PHYS 231 Sept. 9, 2024

Last time:

Relationship between potential diff. & $\vec{E}$:

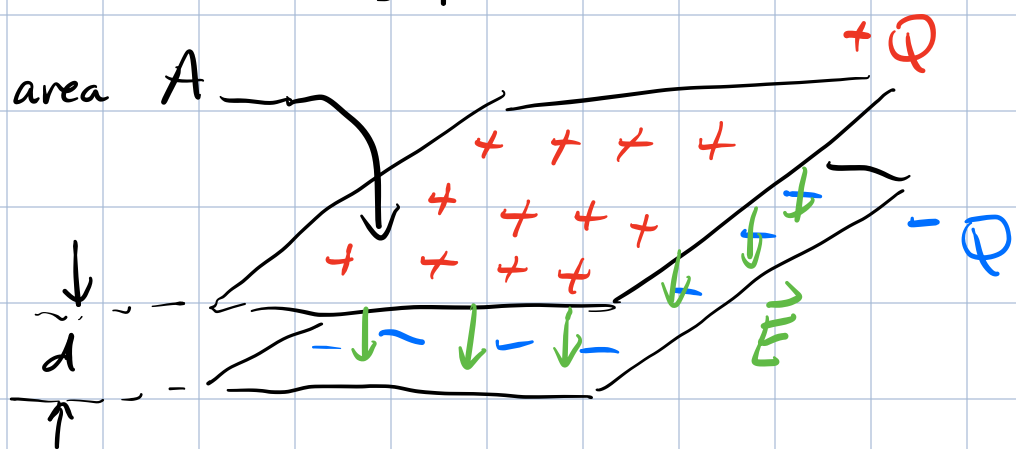
$$\Delta V = V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{l}$$

follows from Work-KE. theorem

$$W = \int_1^2 \vec{F}\cdot d\vec{l}$$

Today: Capacitors & Inductors

Parallel Plate Capacitor

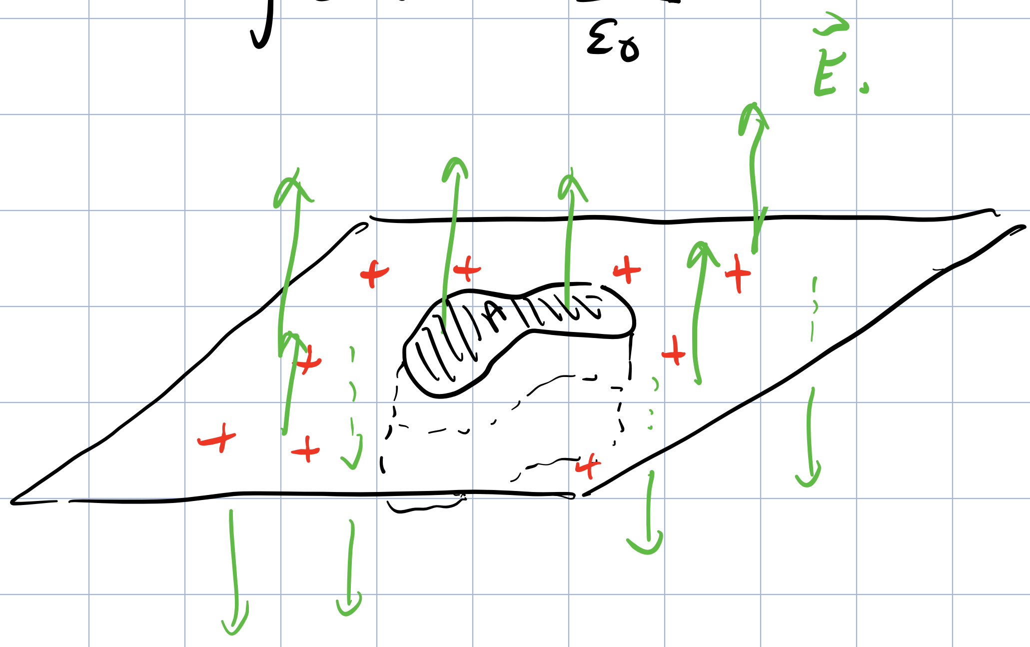


Capacitance is defined to be $C = \dfrac{Q}{|\Delta V|}$

We can use Gauss's law to find $\vec{E}$ due to a sheet of charge w/ charge per unit area:

$$\sigma = \frac{Q}{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$

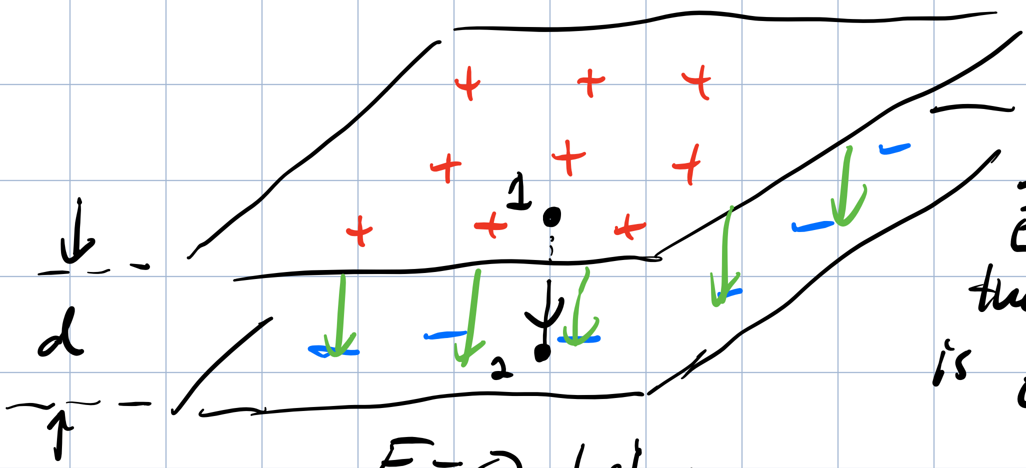


$$2EA = \frac{q_{encl}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$\therefore \; 2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$ Electric field of a uniformly charge sheet.

$E = 0$ above



$\vec{E}$ between the plates is $E = \dfrac{\sigma}{\varepsilon_0}$

$E = 0$ below

$$|\Delta V| = \int_1^2 \vec{E} \cdot d\vec{\ell}$$ volt. diff. between plates.

$$= Ed = \frac{\sigma}{\varepsilon_0} d = \boxed{\frac{Qd}{\varepsilon_0 A} = |\Delta V|}$$

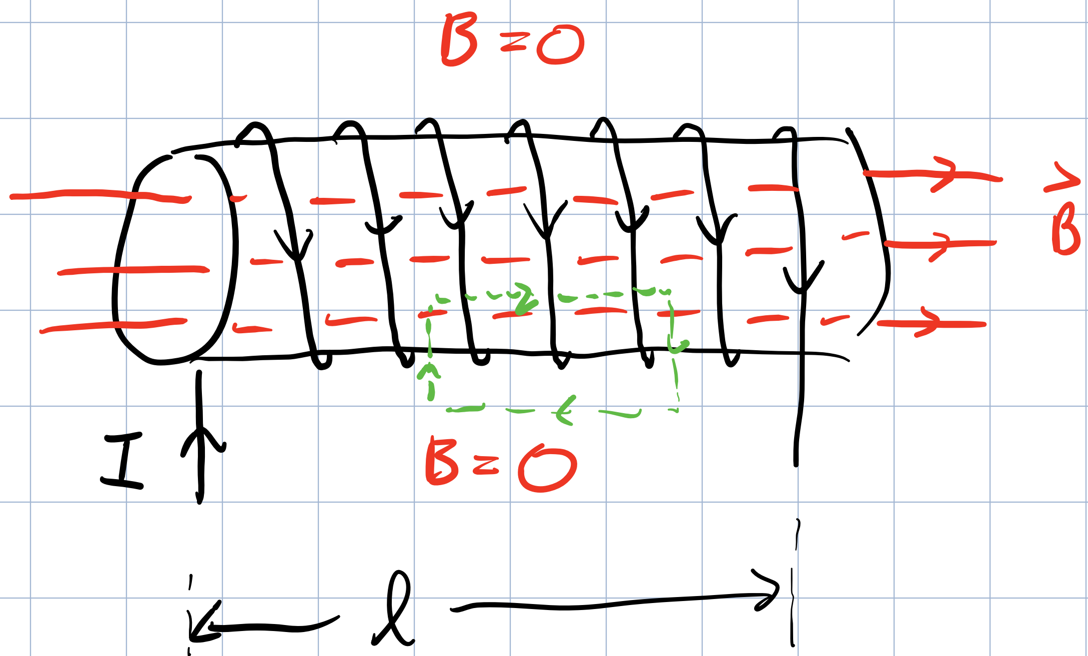
By definition:

$$\boxed{C = \frac{Q}{|\Delta V|} = \varepsilon_0 \frac{A}{d}}$$ Parallel Plate cap. capacitance.

Summary: $|\Delta V_R| = IR = R\dfrac{dQ}{dt}$

$$|\Delta V_C| = \frac{Q}{C}$$

Inductors / Solenoids (coils of wire)



Ampère's Law $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$

$$\boxed{B = \mu_0 \frac{N}{\ell} I}$$

Magnetic field inside solenoid / inductor.

Use Faraday's Law to find voltage across inductor:

$$|\Delta V_L| = \frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

For a single turn of the solenoid

$$\Phi_1 = A\left(\mu_0 \frac{NI}{l}\right)$$

Net flux for entire inductor is:

$$\Phi_B = N\Phi_1 = \mu_0 \frac{N^2}{l} I A$$

From Faraday's Law:

$$|\Delta V_L| = \frac{d}{dt}\Phi_B = \frac{d}{dt}\left(\mu_0 \frac{N^2}{l} I A\right)$$

everything is const. expect possibly for I.

$\therefore$

$$|\Delta V_L| = \underbrace{\left(\mu_0 \frac{N^2}{l} A\right)}_{\equiv L} \frac{dI}{dt}$$

$\equiv L$ the "inductance" of the solenoid.

$$\boxed{\therefore |\Delta V_L| = L\frac{dI}{dt}}$$

If I is const., then $\Delta V_L = 0$.
Get voltage diff only when I is changing.

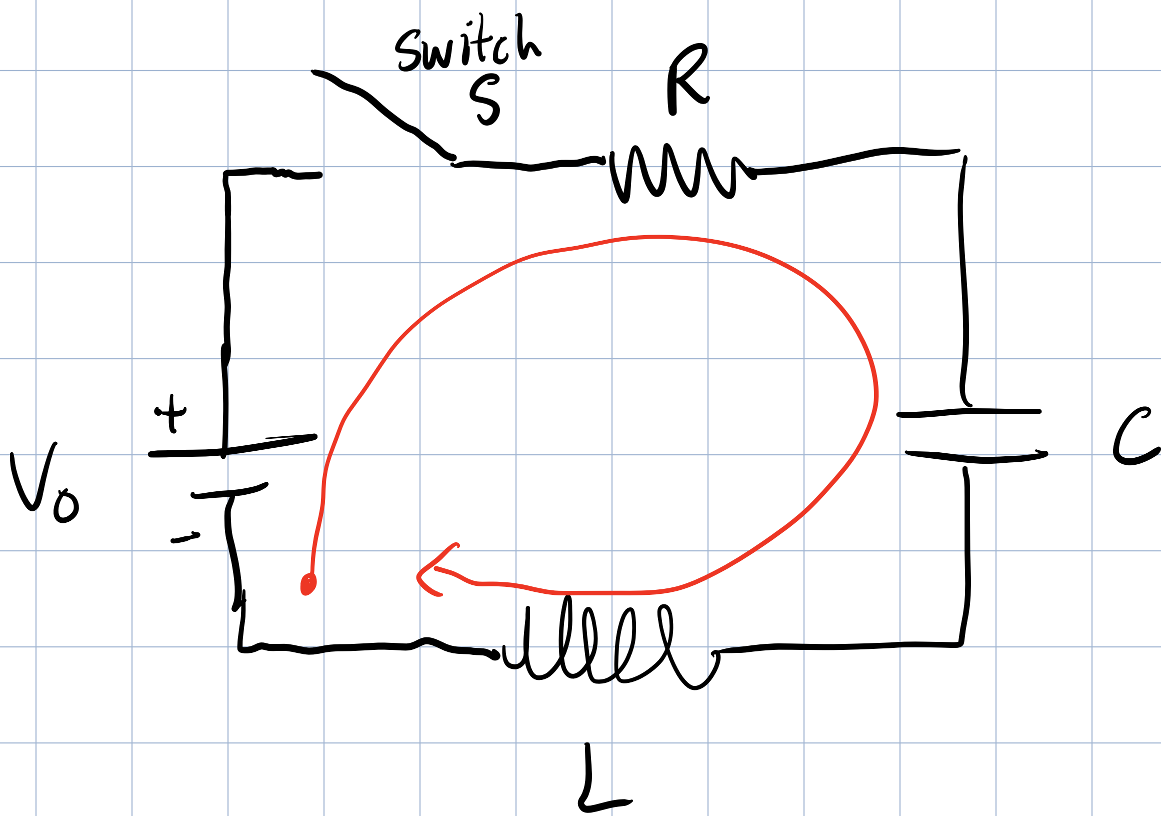
Summary:

Resistors : $|\Delta V_R| = IR = R\dfrac{dQ}{dt}$

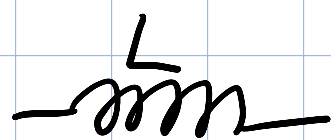
Capacitors : $|\Delta V_C| = \dfrac{1}{C}Q$

Inductors : $|\Delta V_L| = L\dfrac{dI}{dt} = L\dfrac{d^2Q}{dt^2}$

Think about a series circuit w/ an L, C, R.



Circuit symbol for an inductor :



Start w/ switch open ($I=0$) & uncharged capacitor ($Q=0, \Delta V_c = 0$). Then at time $t=0$, close switch.

When switch is closed $I \neq 0$ & $Q \neq 0$.

Kirchoff voltage loop rule requires:

$$V_0 - IR - \frac{Q}{C} - L\frac{dI}{dt} = 0$$

In terms of Q:

$$\boxed{V_0 - \frac{1}{C}Q - R\frac{dQ}{dt} - L\frac{d^2Q}{dt^2} = 0}$$

2nd order diff. eq'n.