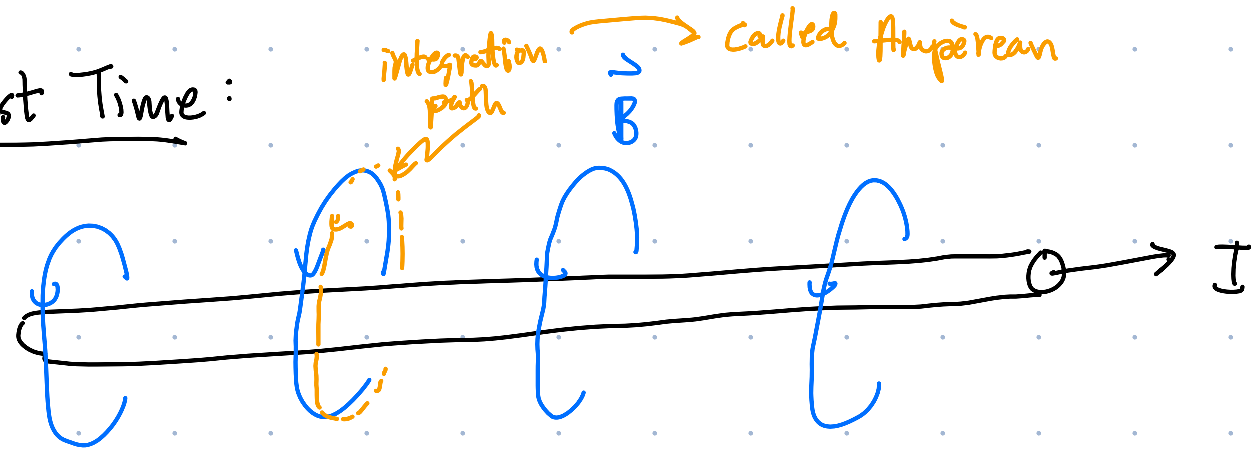
PHYS 121 March 28, 2025

- Next PrairieLearn HW due $\boxed{\text{today}}$
- Labs are done
- Last tutorial next week
- See course website for final exam details (including formula sheet)
- If participating in the hands-on bonus project, send me a link to your YouTube video by 23:59 on April 7.
- Complete the end-of-term survey by 23:59 on April 8 for 0.5 marks towards your final grade. A link to the survey has been provided in Canvas.

Last Time:



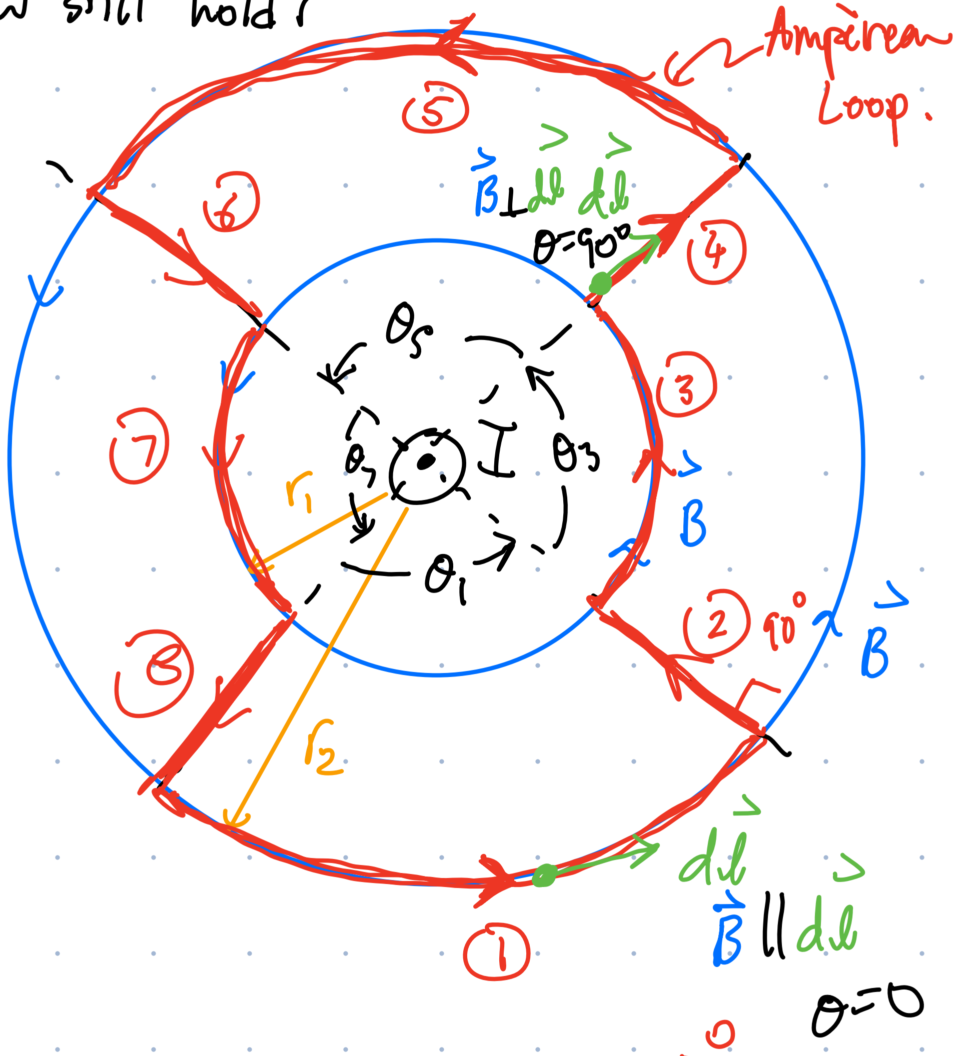
$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$$ Ampère's Law

current that passes through the integration path.

What about an integration path of arbitrary shape.
Does Ampère's Law still hold?

End view of a long straight current.

Can form any arbitrary path around I using small radial sections & circular arcs.



Evaluate $\oint \vec{B} \cdot d\vec{l} = \int_{①} \vec{B} \cdot d\vec{l} + \cancel{\int_{②} \vec{B} \cdot d\vec{l}}^{\,0} + \dots$

closed path

circular arc

radial

$+ \int_{⑦} \vec{B} \cdot d\vec{l} + \cancel{\int_{⑧} \vec{B} \cdot d\vec{l}}^{\,0}$

Consider all the sections that are radial. {2, 4, 6, 8}

$$\vec{B} \cdot d\vec{l} = B\, dl\, \cancel{\cos 90^\circ}^{\,0} \qquad \vec{B} \perp d\vec{l}$$

All radial section evaluate to zero.

Next, consider circular arc paths {1, 3, 5, 7}

In these cases, $\vec{B} \parallel d\vec{l}$ s.t. $\theta = 0$

$$\therefore \vec{B} \cdot d\vec{l} = B\, dl\, \underbrace{\cos 0}_{1} = B\, dl$$

$\vec{B}$ is const. in magn. along any circular arc path.

$$\int_{1} \vec{B} \cdot d\vec{l} = \int_{1} B\, dl = B \underbrace{\int_{1} dl}_{\text{length of arc } 1}$$

$r_2 \theta_1$

$$\int_{①} \vec{B}\cdot d\vec{l} = B(r_2\theta_1)$$

For a long straight wire, know $B = \dfrac{\mu_0 I}{2\pi r}$

The strength of the magnetic field along path ① is $B = \dfrac{\mu_0 I}{2\pi r_2}$

$$\int_{①} \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi \cancel{r_2}}(\cancel{r_2}\theta_1) = \mu_0 I\left(\frac{\theta_1}{2\pi}\right)$$

Likewise, if we repeat this calc for ③, ⑤, ⑦, get similar results.

$$\int_{③} \vec{B}\cdot d\vec{l} = \mu_0 I\left(\frac{\theta_3}{2\pi}\right) \qquad \int_{⑤} \vec{B}\cdot d\vec{l} = \mu_0 I\left(\frac{\theta_5}{2\pi}\right)$$

$$\int_{⑦} \vec{B}\cdot d\vec{l} = \mu_0 I\left(\frac{\theta_7}{2\pi}\right)$$

Finally: $\oint \vec{B}\cdot d\vec{l} = \int_{①} \vec{B}\cdot d\vec{l} + \dots + \int_{⑧} \vec{B}\cdot d\vec{l}$

$$= \mu_0 I \frac{\theta_1}{2\pi} + \mu_0 I \frac{\theta_3}{2\pi} + \mu_0 I \frac{\theta_5}{2\pi}$$

$$+ \mu_0 I \frac{\theta_7}{2\pi}$$

$$= \mu_0 I \left[\frac{\overbrace{\theta_1 + \theta_3 + \theta_5 + \theta_7}^{2\pi}}{2\pi} \right]$$

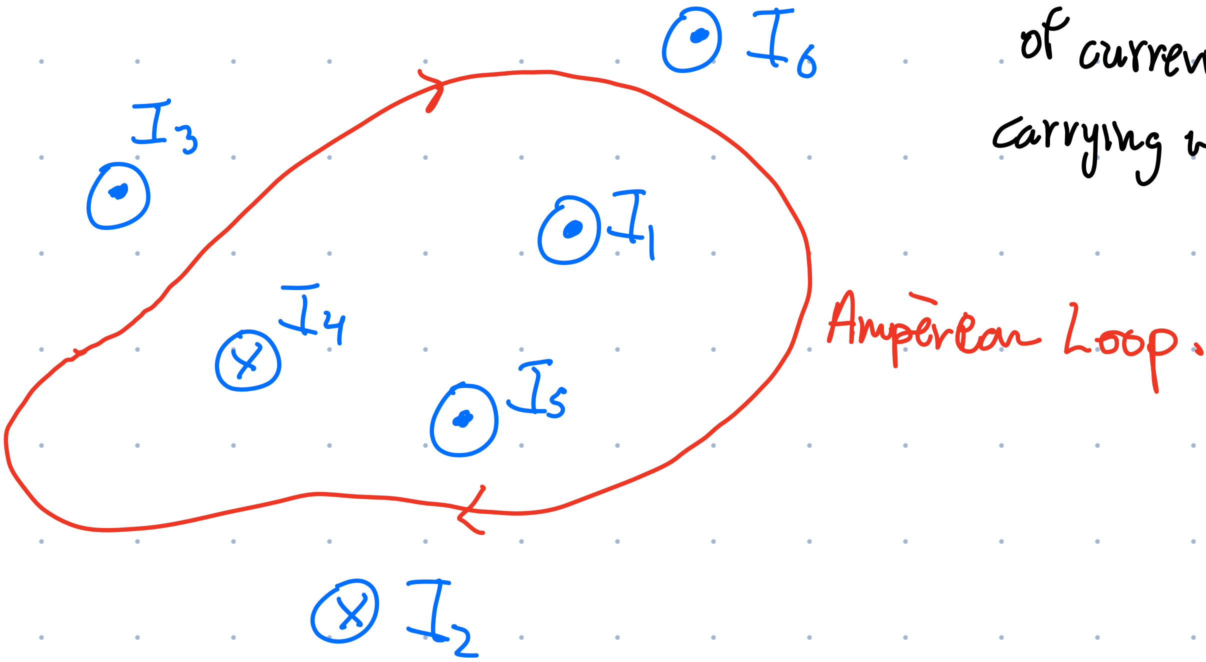
$$\boxed{\therefore \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}}$$

valid for <u>any</u> path around current I.

Ampère's Law.

Ampère's Law Example

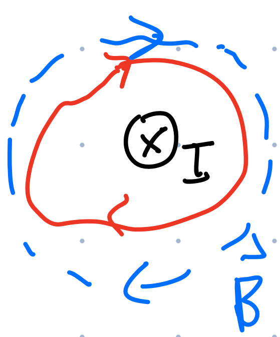
Collection of current-carrying wires.



What is the result of $\oint \vec{B} \cdot d\vec{l}$ for the red integration path / Amperean loop?

Know that for one current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

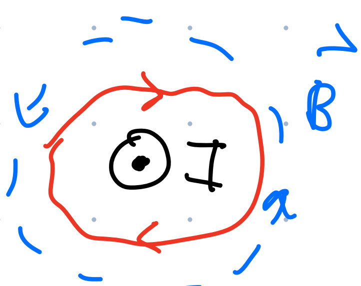


$\vec{B}$ & $d\vec{l}$ $\approx$ parallel

If we reverse the current dir'n

$$\oint \vec{B} \cdot d\vec{l} = -\mu_0 I$$

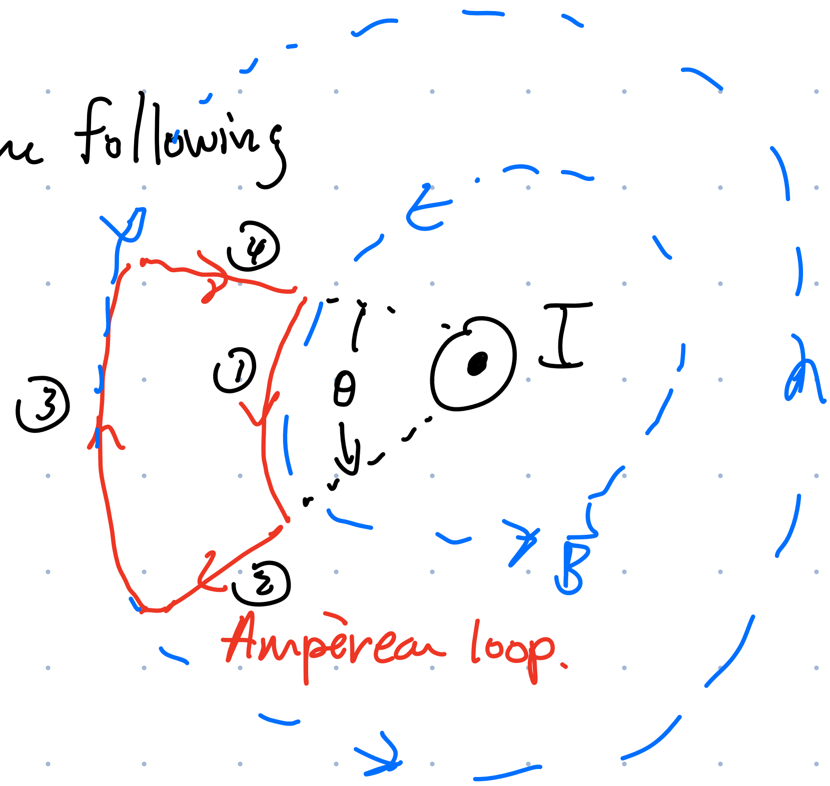
$\vec{B}$ & $d\vec{l}$ $\approx$ antiparallel



Use RHR to determine if $\oint \vec{B} \cdot d\vec{l}$ contribution from current I is pos. or neg.

1. Put thumb in dir'n of I.
2. Fingers give dir'n of integration that contributes pos. values to $\oint \vec{B} \cdot d\vec{l}$

Another case is the following



Radial pieces (2) & (4) contribute nothing to $\oint \vec{B} \cdot d\vec{l}$

(1) contributes $+\mu_0 I \frac{\theta}{2\pi}$

(3) contributes $-\mu_0 I \frac{\theta}{2\pi}$

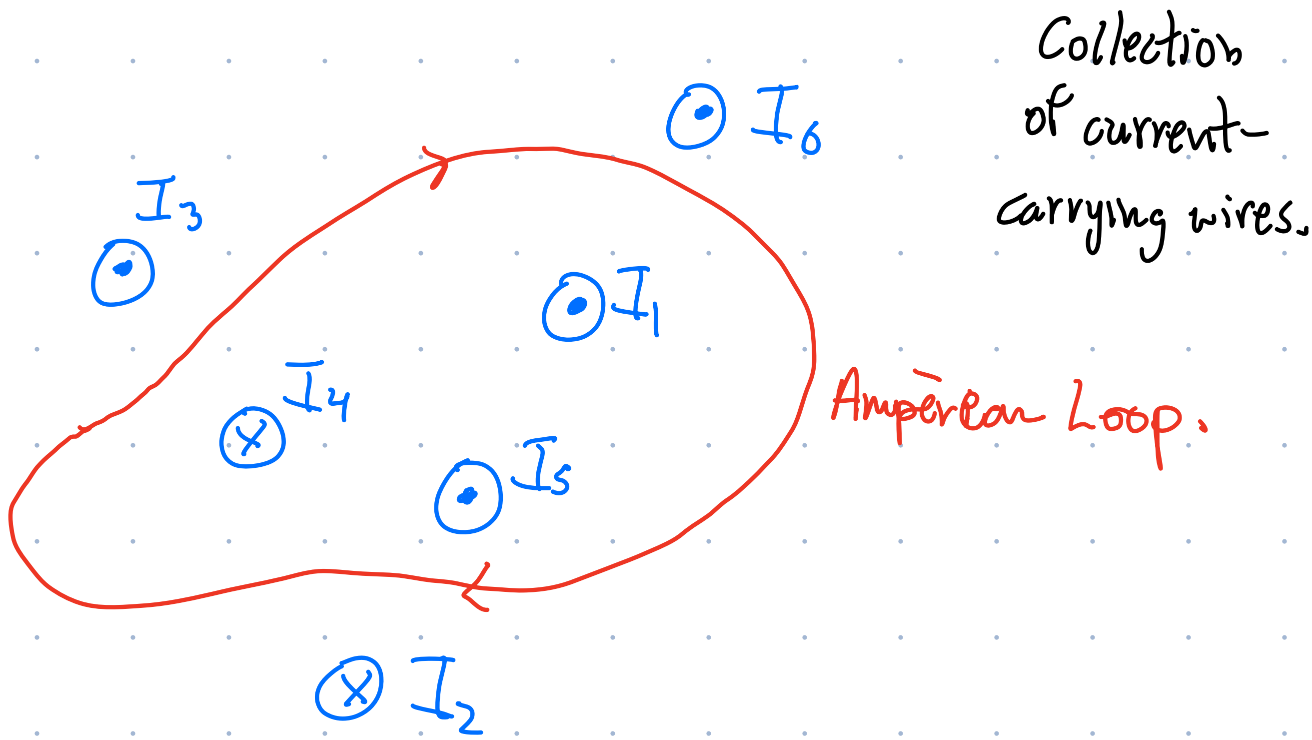
$$\therefore \oint \vec{B} \cdot d\vec{l} = \oint_{(1)} \vec{B} \cdot d\vec{l} + \ldots + \int_{(4)} \vec{B} \cdot d\vec{l} = \underline{\underline{0}}$$

∴ This result is consistent w/ Ampère's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$, since in this case, the Ampèrean loop encloses no current.

$$\therefore I_{encl} = 0.$$

Return to orig. problem:



I_2, I_3, I_6 do not pass through loop.

$\therefore$ They do not contribute to I_{encl}.

I_1, I_4, I_5 do contribute to I_{encl}. But, some contribute pos. & others neg.

By RHR:

- Curl fingers in dir'n of integration path/Ampērean loop
- Thumb gives dir'n for currents that make pos contributions to I_{encl}.

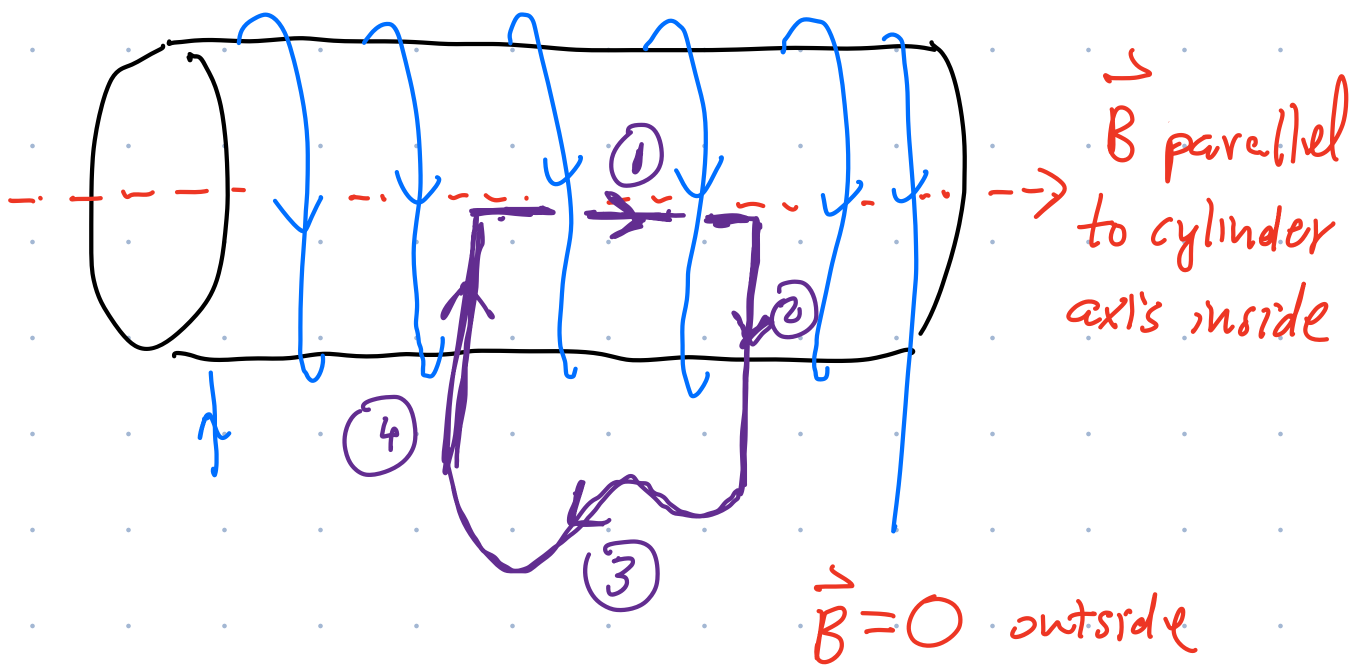
In the example above, currents into the screen (I_4) make pos. contributions.

$$\therefore I_{encl} = I_4 - I_1 - I_5$$

$$\boxed{\therefore \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} = \mu_0 (I_4 - I_1 - I_5)}$$

A more sophisticated application of Ampère's Law.

Use Ampère's Law to find $\vec{B}$ due to a solenoid.



To evaluate $\oint \vec{B}\cdot d\vec{l}$, want integration path to be $\parallel$ or $\perp$ to $\vec{B}$. $\therefore$ inside solenoid, path should vertical and/or horizontal.

Next time, we will evaluate

$$\oint \vec{B}\cdot d\vec{l} = \int_{①} \vec{B}\cdot d\vec{l} + \int_{②} \vec{B}\cdot d\vec{l} + \int_{③} \vec{B}\cdot d\vec{l} + \int_{④} \vec{B}\cdot d\vec{l}$$

to find $\vec{B}$ due to the solenoid.