PHYS 121 April 7, 2025

- Next PrairieLearn HW due **Tues. Apr. 8**
- See course website for final exam details (including formula sheet)
- If participating in the hands-on bonus project, send me a link to your YouTube video by 23:59 **today**

- Complete the end-of-term survey by 23:59 on April 8 for 0.5 marks towards your final grade. A link to the survey has been provided in Canvas.

Last Time:

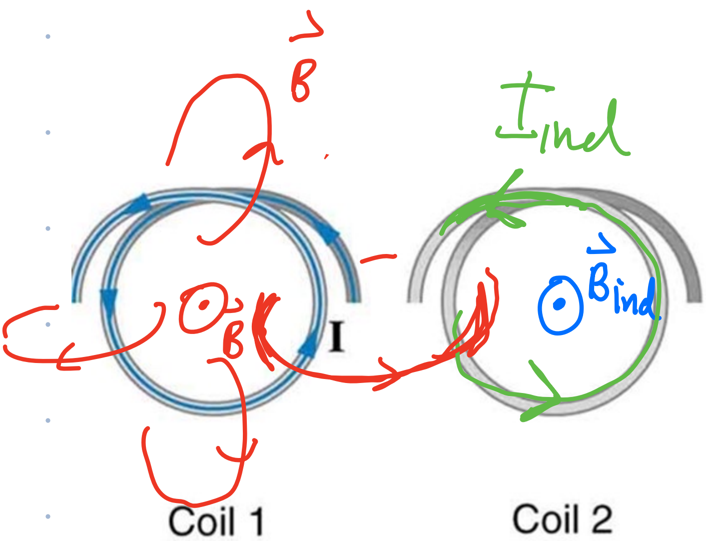
Faraday's Law: $\mathcal{E} = \left| \frac{d\Phi_B}{dt} \right|$ where $\Phi_B = \int \vec{B} \cdot d\vec{a}$

Lenze's Law:

The induced magnetic field $\vec{B}_{ind}$ has a dir'n that tends to maintain the original magnetic flux.

- If Φ_B is increasing,
  $\vec{B}_{ind}$ is in opp. dir'n of $\vec{B}$
- If Φ_B is decreasing,
  $\vec{B}_{ind}$ is in same dir'n as $\vec{B}$.

1: Referring to the figure, what is the direction of the current induced in coil 2: (a) If the current in coil 1 increases? (b) If the current in coil 1 decreases? (c) If the current in coil 1 is constant?



(a) $\vec{B}$ due to coil 1 creates a magnetic flux in 2 that is directed into the page. Since I is increasing, Φ_B through 2 is increasing into the page. To oppose this increasing flux, $\vec{B}_{ind}$ is out of the screen through the centre of coil 2.

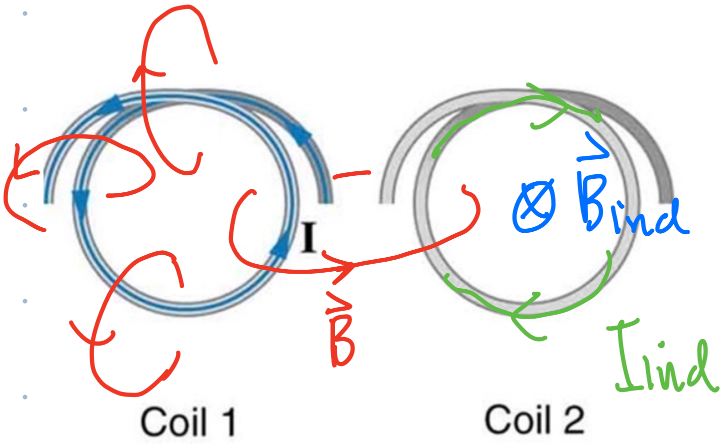
Require I_{ind} to be CCW by RHR to make $\vec{B}_{ind}$ out of screen.

(c) If I in coil 1 in const, then it creates a constant $\vec{B}$ s.t.

Φ_B through coil 2 is also constant.

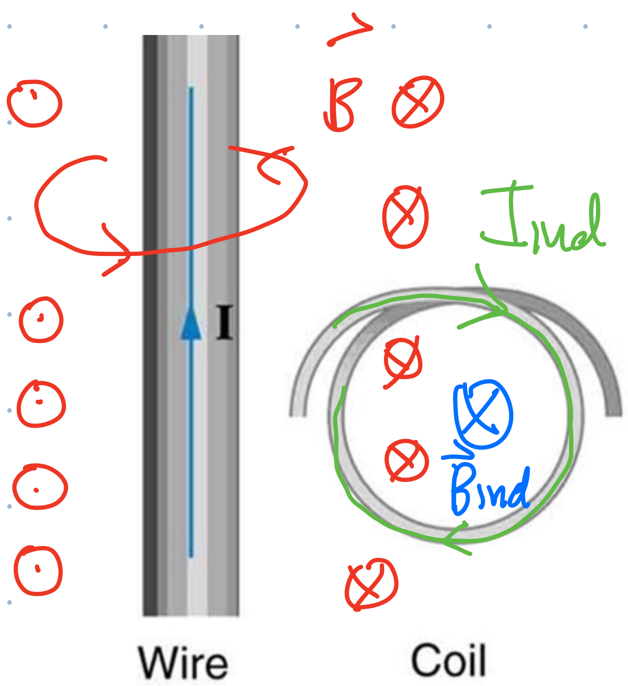
But, $\varepsilon = \left| \frac{d\Phi_B}{dt} \right| = 0$

$\therefore I_{ind} = 0$, $\vec{B}_{ind} = 0$ in coil 2.



(b) $\vec{B}$ in coil 2 due to coil 1 is into the screen (like in (a)). However, since I is decreasing, $\vec{B}_{ind}$ is also into the screen to maintain original flux. This $\vec{B}_{ind}$ is created by a CW I_{ind} in coil 2.

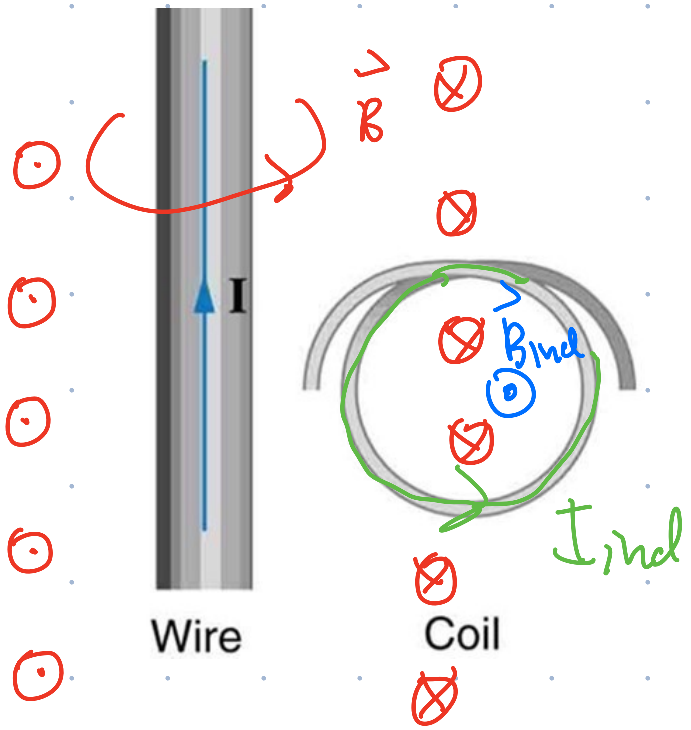
2: Referring to the figure, what is the direction of the current induced in the coil: (a) If the current in the wire increases? (b) If the current in the wire decreases? (c) If the current in the wire suddenly changes direction?



(b) $\vec{B}$ loop around long straight currents. By RHR, this creates Φ_B that is into the screen through coil. If I in decreasing, then Φ_B is decreasing. To compensate for decreasing Φ_B, $\vec{B}_{ind}$ is into screen. RHR tells us that I_{ind} is CW.

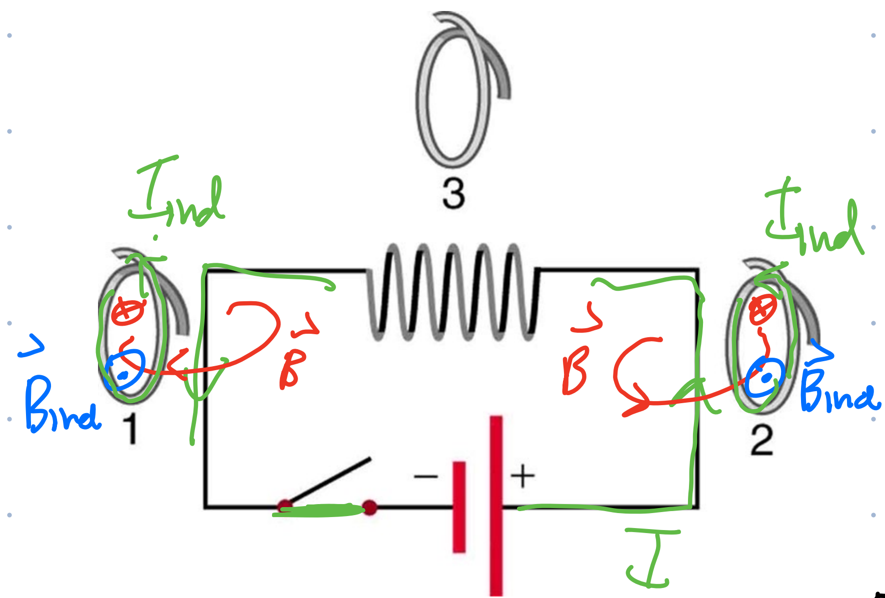
(c) Initially Φ_B is into page. When I reverse dir'n, get Φ_B out of page.

System tries to maintain original flux into page by creating $\vec{B}_{ind}$ that is into the page. Like in (b), this requires I_{ind} CW.



(a) Like in (b), $\vec{B}$ through coil due to current I in wire is into the screen. Since I is increasing, Φ_B through the coil is increasing. $\vec{B}_{ind}$ is out of the screen to counter the increase in flux. This $\vec{B}_{ind}$ is created by a CCW I_{ind} in the coil.

3: Referring to the figure, what are the directions of the currents in coils 1, 2, and 3 (assume that the coils are lying in the plane of the circuit): (a) When the switch is first closed? (b) When the switch has been closed for a long time? (c) Just after the switch is opened?



(a) Just after switch is closed, est. a current that is ccw in circuit.

In coils 1 & 2, $\vec{B}$ due to I in into screen through the loops.

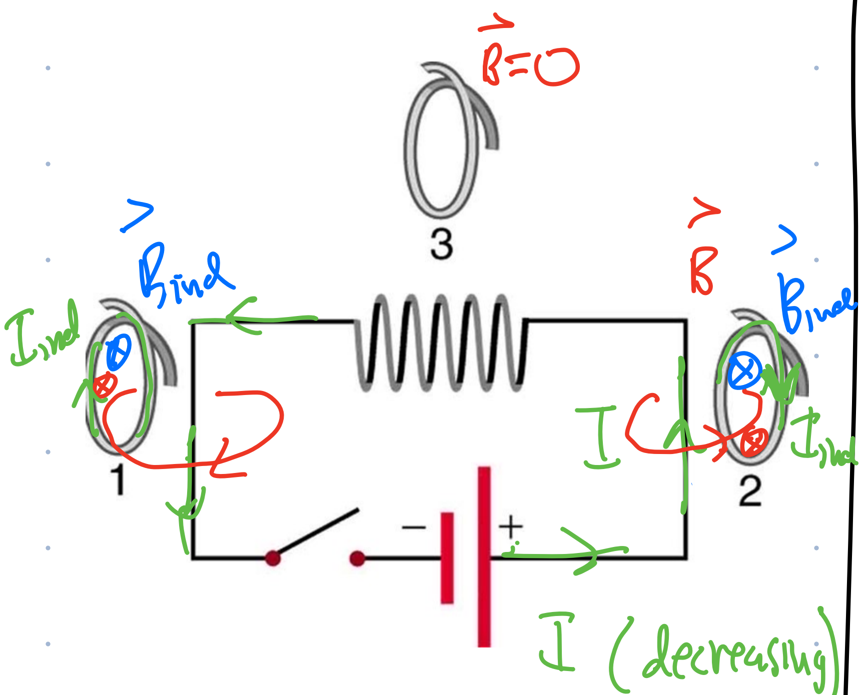
Since Φ_B was initially zero, $\vec{B}_{ind}$ oppose increasing Φ_B into page by creating $\vec{B}_{ind}$ that is out of page. $\vec{B}_{ind}$ out of page requires I_{ind} ccw.

Coil 3 is next to a solenoid w/ curren I. We know that $\vec{B}$ is uniform inside the solenoid & zero outside the solenoid. Since coil 3 sits outside the solenoid, it has no flux through it. $\therefore$ $\mathcal{E} = \left| \frac{d\Phi_B}{dt} \right| = 0$ $\therefore$ $I_{ind} = 0$, $\vec{B}_{ind} = 0$.

(b) When switch has be closed for a long time, the current in the circuit is constant.

∴ Φ_B through all three coils is const.

Since $\mathcal{E} = \left| \frac{d\Phi_B}{dt} \right| = 0$, there are no induced currents.



(c) Like in (a), the magnetic field outside the solenoid is zero.

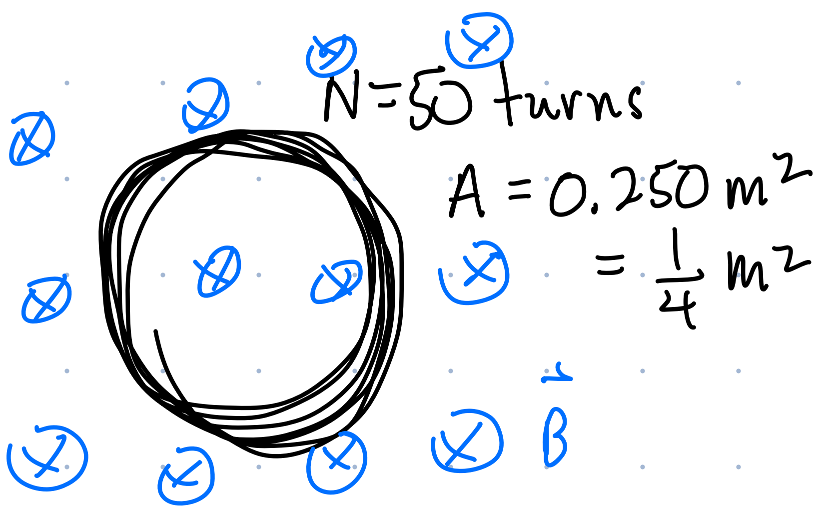
∴ Coil 3 always has zero Φ_B & no induced current.

Flux through coils 1 & 2 is into the screen and decreasing as the current I goes to zero.

∴ $\vec{B}_{ind}$, in both cases, is into the screen. These $\vec{B}_{ind}$ are established by CW I_{ind} in coils 1 & 2.

4: Suppose a 50-turn coil lies in the plane of the page in a uniform magnetic field that is directed into the page. The coil originally has an area of 0.250 m^2. It is stretched to have no area in 0.100 s. What is the direction and magnitude of the induced emf if the uniform magnetic field has a strength of 1.50 T?

Initial



$\Phi_{B,i} = NBA_i$ ← get a flux of BA through each of the 50 loops

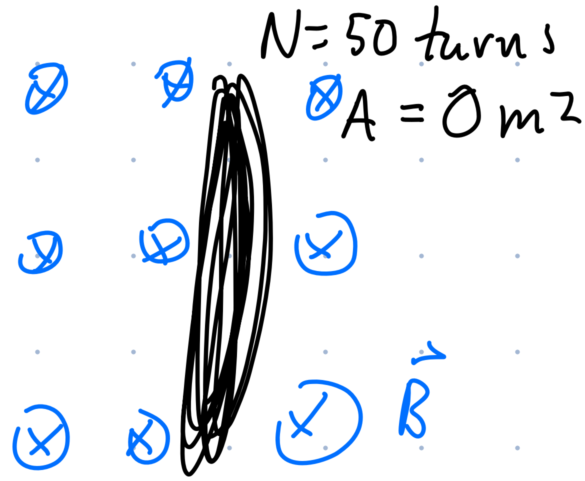
$\therefore \Phi_{B,i} = 50\,(1.5\,T)\,\frac{1}{4}\,m^2$

$= (12.5)(1.5)\ T\cdot m^2$

$= (12.5 + 6.25)\ T\cdot m^2$

$= 18.75\ T\cdot m^2$

Final



$\Phi_{B,f} = NBA_f$, with $A_f = 0$

$\therefore \Phi_{B,f} = 0$

$$\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \left|\frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t}\right| = \frac{18.75\ \text{Tm}^2}{0.1\ \text{s}}$$

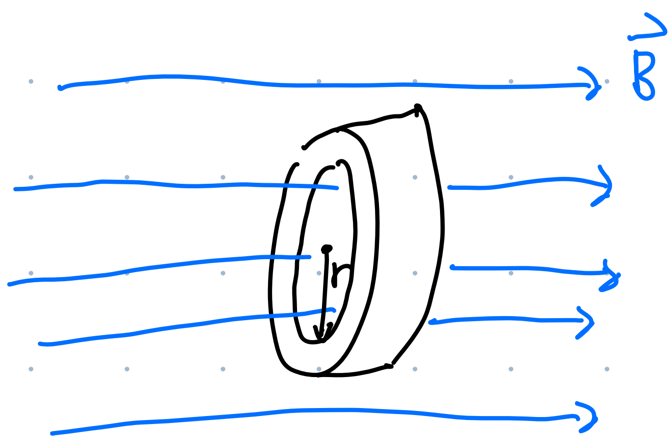
$$= 187.5\ \frac{\text{Tm}^2}{\text{s}}$$

$$\boxed{\mathcal{E} = 187.5\ \text{V}}$$

5: (a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field.

(a) Initially, the ring is in zero field. $\therefore \Phi_{B,i} = 0$

Later, the ring is in a non-zero magnetic field



$\Phi_{B,f} = BA$
$= B\pi r^2$

$\therefore \Delta\Phi_B = \Phi_{B,f} - \Phi_{B,i} = B\pi r^2$

$r = \frac{d}{2} = 1.10\ cm = 1.10\times10^{-2}\ m$

$$\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{B\pi r^2}{\Delta t} = \frac{(2T)\pi(0.0110\ m)^2}{0.250\ s}$$

$= \boxed{3.04\ mV}$ (small)

6: Referring to the situation in the previous problem: (a) What current is induced in the ring if its resistance is 0.0100 Ω? (b) What average power is dissipated? (c) What magnetic field is induced at the center of the ring? (d) What is the direction of the induced magnetic field relative to the MRI's field?

(a) $I = \frac{\varepsilon}{R} = \frac{3.04\text{ mV}}{10\text{ m}\Omega} = \boxed{0.304\text{ A}}$

(b) $P = I^2R = 9.2\times10^{-4}\text{ W} \simeq \boxed{0.92\text{ mW}}$ small.

(c) The magnetic field at the centre of a current loop is given by $B = \frac{\mu_0 I}{2r}$

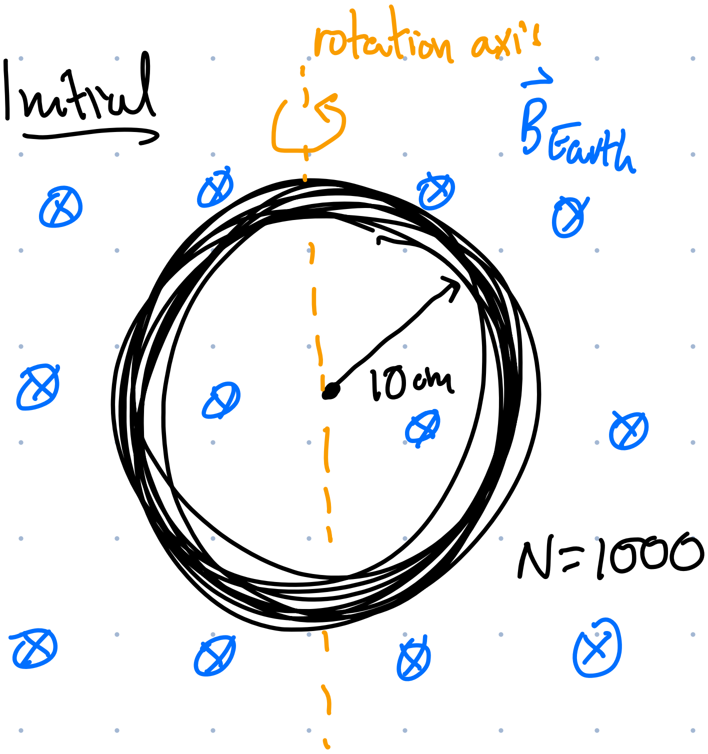
$\therefore B = 1.74\times10^{-5}\text{ T}$

$= \boxed{17.4\ \mu\text{T}}$

(d) Since Φ_B increased through the ring, $\vec{B}_{ind}$ acts so as to oppose this increase in the flux.
$\therefore \vec{B}_{ind}$ is antiparallel to $\vec{B}$ from the MRI machine.

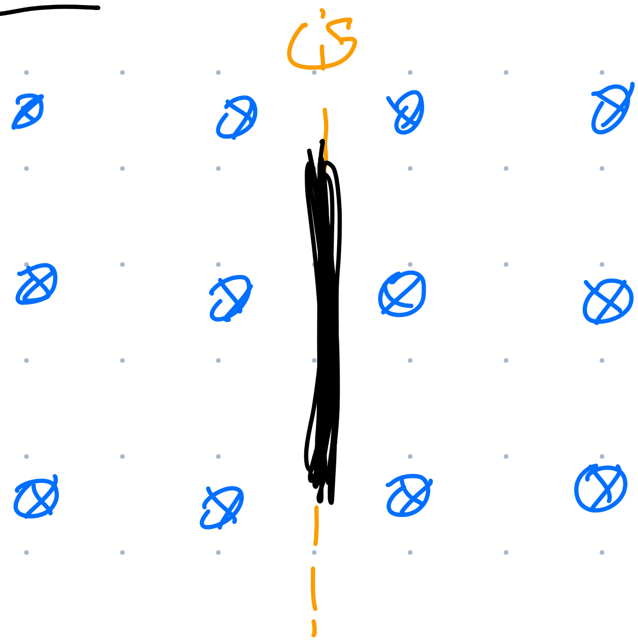
7: An emf is induced by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's 5.00×10^{-5} T magnetic field. What average emf is induced, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

Initial



$$\Phi_{B,i} = NBA$$
$$= NB_{Earth}\pi r^2$$

Final



$$\Phi_{B,f} = 0$$

$$|\Delta\Phi_B| = |\Phi_{B,f} - \Phi_{B,i}| = NB_{Earth}\pi r^2$$

$$\mathcal{E} = \left| \frac{\Delta \Phi_B}{\Delta t} \right| = \frac{N B_{Earth} \pi r^2}{\Delta t} = \frac{10^3 (5 \times 10^{-5} \, T) \pi (0.10 \, m)^2}{0.01 \, s}$$

$$= \frac{5 \times 10^{-4} \pi}{10^{-2}} \, V$$

$$= 5 \times 10^{-2} \pi \, V$$

$$= \boxed{0.157 \, V}$$

8: A 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.) Find the magnetic field strength needed to induce an average emf of 10,000 V.

This problem is essentially identical to the previous one (#7).

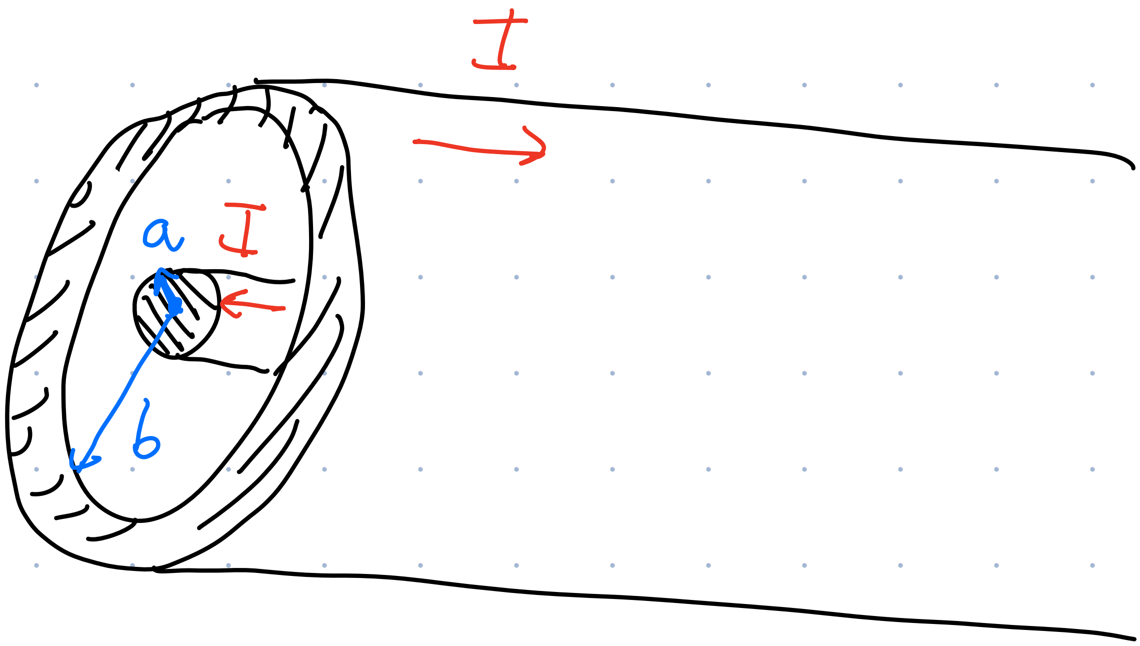
$$\mathcal{E} = \frac{NB\pi r^2}{\Delta t} \qquad \text{want to find } B.$$

$$B = \frac{\mathcal{E}\Delta t}{N\pi r^2} = \frac{(10^4\,V)(4.17\times10^{-3}\,s)}{(5\times10^2)\pi\left(\frac{1}{4}\,m\right)^2}$$

$$= \frac{(16)(4.17\times10^{-1})}{5\pi}\,T$$

$$\therefore B \approx 0.425\,T$$

Ampère's Law Example: Coaxial Cable.



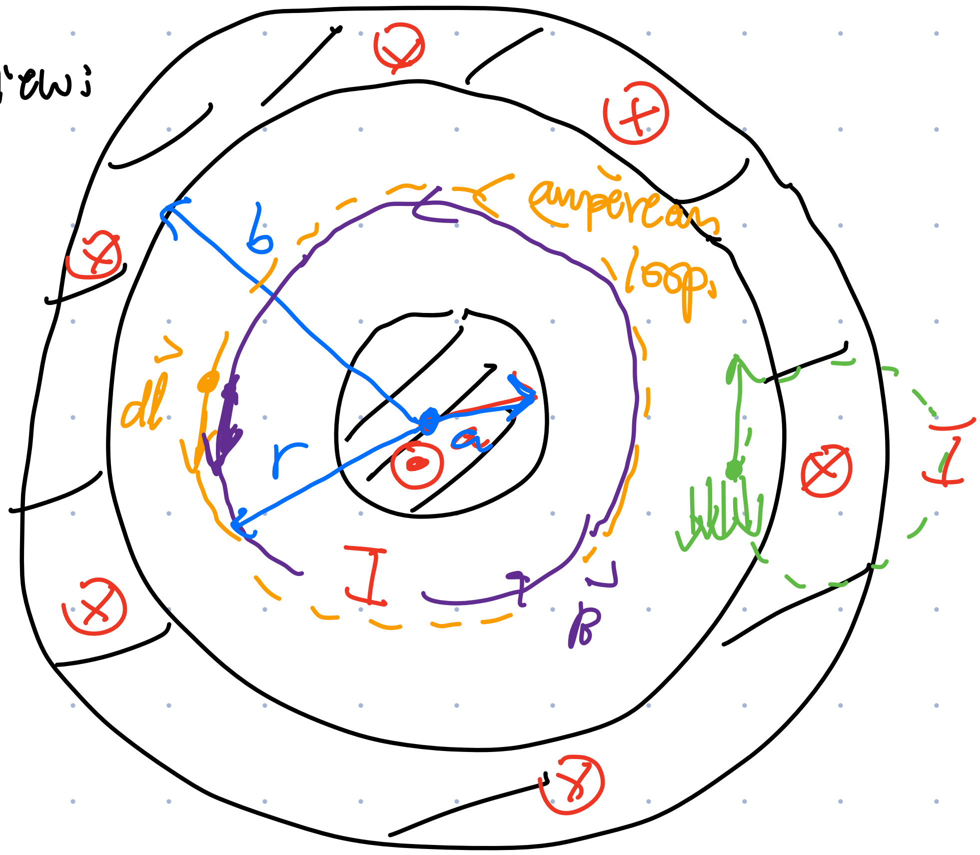
"Outer conducting" cylindrical shell of radius b carrying current I right. The centre conductor is inside outer conductor & has radius a & current I to the left.

(a) Find $\vec{B}$ at a position $a < r < b$.
(Point between the inner & outer conductors).

Ampère's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

Select an Ampèrean loop / integration path.

Side view:



$\vec{B}\cdot\vec{dl} = B\,dl$ since $\vec{B} \parallel \vec{dl}$ everywhere.

Expect B is const. everywhere on Ampèrean loop.

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl = B\underbrace{\int dl}$$

loop circumference $= 2\pi r$

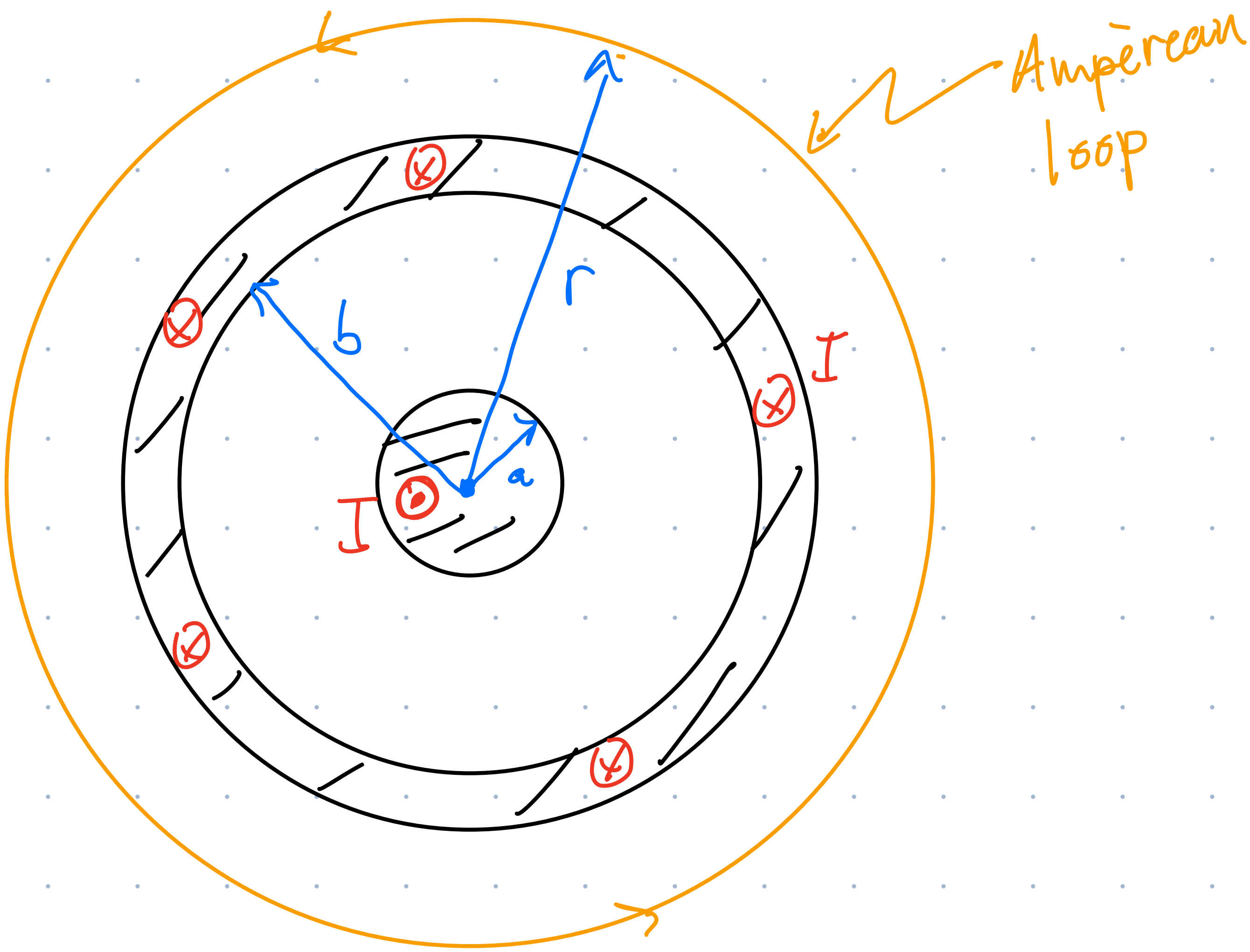
① $\oint \vec{B}\cdot\vec{dl} = B(2\pi r)$

② $\mu_0 I_{encl}$ only the current on inner conductor passes through loop $\therefore I_{encl} = I$

① = ② $\Rightarrow$ $B(2\pi r) = \mu_0 I$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$

(b) Find the magnetic field at a position $r > b$ (outside the outer conductor).



Like in (a) $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$

inner conductor

However, in this case, $I_{encl} = I - I = 0$

outer conductor

$\therefore \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$\Downarrow$

$B(2\pi r) = 0$

$\therefore$ the only possibility is $\boxed{B = 0}$