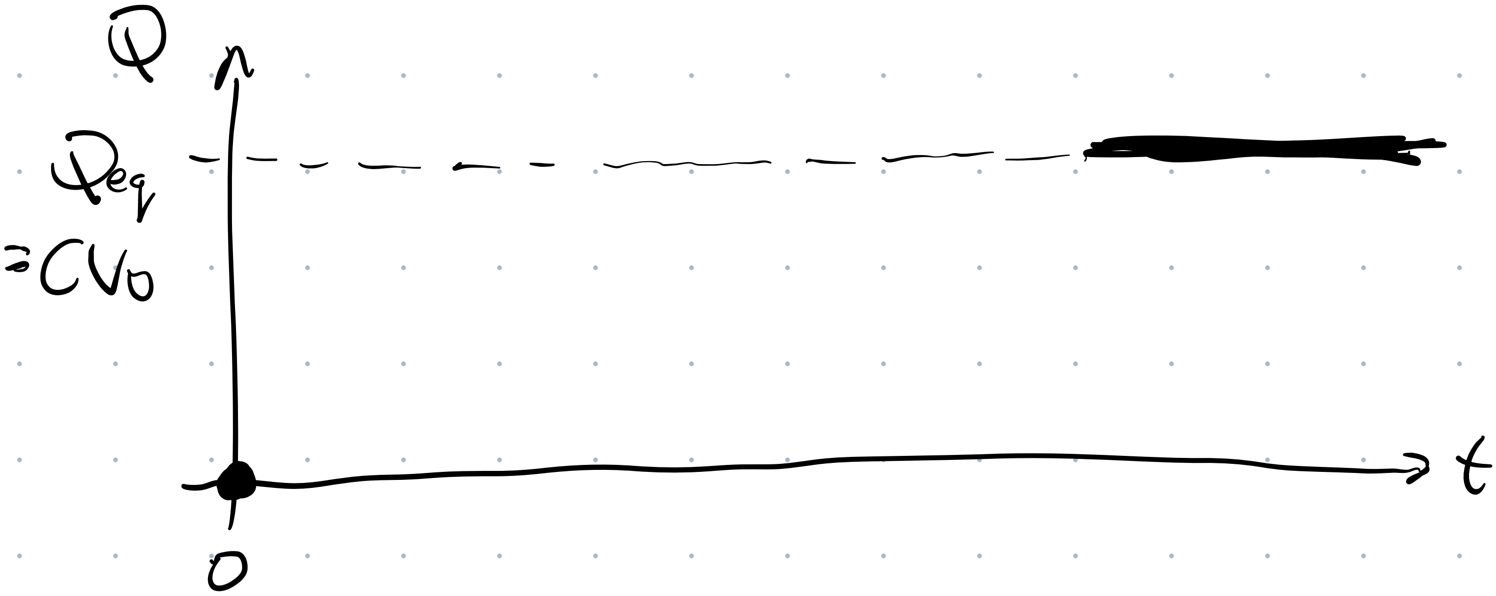
$$V_0 - R\frac{dQ}{dt} - \frac{1}{C}Q = 0$$

$$\boxed{\therefore \quad \frac{dQ}{dt} + \frac{1}{RC}Q = \frac{V_0}{R}}$$

First order diff. eq'n. want to solve for $Q(t)$.

Long time after switch up, $\frac{dQ}{dt} = 0$

$$\therefore \quad \frac{1}{\cancel{R}C}Q_{eq} = \frac{V_0}{\cancel{R}} \Rightarrow Q_{eq} = CV_0$$



Q
Qeq
=CV0
0
t