PHYS 121 March 14, 2025

- Complete PrairieLearn HW by 23:59 $\boxed{\text{today}}$
- Complete Pre-Lab #7 before the start of your lab.

- Quiz #2 will be March 19. See the course website for details / formula sheet.

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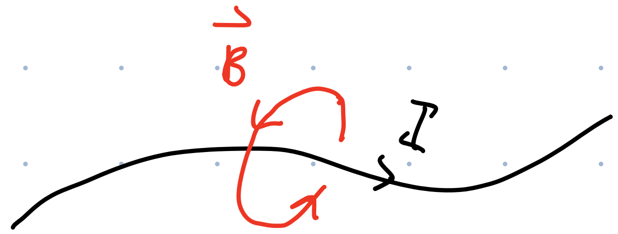
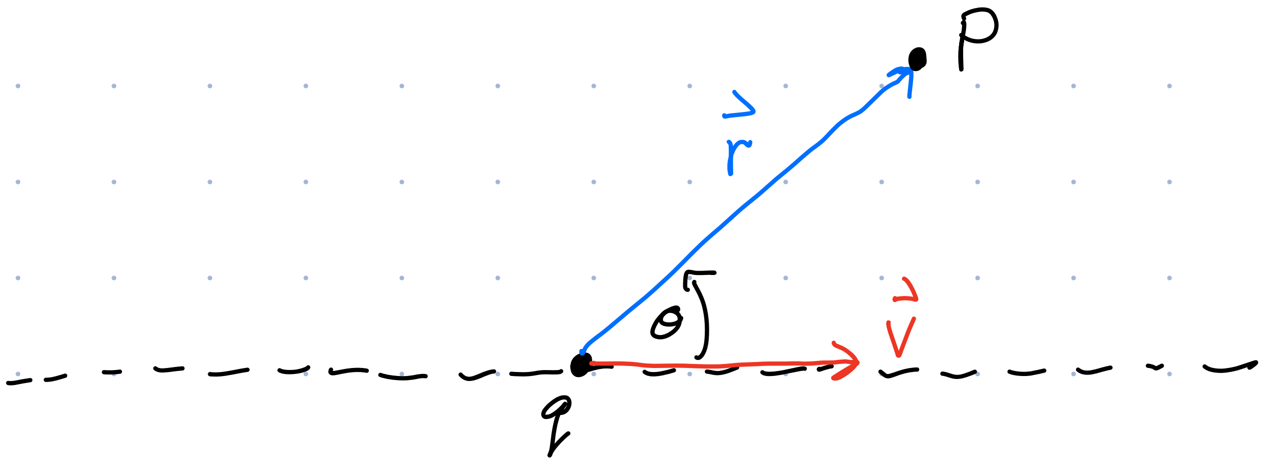
Last Time:

- Magnetic poles come in North - South pairs
- Opp. poles attract, like poles repel.
- moving charge/current is a source $\vec{B}$-fields.
- Magnetic fields loop around currents




dir'n of $\vec{B}$ is given by RHR (thumb in dir'n of $I$, fingers give dir'n of $\vec{B}$)




@ P, the magnitude of $\vec{B}$ due to q moving with velocity $\vec{v}$ is given by:

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{q v \sin\theta}{r^2}}$$

$\mu_0$ is called the "permeability of free space"

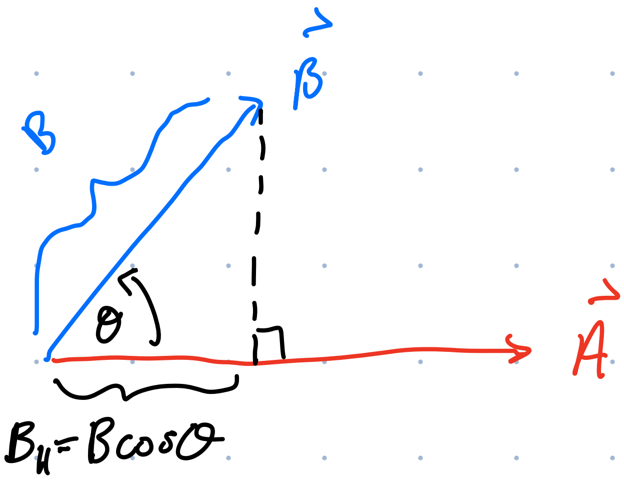
$$\mu_0 = 4\pi \times 10^{-7} \ \frac{T \cdot m}{A}$$

T = 1 Tesla, the unit of magnetic fields

Today: Start w/ the "cross-product" $\vec{A} \times \vec{B}$

The cross-product is a product between two vectors.

First, recall the dot product between two vectors




$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

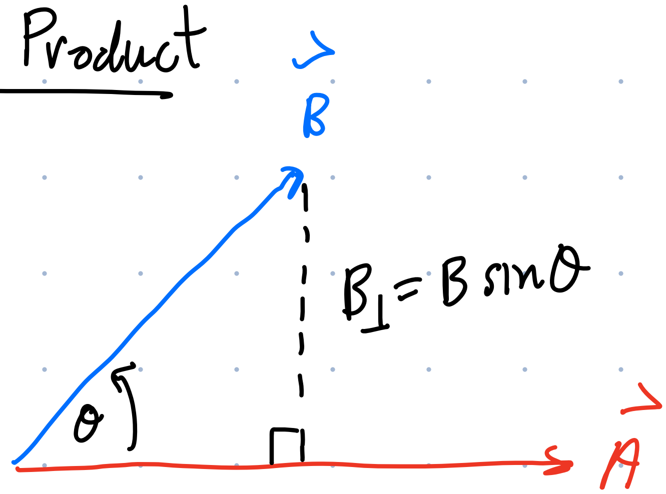
$$AB\cos\theta$$

$$\vec{A} \cdot \vec{B} = AB_{\parallel}$$

The dot product selects the component of $\vec{B}$ that is parallel to $\vec{A}$ & evaluates $AB_{\parallel}$

Note that the result of $\vec{A} \cdot \vec{B}$ is a scalar quantity (just a number).

# Cross Product




While the dot product selects $\parallel$ component of $\vec{B}$, the cross product selects the $\perp$ component of $\vec{B}$.

The magnitude of $\vec{A}\times\vec{B}$ is:

$$\left|\vec{A}\times\vec{B}\right| = AB_\perp = AB\sin\theta$$

not ordinary multiplication,
it is the cross product between vectors
$\vec{A}$ & $\vec{B}$.

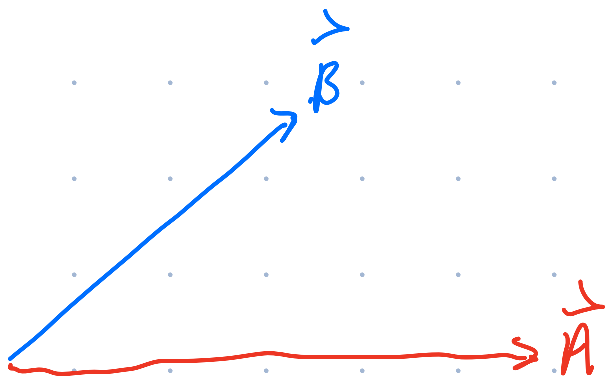
One important difference between dot & cross products is that the result of the cross product is another vector.

In general $\vec{A} \times \vec{B} = \vec{C}$

new vector

$|\vec{C}| = AB\sin\theta$

need to specify the dir'n of $\vec{C}$.




The vectors $\vec{A}$ & $\vec{B}$ define a plane which, in this case, is the plane of screen.

Vector $\vec{C}$ will be $\perp$ to this plane or, equivalently, $\vec{C}$ will be $\perp$ to both $\vec{A}$ & $\vec{B}$.

To determine whether $\vec{C}$ is into or out of the screen, we use a 2nd RHR.

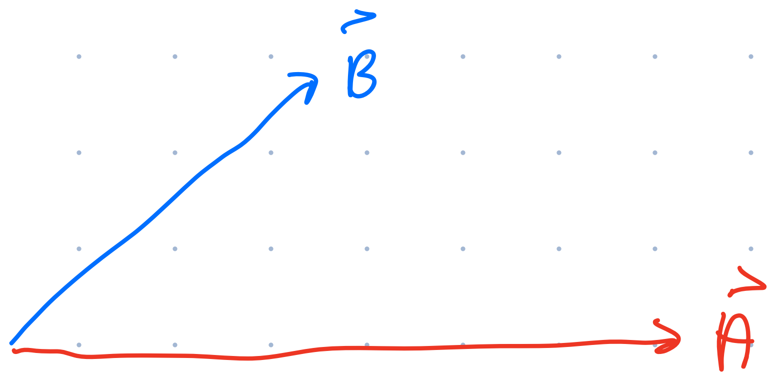
- Place right hand/arm in dir'n of $\vec{A}$ (the first vector in the cross product $\vec{C} = \vec{A} \times \vec{B}$)

- Curl fingers of right hand so that they are parallel to $\vec{B}$ (second vector in the cross product).

- Thumb pts in the dir'n of $\vec{C}$

$\vec{C} = \vec{A} \times \vec{B}$

$\vec{C}$ points out of the screen.




reverse order of $\vec{A}$ & $\vec{B}$

If we evaluate $\vec{D} = \vec{B} \times \vec{A}$

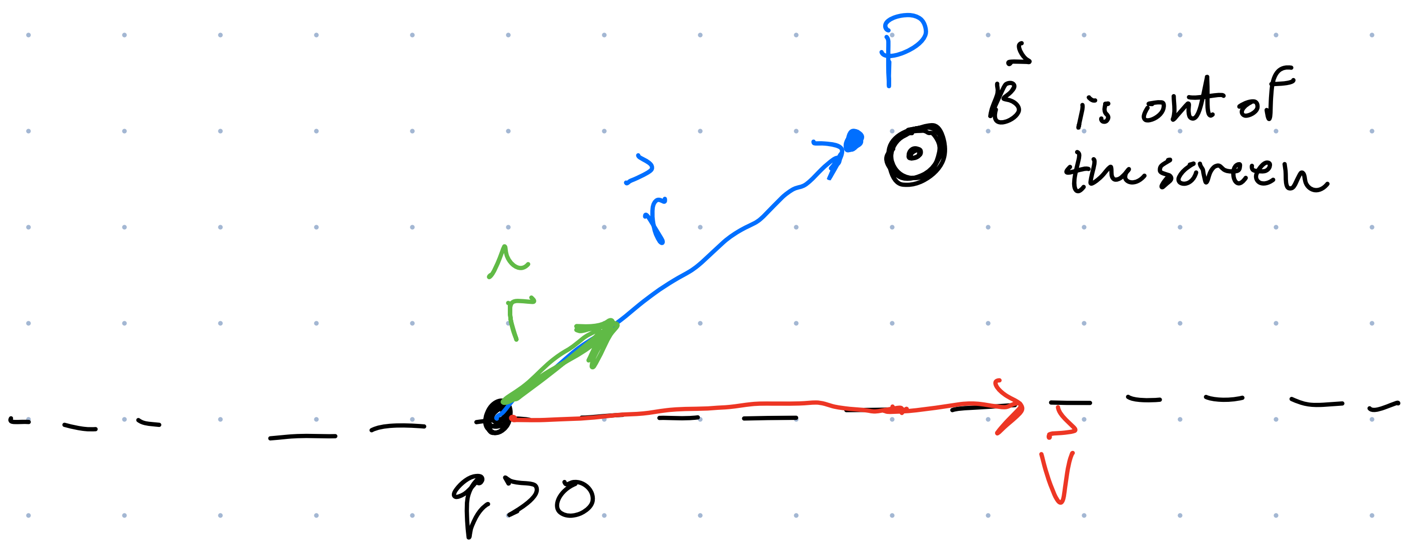
$$|\vec{D}| = AB\sin\theta = |\vec{C}|$$

the magnitudes of $\vec{C}$ & $\vec{D}$ are same.

$\vec{D}$ points into the screen & is anti-parallel to $\vec{C}$.

In fact, $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$

Return to moving pt. charge creating magnetic field $\vec{B}$ @ pt. P.




$\odot$ vector coming out of screen (tip of an arrow)

$\otimes$ vector into the screen (feathers of an arrow)

Define a unit vector $\hat{r}$ $(|\hat{r}|=1)$ in the dir'n of position vector $\vec{r}$

$$\vec{r} = r\,\hat{r}$$

Consider the cross product

$$\vec{v} \times \hat{r}$$

magnitude: $\boxed{|\vec{v} \times \hat{r}|} = \underbrace{|\vec{v}|}_{v}\underbrace{|\hat{r}|}_{1}\sin\theta = \boxed{v\sin\theta}$

direction: $\vec{v} \times \hat{r}$ by RHR #2 points out of the screen which is the same dir'n as $\vec{B}$.

Last time, we found that

$|\vec{v} \times \hat{r}|$

$$B = \frac{\mu_0}{4\pi}\frac{q\,\boxed{v\sin\theta}}{r^2}$$

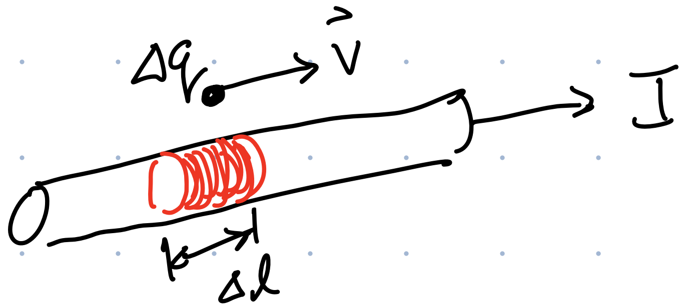
Furthermore $\vec{v} \times \hat{r}$ gives correct dir'n for $\vec{B}$

Complete vector eq'n for $\vec{B}$ due to a moving pt. q is:

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} \frac{q\, \vec{v} \times \hat{r}}{r^2}}$$

Gives dir'n & mag. of $\vec{B}$. (1)

Want to re-express (1) in terms of a current I.




Consider the total charge Δq in the red section of length Δl moving w/ velocity $\vec{v}$

Current can be expressed as:

$$I = \frac{\Delta q}{\Delta t}$$

mult. by $1 = \frac{\Delta l}{\Delta l}$

$$I = \frac{\Delta q}{\Delta t}\frac{\Delta l}{\Delta l} = \frac{\Delta q}{\Delta l}\underbrace{\frac{\Delta l}{\Delta t}}_{v}$$

$$\therefore \quad I = \frac{\Delta q}{\Delta l} v$$ solve for $\Delta q\, v$

$$\therefore \quad \Delta q\, v = I \Delta l$$

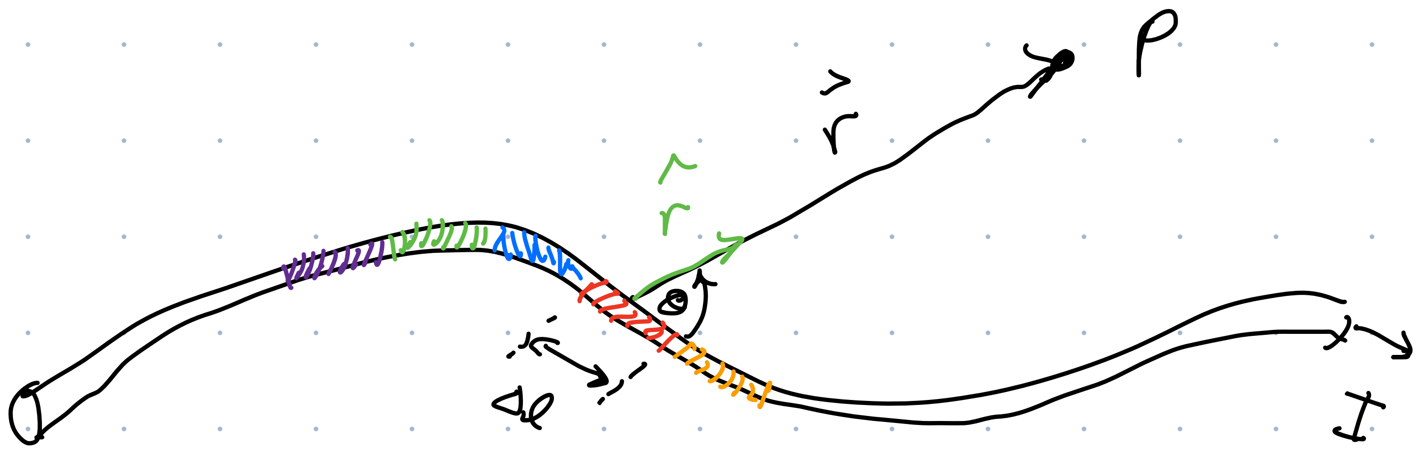
To make this a vector eq'n, use $v \to \vec{v}$ & we'll give $\Delta l$ a dir'n that is parallel to $\vec{v}$ or, equivalently, in dir'n of current $I$.

$$\therefore \quad \Delta q\, \vec{v} = I \vec{\Delta l}$$

replace $q\vec{v}$ is Eq 1 w/ $I \vec{\Delta l}$

$$\boxed{\therefore \quad \vec{B} = \frac{\mu_0}{4\pi}\, \frac{I \vec{\Delta l} \times \hat{r}}{r^2}} \quad 2$$

Eq'n ② is called the Biot-Savart law,
gives the magnetic field at Pt. P due to
a current segment of length $\delta l$.




To get the total magnetic field @ P due to
entire length of wire, need to sum the
contributions from all current segments that
make up the entire wire.