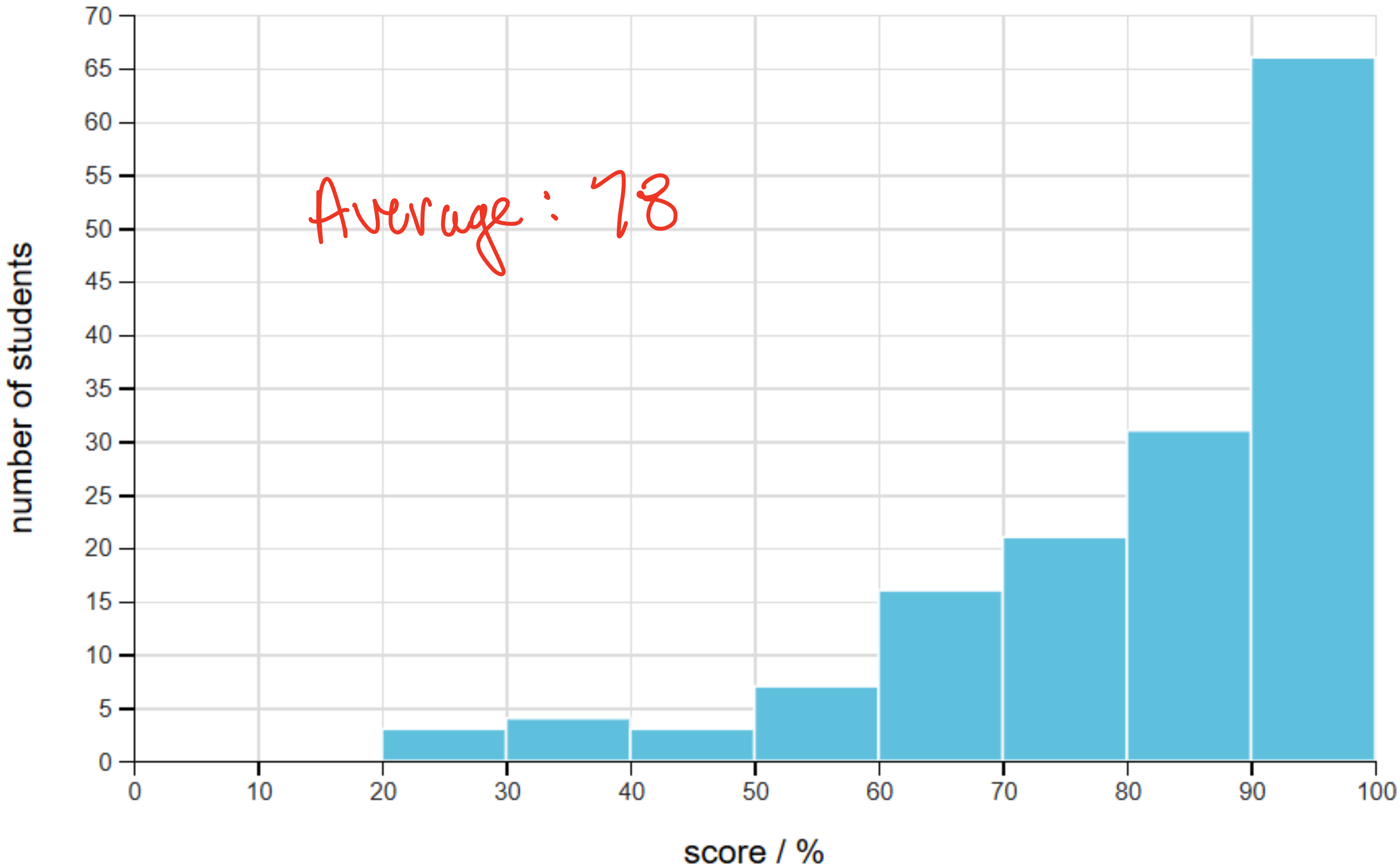
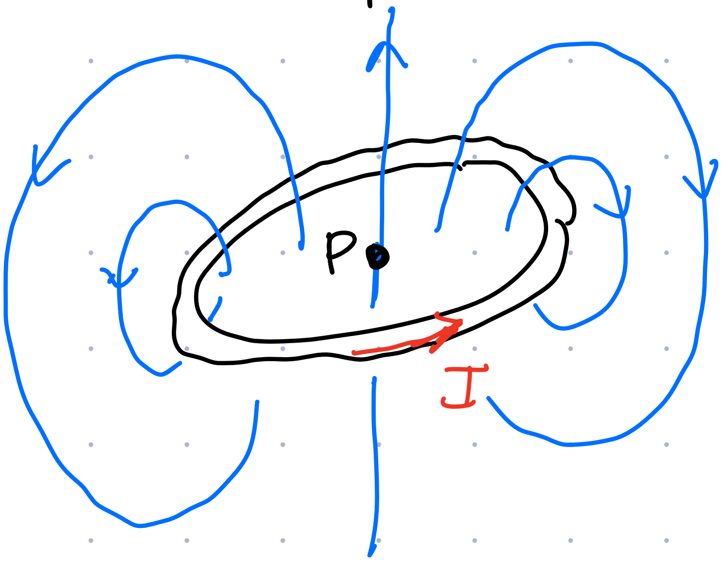
PHYS 121

March 21, 2025

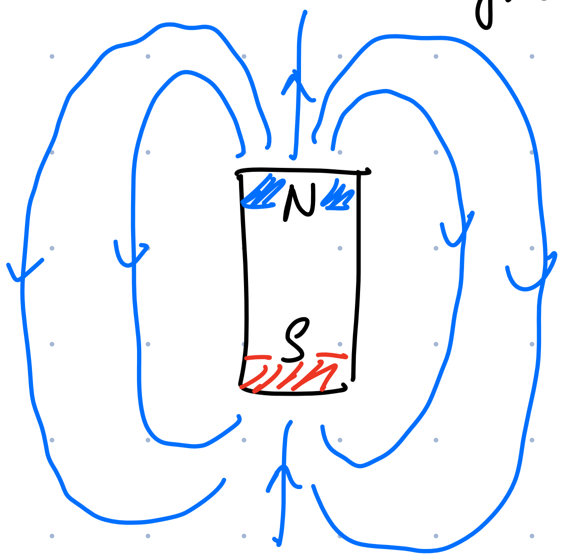
- Next PrairieLearn HW due Mar. 28
- Complete Pre-Lab #8 before the start of your lab.
- office hours: 11:00–12:00 today (instead of 11:30–12:30)



Complete $\vec{B}$-field due to current loop



c.f. bar magnet

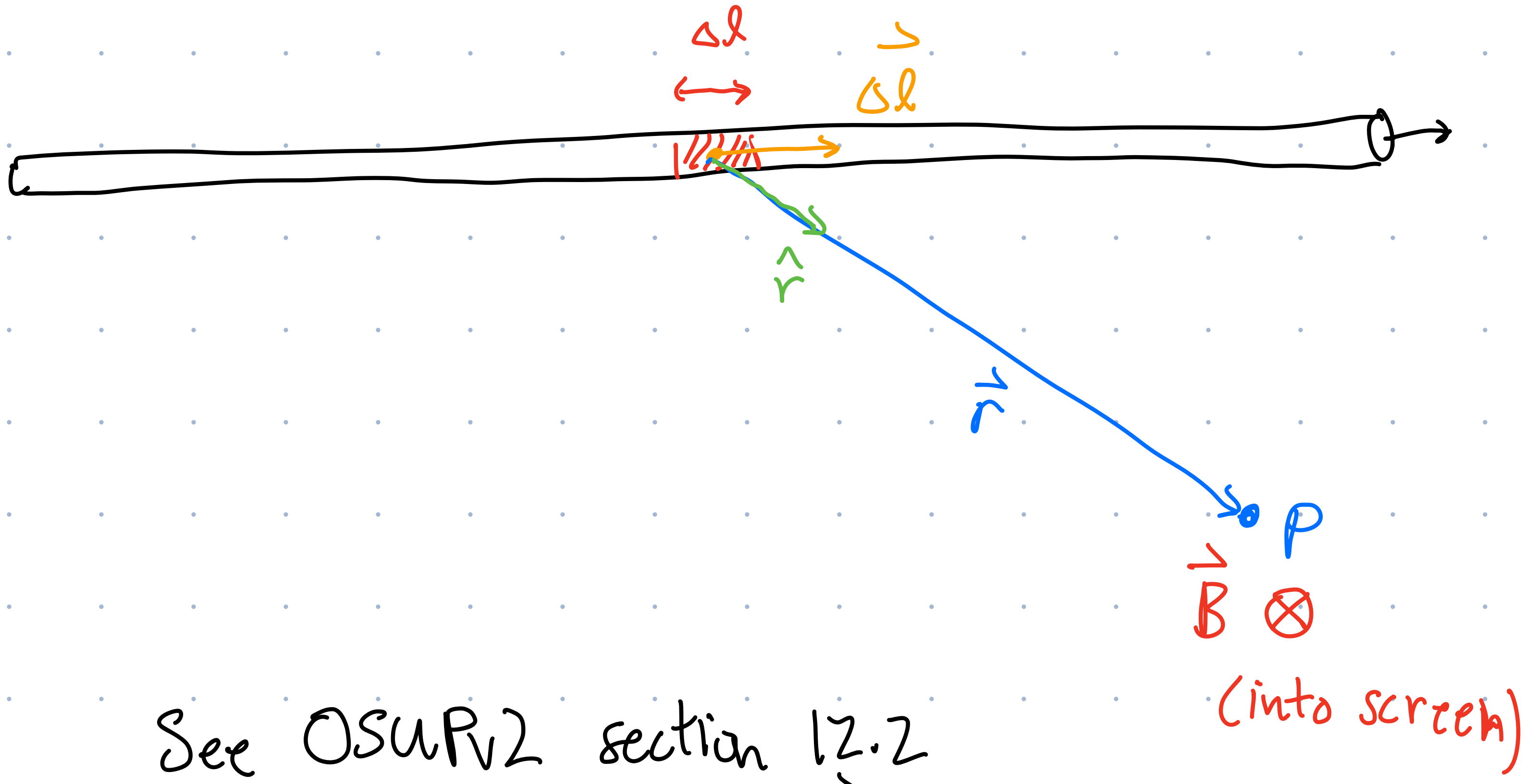


Using the Biot-Savart law: $\vec{B} = \frac{\mu_0}{4\pi} \frac{I \vec{\Delta l} \times \hat{r}}{r^2}$,

showed that $|\vec{B}|$ at the centre of loop (pt. P above) is given by:

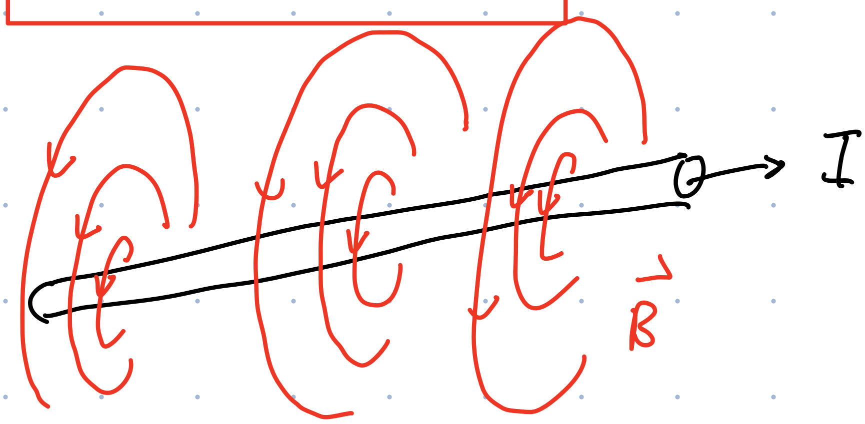
$$\boxed{B_{loop} = \frac{\mu_0 I}{2r}}$$

Biot-Savart law can also be used to find $|\vec{B}|$ due to an infinitely long current.

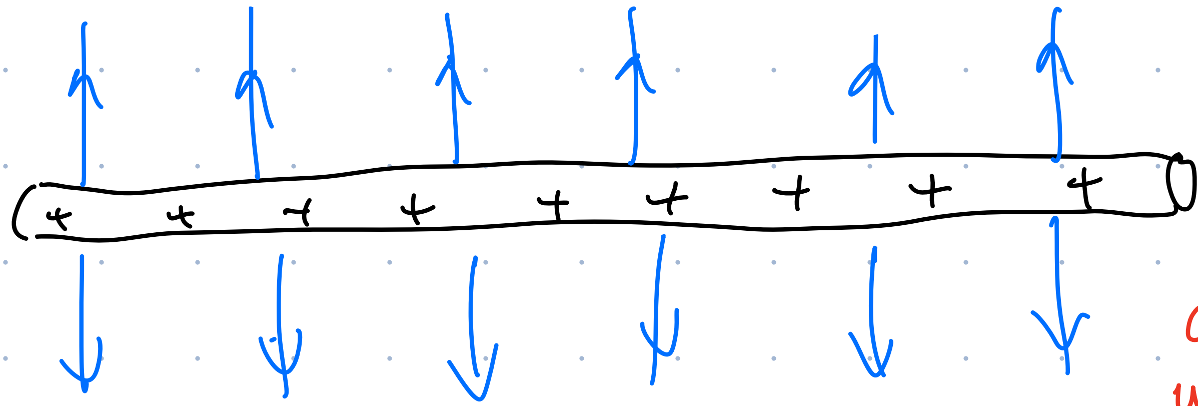


See OSUPv2 section 12.2 for a detailed calc. of $\vec{B}$ due to infinite wire.

The result is: $\boxed{|\vec{B}| = \frac{\mu_0 I}{2\pi r}}$



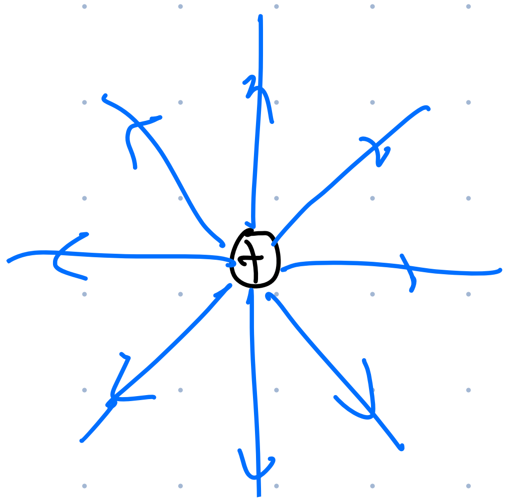
Compare to $\vec{E}$ due to infinitely-long, uniformly-charged rod



side view

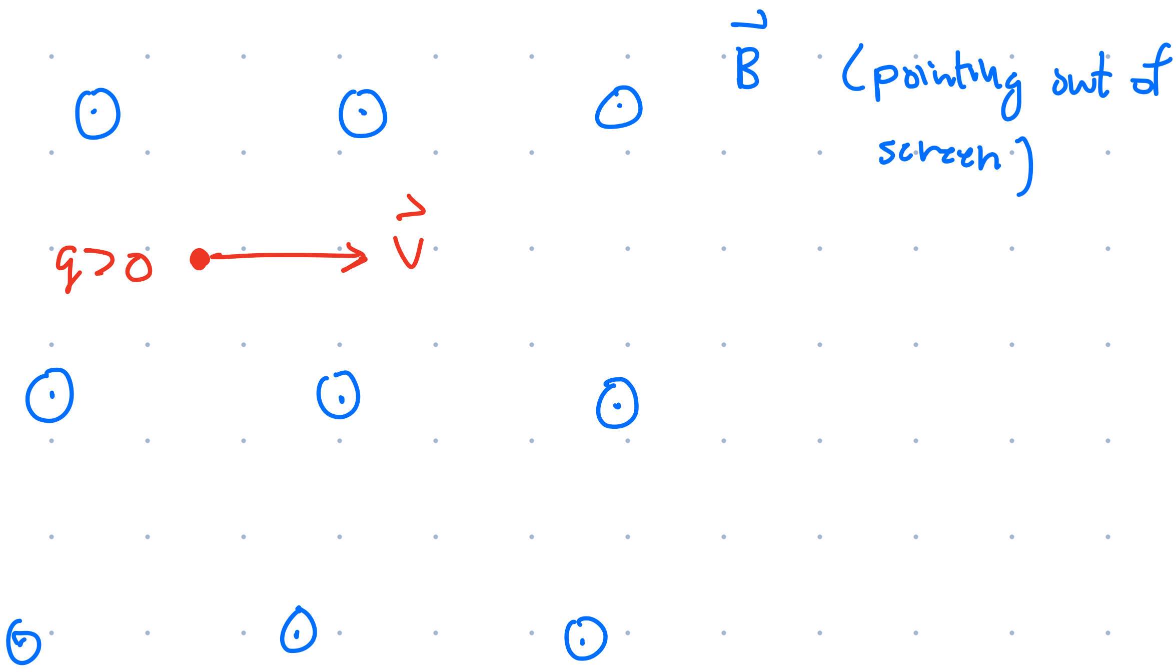
charge per unit length

$$\boxed{|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 r}}$$



end view

Force on a moving charge in a magnetic field $\vec{B}$



<u>Observations</u>:

① Only <u>moving</u> charges experience a force due to $\vec{B}$.

- $F \propto v$
- If $|\vec{v}|=0$, $|\vec{F}|=0$ no force on stationary charges.

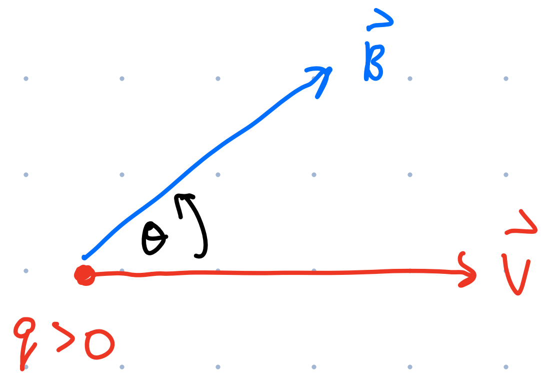
② Force changes in proportion to the value of q.

$$F \propto q$$

③ Force is proportional to $|\vec{B}|$

$$F \propto B$$

④ The value of the force depends on the angle between $\vec{v}$ & $\vec{B}$.



$$F \propto \sin\theta$$

When $\theta = 0$ ($\vec{B} \parallel \vec{v}$) and $\theta = 180°$ ($\vec{B}$ antiparallel to $\vec{v}$) the force is zero.

$|\vec{F}|$ is maximum when $\theta = 90°$ or $270°$ ($\vec{B} \perp \vec{v}$)

The magnitude of the force on q is

$$\boxed{|\vec{F}| = qvB\sin\theta}$$

We can write $vB\sin\theta = |\vec{v}\times\vec{B}|$

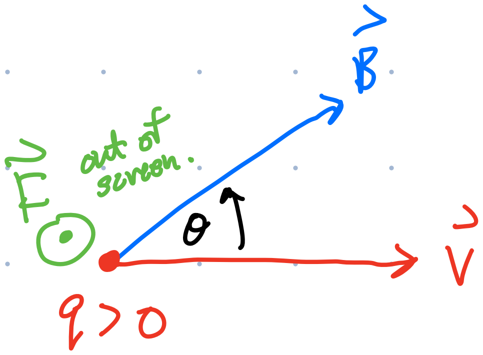


Fig. 1

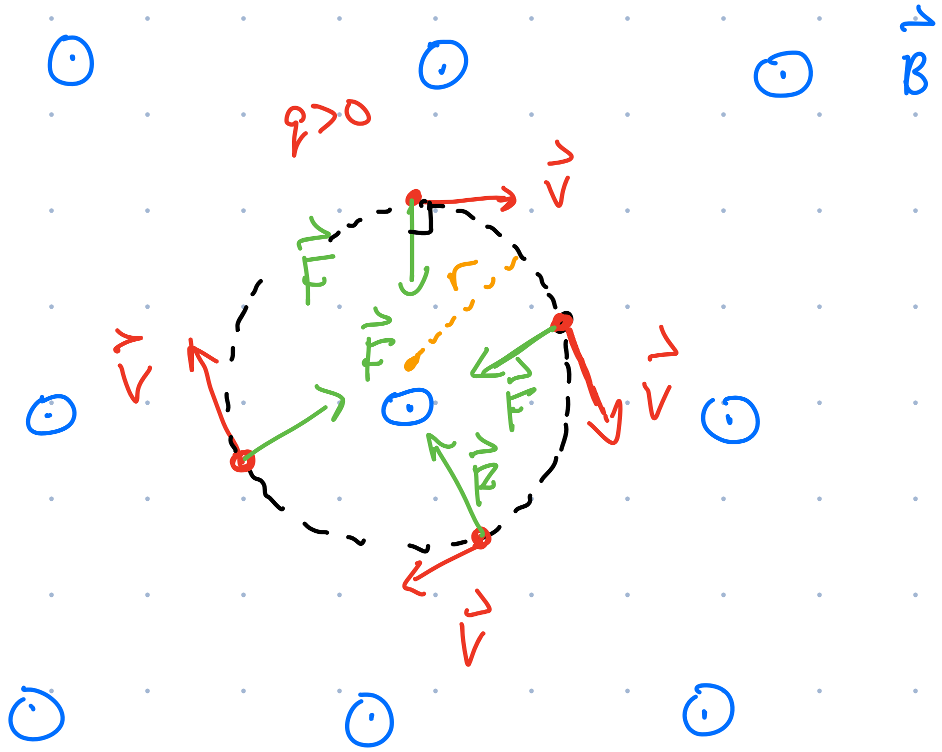
$$\therefore |\vec{F}| = q|\vec{v}\times\vec{B}|$$

Direction of $\vec{F}$ on q is $\perp$ to both $\vec{v}$ & $\vec{B}$. In the figure above (Fig. 1), $\vec{v}$ & $\vec{B}$ lie in the plane of the screen. $\therefore$ $\vec{F}$ is either into or out of the screen. To determine which, use RHR for cross products. i.e. the dir'n of $\vec{v}\times\vec{B}$ gives the dir'n of $\vec{F}$.

$$\boxed{\vec{F} = q\,\vec{v}\times\vec{B}}$$

Gives correct mag. & dir'n of $\vec{F}$ on q.

Simplest example is a charge moving $\perp$ to a uniform $\vec{B}$.



Since $\vec{v} \perp \vec{B}$, $\theta = 90°$ & $\sin\theta = 1$

$$\therefore F = q|\vec{v}\times\vec{B}| = qvB$$

$\vec{F}$ is $\perp$ to $\vec{v}$. Since $\vec{F} = m\vec{a}$, $\vec{a} \perp \vec{v}$.
If $\vec{a} \perp \vec{v}$, speed of q remains const, but it's dir'n changes.

Our qualitative analysis has revealed that a charge moving w/ const. speed through a uniform magnetic field $\vec{B}$ undergoes circular motion.

centripetal acceleration for circular motion $a_c = \frac{v^2}{r}$

Know $F = qvB = m a_c$

$$\therefore q\cancel{v}B = m\frac{v^{\cancel{2}}}{r}$$

solve for the radius

$$\boxed{r = \frac{mv}{qB}}$$

The time for the charge to complete on circular loop is given by:

circumference

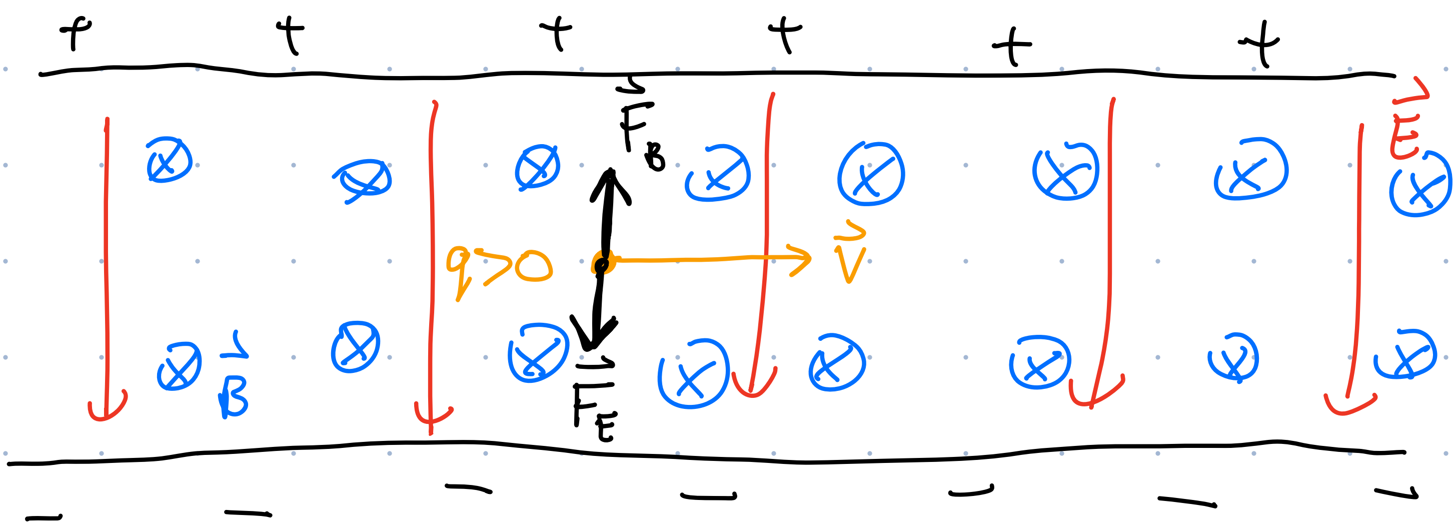
$$T = \frac{\text{dist.}}{\text{speed}} = \frac{2\pi r}{v}$$

period

$$\boxed{\therefore T = \frac{2\pi}{\cancel{v}}\left(\frac{m\cancel{v}}{qB}\right) = \frac{2\pi m}{qB}}$$

Design a velocity selector (application)

Start w/ a charge capacitor



Next, apply a uniform magnetic field $\vec{B} \perp$ to $\vec{E}$.

Now, fire a charged particle through the cap. s.t. $\vec{v} \perp \vec{E}, \vec{B}$.

The electric force on q is given by:

$$\vec{F}_E = q\vec{E} \qquad |\vec{F}_E| = qE$$

The magnetic force on q is given by:

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad |\vec{F}_B| = qvB$$ since $\sin\theta = 1$

For q to pass through velocity selector undeflected, we require

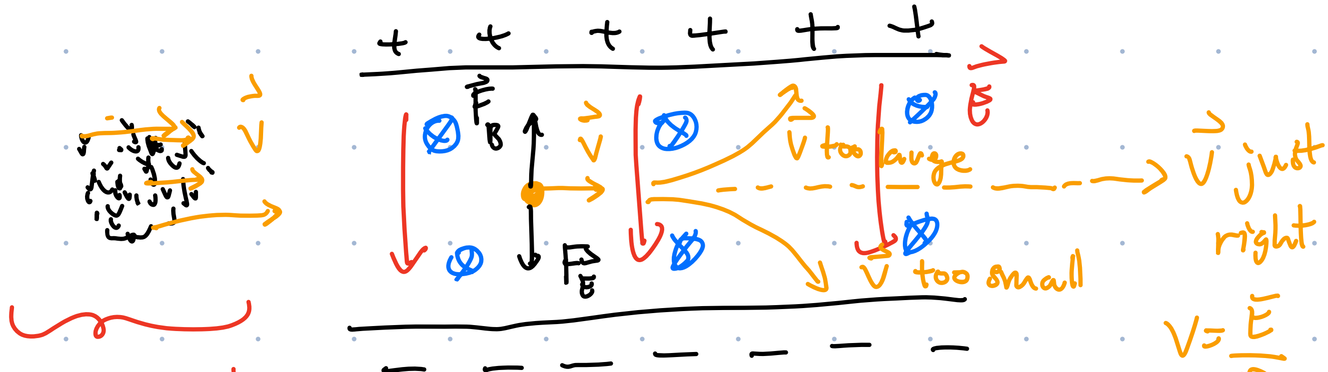
$$|\vec{F}_E| = |\vec{F}_B|$$

$$\cancel{q}E = \cancel{q}vB$$

$$\boxed{\therefore \quad v = \frac{E}{B}}$$

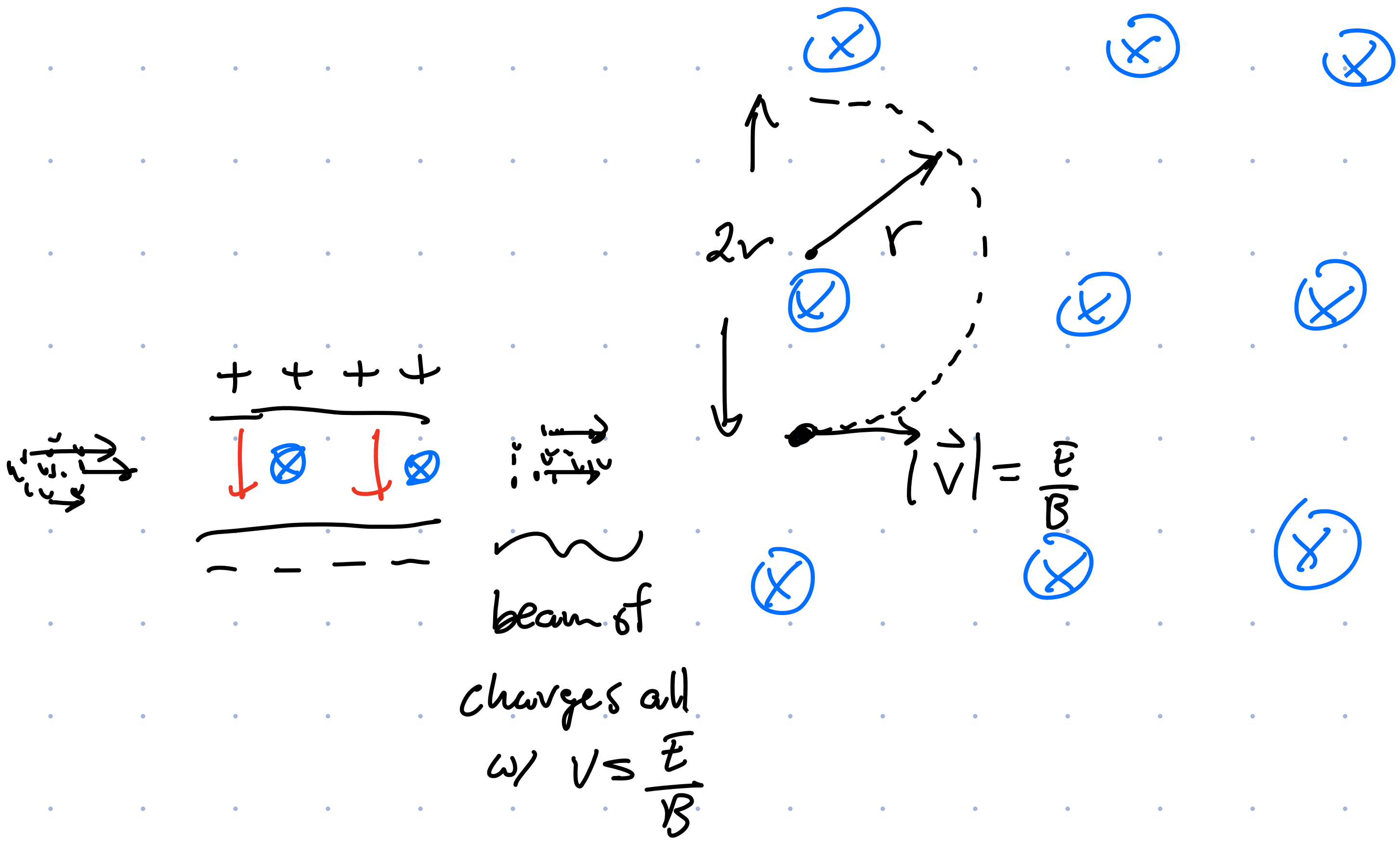
Only particles w/ speed $v = \frac{E}{B}$ make it through the velocity selector.

Application



collection of charged particles w/ a distribution of speeds

Another application — Mass spectrometer



Measure to radius of particle trajectory in the uniform magnetic field.

Know $\quad r = \dfrac{mv}{qB}$

and w/ know $\quad v = \dfrac{E}{B}$

$$\therefore \; m = \frac{qBr}{v} = \frac{qBr}{E/B} = \frac{qB^2r}{E}$$

If we know q, B, $\bar{E}$, and measure r

can calc. m $\rightarrow$ principle of mass spectroscopy.