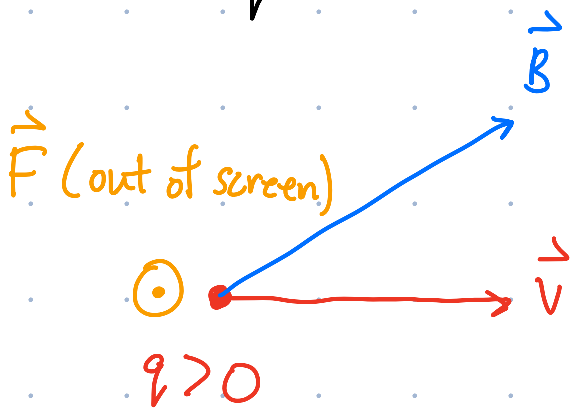
PHYS 121 March 26, 2025

- Next PrairieLearn HW due Mar. 28
- Complete Pre-Lab #8 before the start of your lab.
- See course website for final exam details (including formula sheet)

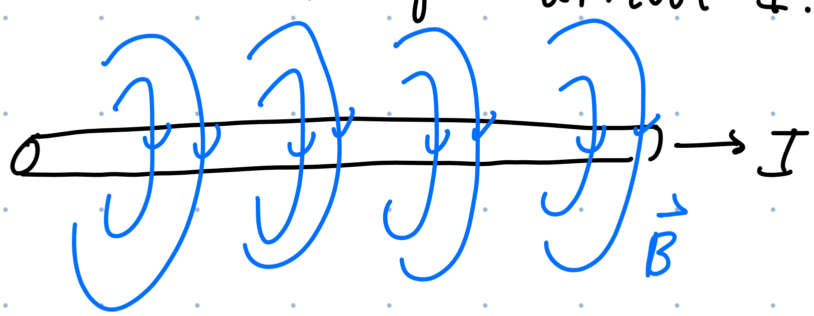
<u>Last week</u>: Force on a charge moving through a magnetic field.

$$\vec{F} = q\vec{v} \times \vec{B}$$



Magnetic field due to infinite straight current I.

$$B = \frac{\mu_0 I}{2\pi r}$$

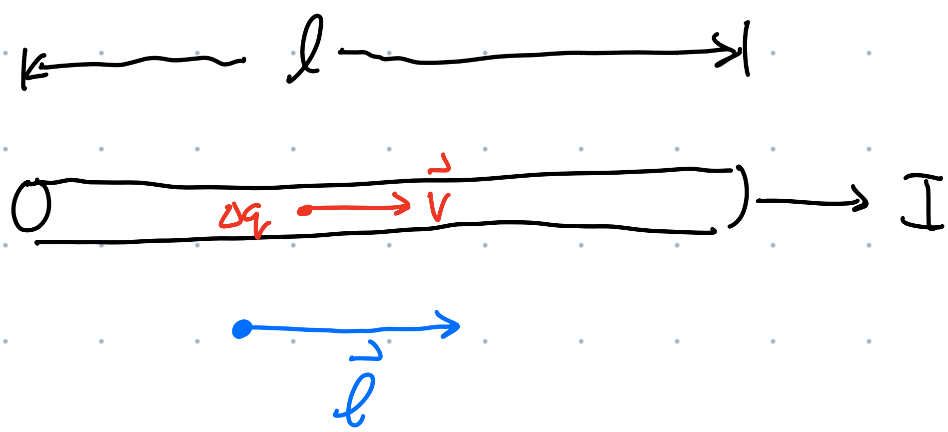


Magnetic Force on a Current.

Know for a single pt. charge in $\vec{B}$, the force is:

$$\vec{F} = q\vec{v} \times \vec{B} \quad ①.$$

What is the force on a current (collection of moving charges) in a magnetic field?



The time Δt for a collection of charges Δq to move the length of the wire is:

$$\Delta t = \frac{l}{v}$$

v ← speed of charges.

Current $I = \dfrac{\Delta q}{\Delta t} = \dfrac{\Delta q}{\left(\frac{l}{v}\right)} = \dfrac{v \Delta q}{l}$

or: $\Delta q \, v = I l$

define $\vec{l}$ s.t. its dir'n is given by the dir'n of the current I.

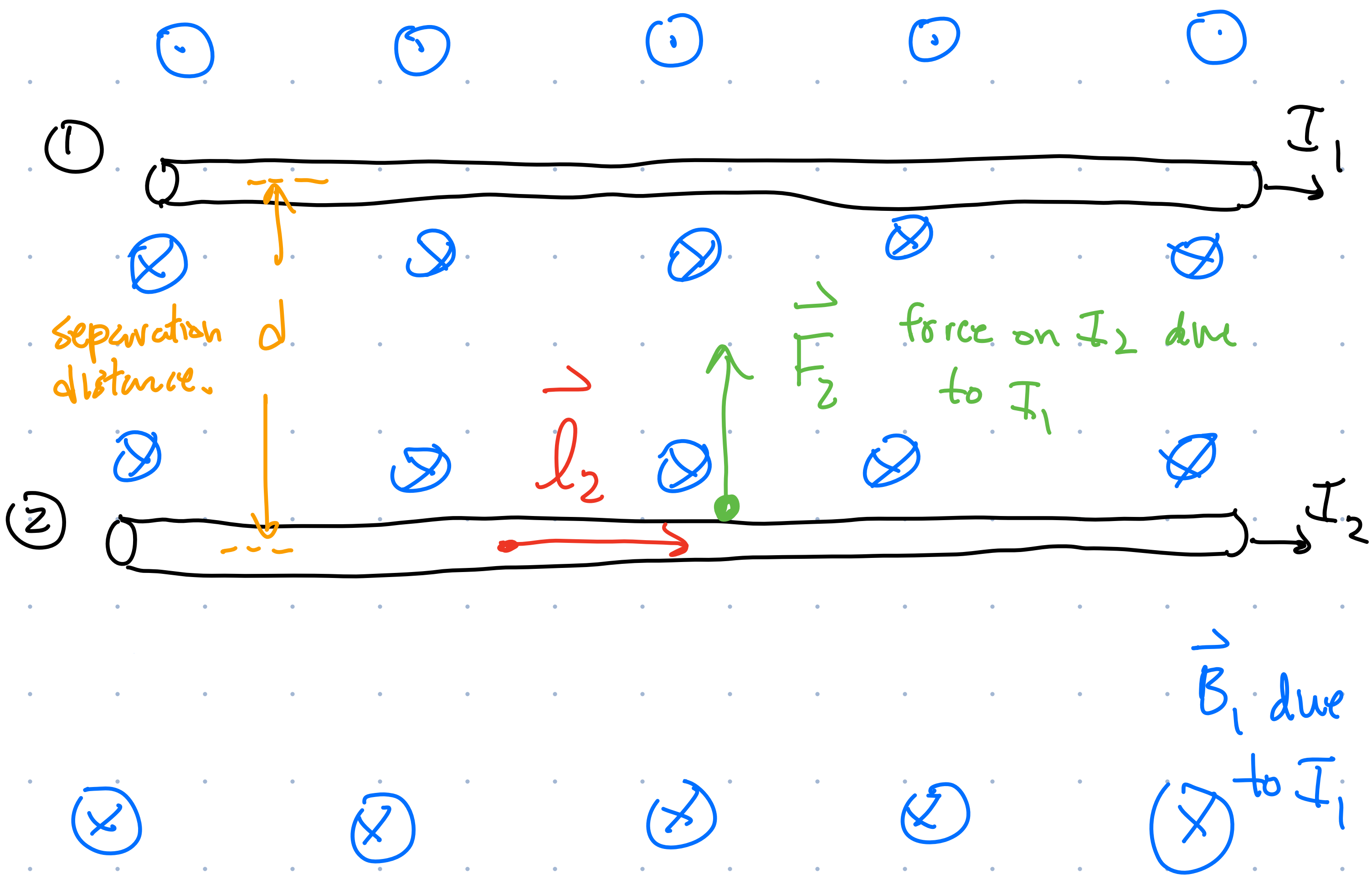
$$\Delta q \, \vec{v} = \underbrace{I \vec{l}}_{\text{sub into (1)}}$$

$$\vec{F} = \underbrace{q\vec{v}}\times\vec{B} \quad (1)$$

$$\boxed{\therefore \ \vec{F} = I\vec{l}\times\vec{B}}$$

Force on a current in magnetic field $\vec{B}$.

Example: Force between Parallel Currents.



What is the force on I_2 due to magnetic field of I_1?

I_1 make a magnetic field $\vec{B}_1$ that is <u>into</u> the screen at the position of I_2.

Force on I_2 is given by: $\vec{F_2} = I_2 \vec{l}_2 \times \vec{B}_1$

$\vec{B}$ due to I_1

By RHR, $\vec{l}_2 \times \vec{B}_1$ is towards top of the screen.
By Newton's 3rd Law, force on I_1 due to the magnetic field of I_2 is down.

$\Longrightarrow$ $\boxed{\text{Parallel currents attract.}}$

Using the same logic, can deduce that <u>antiparallel currents repel</u>.

Let's work out the magnitude of $\vec{F_2} = I_2 \vec{l}_2 \times \vec{B}_1$

$\vec{l}_2 \perp \vec{B}_1 \Rightarrow \theta = 90^\circ$

$$|\vec{F}_2| = I_2 |\vec{l}_2 \times \vec{B}_1| = I_2 l_2 B_1 \cancelto{1}{\sin 90^\circ}$$

$$|\vec{F}_2| = I_2 l B_1$$

Since I_1 is a long, straight current, know

$$B_1 = \boxed{\frac{\mu_0 I_1}{2\pi d}}$$

d ← dist. from I_1 to I_2

$$\therefore |\vec{F}_2| = I_2 l \left(\frac{\mu_0 I_1}{2\pi d}\right)$$

$$\therefore \boxed{|\vec{F}_1| = |\vec{F}_2| = \frac{\mu_0 I_1 I_2 l}{2\pi d}}$$

attractive if I_1 & I_2 parallel

repulsive if I_1 & I_2 antiparallel.

Ampère's Law

- Gives us a way to calc. $\vec{B}$ due to certain symmetric current distributions.
- Analogue of Gauss's Law, but for magnetic fields.

We will find that:

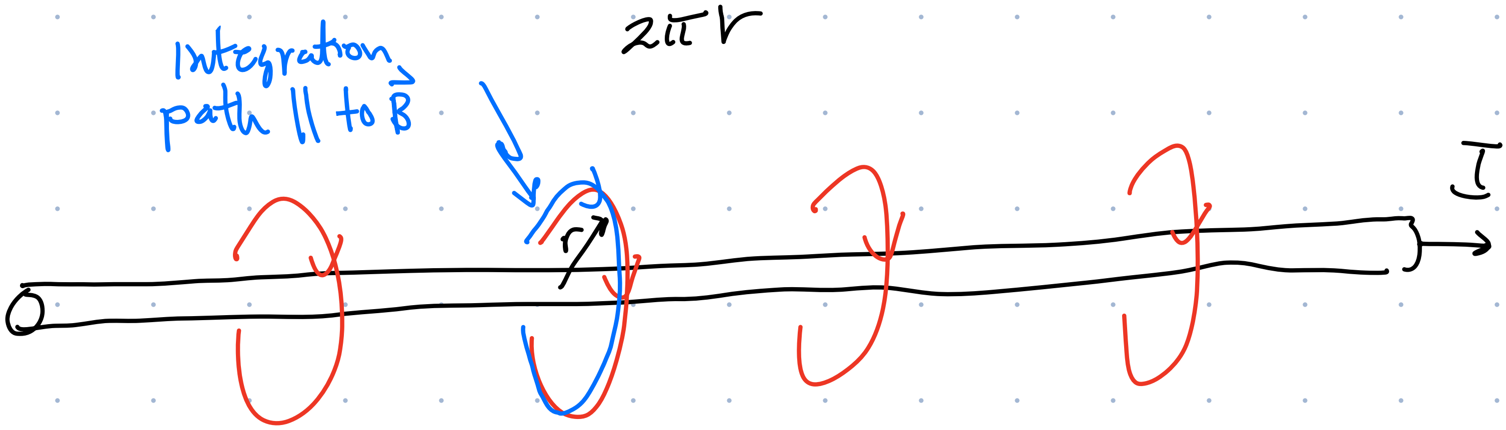
$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$$

Ampère's Law

integrate around a closed path/loop.

Recall that $\vec{B}$ due to a long, straight current is given by

$$B = \frac{\mu_0 I}{2\pi r}$$



Let's evaluate $\oint \vec{B}\cdot d\vec{l}$ for this long straight current. Integrate around a loop that is always parallel to $\vec{B}$.

$$\theta = 0$$

In this case, $\vec{B}\cdot d\vec{l} = B\,dl\,\underbrace{\cos\theta}_{1}$

$$= B\,dl$$

$$\oint \vec{B}\cdot d\vec{l} = \int B\,dl$$

For our chose path, dist. from I to the loop is alway r. $\Rightarrow$ B is const. on our path.

$$= B\underbrace{\int dl}_{\text{circumference of path } (2\pi r)} = 2\pi r B$$

$$\therefore \oint \vec{B}\cdot d\vec{l} = 2\pi r B$$

Already know from Biot-Savart law that $B = \dfrac{\mu_0 I}{2\pi r}$ for a long straight current

$$\therefore \oint \vec{B}\cdot d\vec{l} = \cancel{2\pi r}\left(\frac{\mu_0 I}{\cancel{2\pi r}}\right)$$

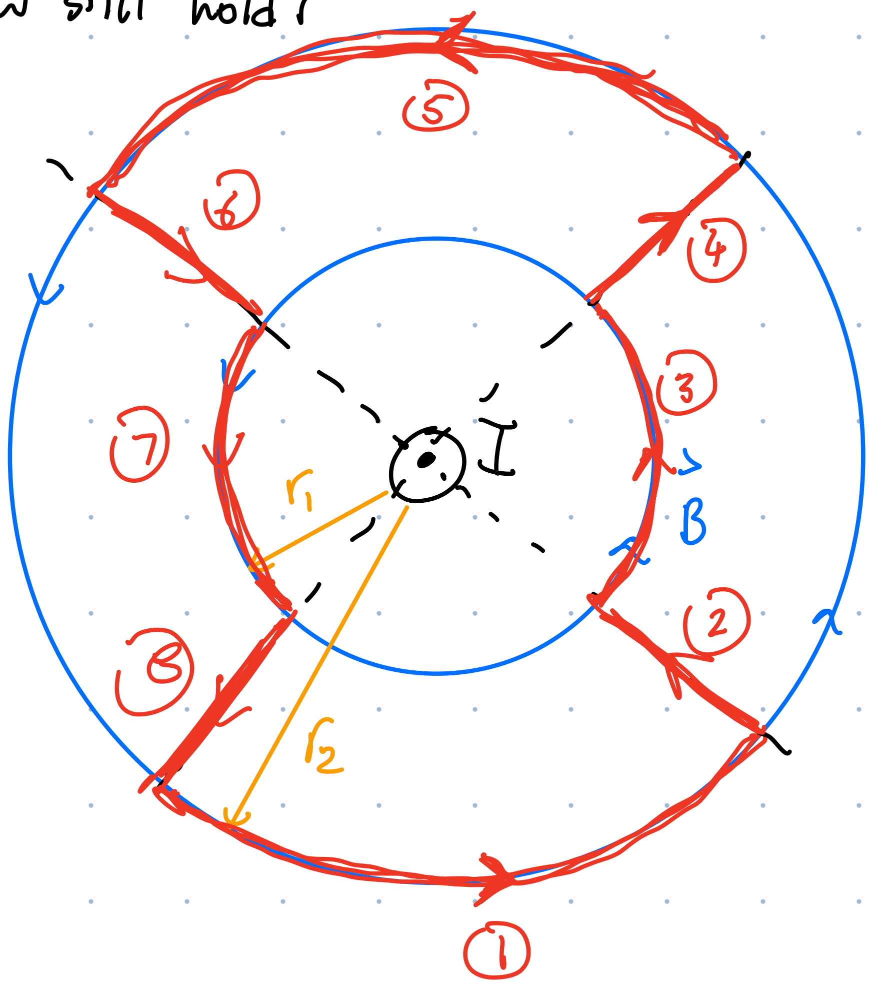
$$\boxed{\therefore \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{end}}$$

we've recovered Ampère's Law.

current that passes through the integration path. Usually labelled as I_{end}.

What about an integration path of arbitrary shape? Does Ampère's Law still hold?

End view of a long straight current.



Can form any arbitrary path around I using small radial sections & circular arcs.