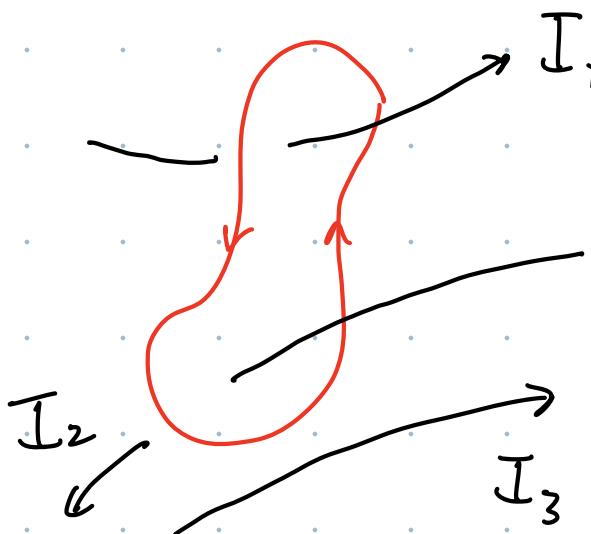


- Next PrairieLearn HW due Tues. Apr. 8
- Labs are done
- Last tutorial this week
- See course website for final exam details  
(including formula sheet)
- If participating in the hands-on bonus project, send me a link to your YouTube video by 23:59 on April 7.
- Complete the end-of-term survey by 23:59 on April 8 for 0.5 marks towards your final grade.  
A link to the survey has been provided in Canvas.

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Last Time: Ampère's Law  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$

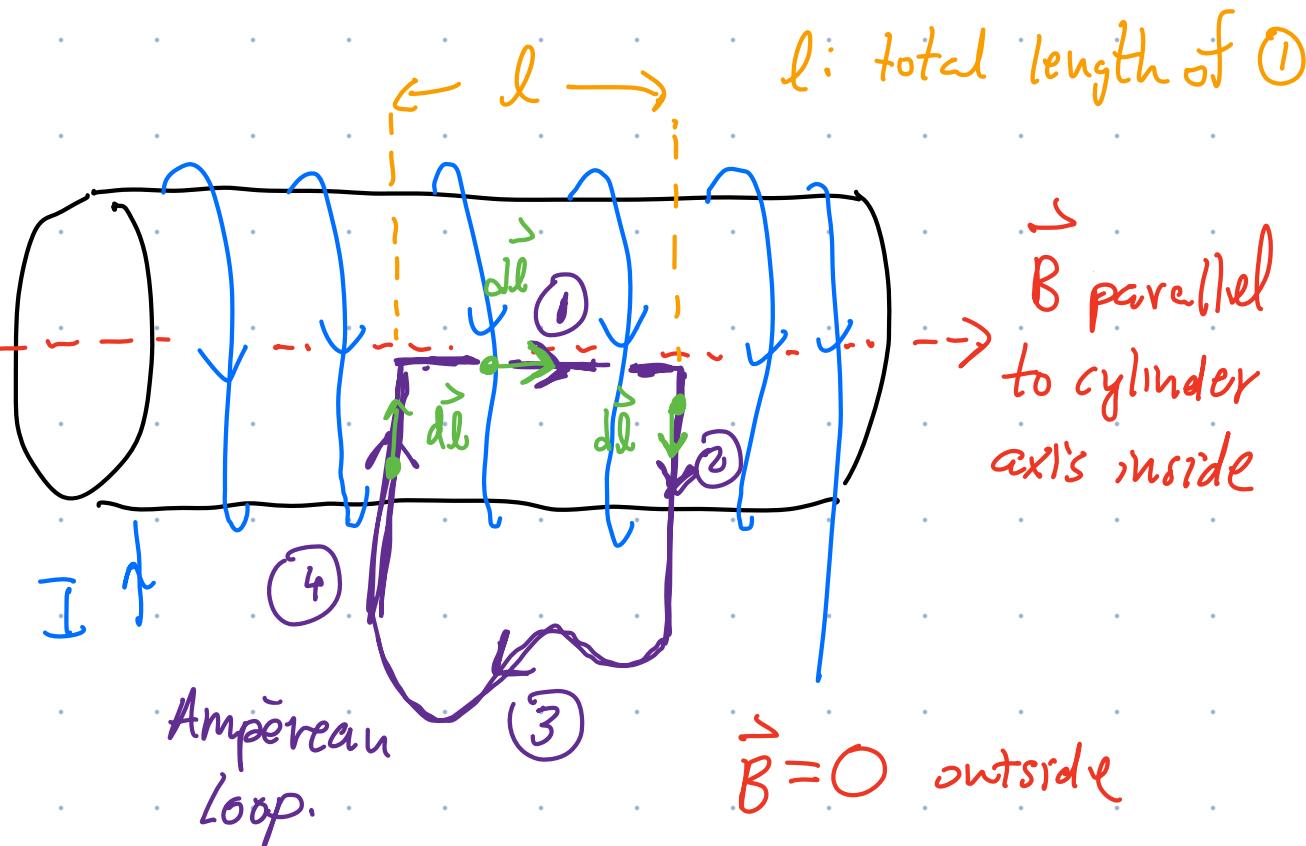


Valid for any integration path { any arrangement of current(s)

$$I_{\text{enc}} = I_1 - I_2$$

Eg. Use Ampère's Law to find  $\vec{B}$  due to a solenoid.

N loops of wire pass through Ampèrean loop.  
 → total  $I_{\text{end}} = NI$



To evaluate  $\oint \vec{B} \cdot d\vec{l}$ , want integration path to be  $\parallel$  or  $\perp$  to  $\vec{B}$ . ∵ inside solenoid, path should vertical and/or horizontal.

$$\therefore \oint \vec{B} \cdot d\vec{l} = \int_1 \vec{B} \cdot d\vec{l} + \int_2 \vec{B} \cdot d\vec{l} + \int_3 \vec{B} \cdot d\vec{l} + \int_4 \vec{B} \cdot d\vec{l}$$

either  $\vec{B} = 0$   
 or  $d\vec{l} \perp \vec{B}$   
 $\therefore \vec{B} \cdot d\vec{l} = 0$

b/c  $\vec{B} = 0$   
 outside solenoid.  
 $\therefore \vec{B} \cdot d\vec{l} = 0$

either  $\vec{B} = 0$   
 or  $d\vec{l} \perp \vec{B}$   
 $\therefore \vec{B} \cdot d\vec{l} = 0$

$$\oint \vec{B} \cdot d\vec{l} = \int_{\textcircled{1}} \vec{B} \cdot d\vec{l}$$

only part ① of loop  
makes a non-zero contribution.

For ①  $\vec{B} \parallel d\vec{l} \therefore \vec{B} \cdot d\vec{l} = B d\vec{l}$

$\vec{B}$  is const. everywhere along ①.

$$\int_{\textcircled{1}} \vec{B} \cdot d\vec{l} = \int_{\textcircled{1}} B d\vec{l} = B \int_{\textcircled{1}} d\vec{l} = Bl$$


$$\therefore \oint \vec{B} \cdot d\vec{l} = Bl = \mu_0 I_{\text{enc}}$$

$I_{\text{enc}}$  is the net current passing through  
Ampèrean loop.  $\rightarrow I_{\text{enc}} = NI$

$$\therefore Bl = \mu_0 NI \quad \text{or}$$

$$B = \mu_0 \left( \frac{N}{l} \right) I$$

Strength of the magnetic  
field inside a solenoid.

Note that  $B = \mu_0 \left( \frac{N}{l} \right) I$  anywhere inside the solenoid. In other words, the magnetic field inside a solenoid is uniform/constant.

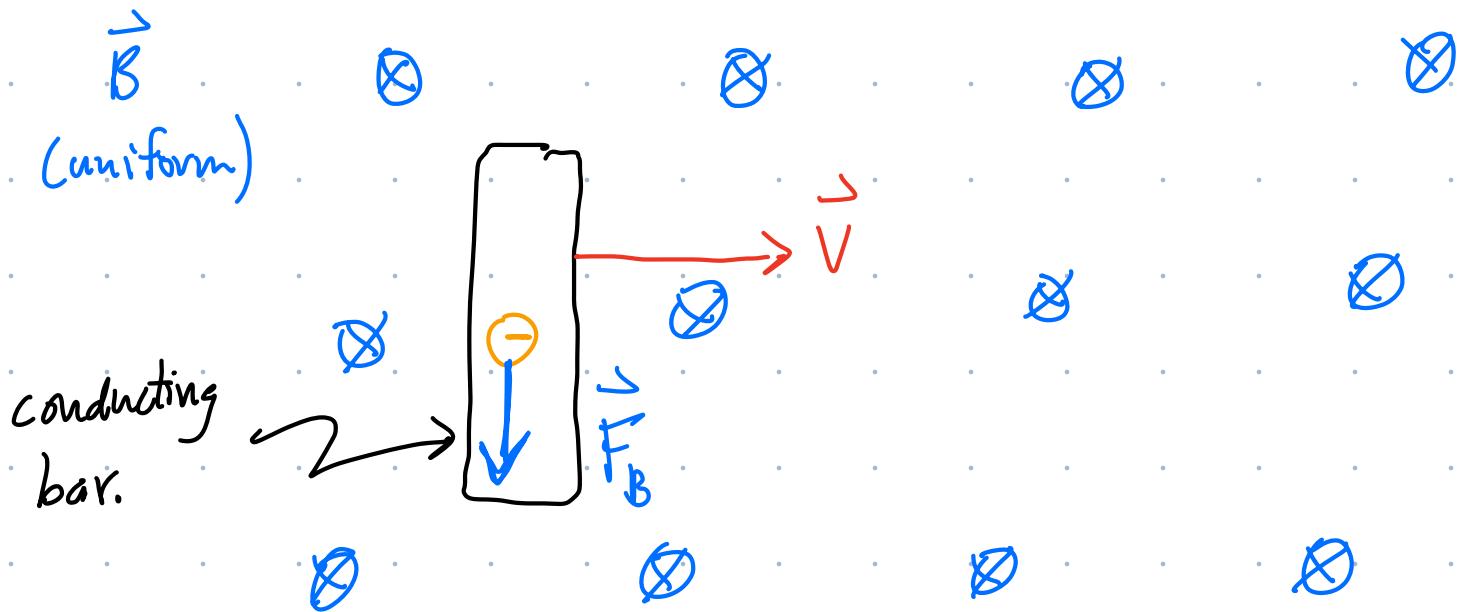
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## Chapter 13 Sec. 3 of OSUPV2

Motional Emf  $\rightarrow$  Induced Voltages

"electromotive force"

$\rightarrow$  Want to use "induced" voltages to power devices.



Pull a conducting bar perpendicularly through a uniform magnetic field w/ speed  $v$ .

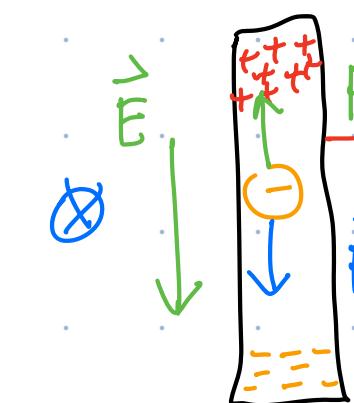
The mobile electrons in the conductor also move to the right w/ speed  $v$ .  $\therefore$  these  $e^-$  experience a magnetic force:

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

B/c  $e^-$  are neg., the force they experience is downwards.

As we keep pulling the rod to the right, electrons pile up at the btm of the rod & leave behind excess pos. charge due to the atomic nuclei inside the solid rod.

This separation charge establishes an electric field inside the rod that pts. from the pos. to the neg. charge distns.



The electric field exerts a force  $\vec{F}_E = q\vec{E}$  that opposes  $\vec{F}_B$ .

In equilibrium, the electric & magnetic forces balance & the mobile  $e^-$  stop migrating to the btm of the rod.

Equil. condition :  $F_E = F_B$

$\Downarrow$

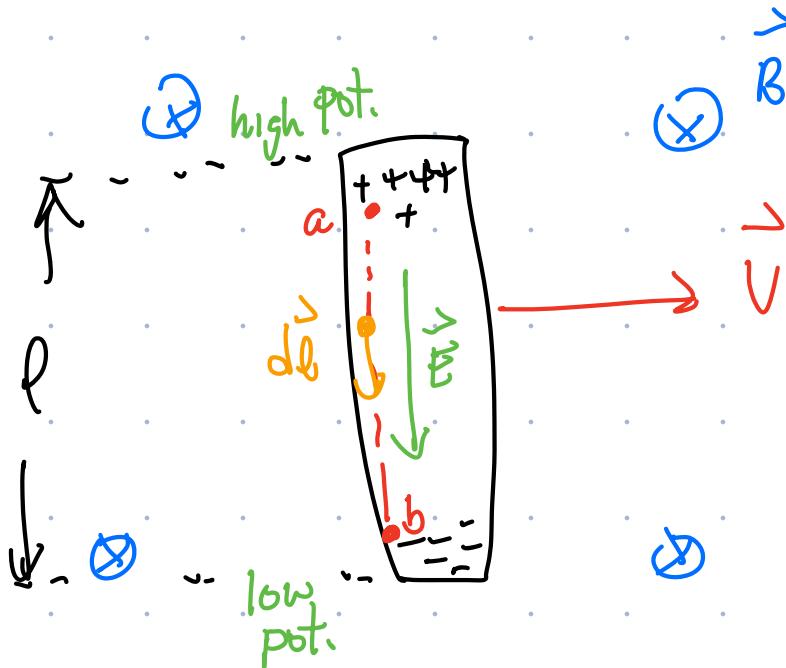
~~$qE = qvB$~~

$$\therefore V = \frac{E}{B}$$

similar to what we found for the velocity selector.

∴ Solve for  $E$ :

$$\vec{E} = VB \quad ①$$



To calc. the potential difference or voltage across the rod, use:

$$\Delta V = - \int \vec{E} \cdot d\vec{l}$$

Since  $\vec{E} \parallel d\vec{l}$ ,  $\vec{E} \cdot d\vec{l} = E dl$

Assume that  $\vec{E}$  is const inside the rod.

$$\therefore |\Delta V| = \int \vec{E} \cdot d\vec{l} = \int E dl = \bar{E} \int dl = El$$

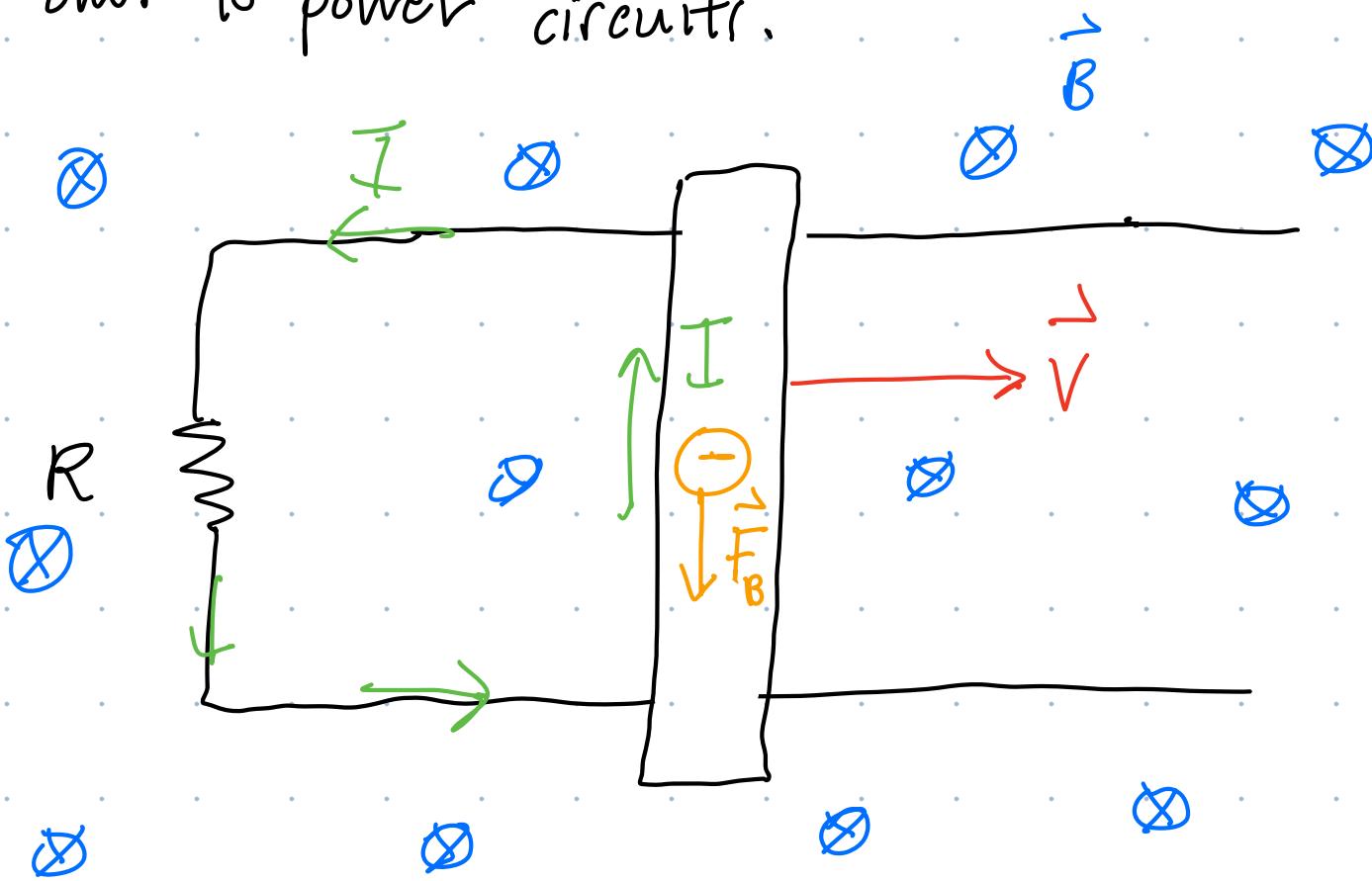
VB from ①

$$\therefore |\Delta V| = vBL$$

(2)

Motional emf (or induced voltage) created by pulling a conductor through a magnetic field.

We can use the voltage due to motional emf to power circuits.



Rod slides on a pair of conducting tracks/wires that are joined on one end by a resistor  $R$ .

$e^-$  flow CW around out circuit which corresponds to a CCW current.

$$I = \frac{\Delta V}{R} = \frac{VBl}{R}$$

(3)